Exploration in RL: Contextual Bandit

Recap: MAB

Interactive learning process:

For $t = 0 \rightarrow T - 1$

(# based on historical information)

1. Learner pulls arm $I_t \in \{1, \ldots, K\}$

2. Learner observes an i.i.d reward $r_t \sim \nu_{I_t}$ of arm I_t

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Arm distributions are fixed across learning..

Question for Today:

Incorporate contexts into the interactive learning framework

Outline for today:

1. Introduction of the model

2. Algorithm

3. Theory and some practical considerations

Make the framework Context Dependent:

Interactive learning process:

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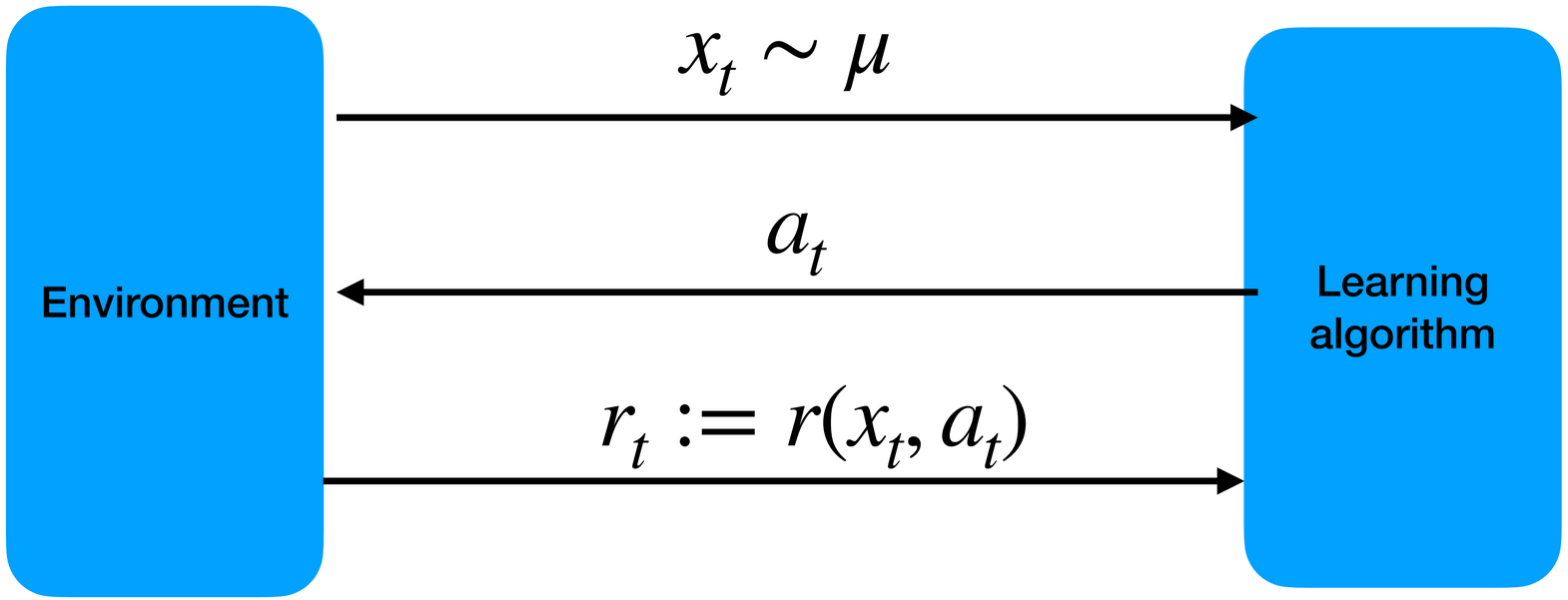
3. Learner observes an reward $r_t := r(x_t, a_t)$

Reward is context and arm dependent now!

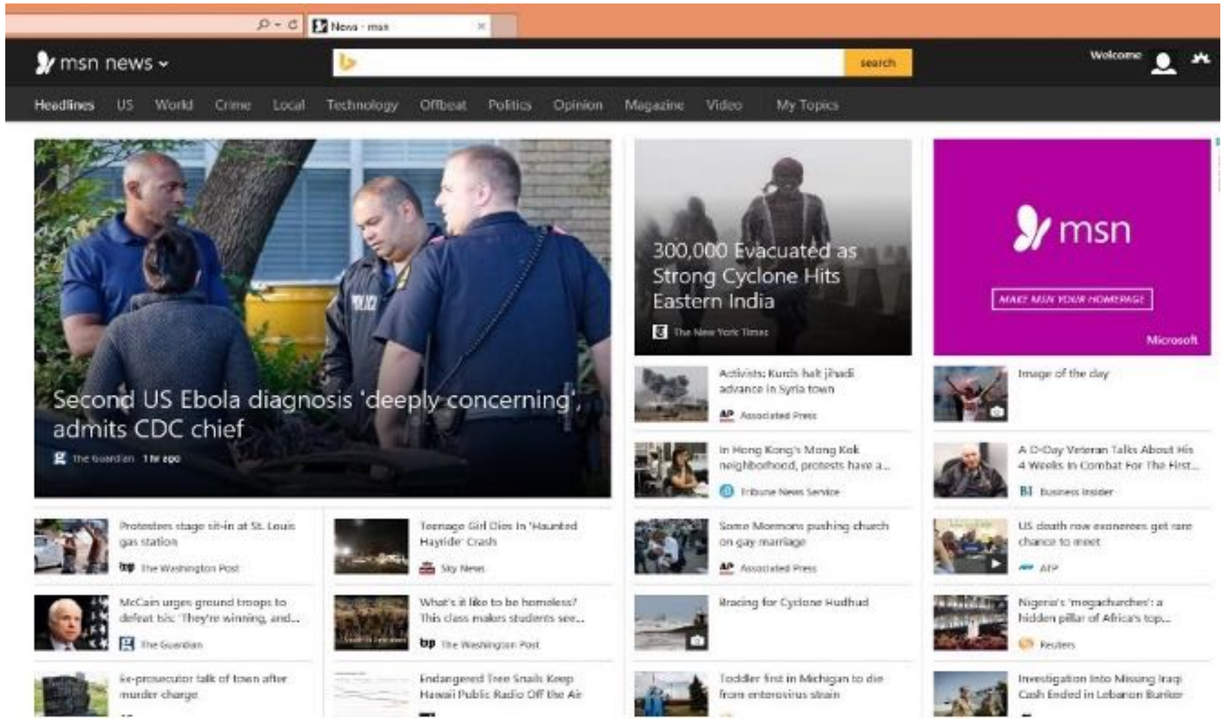
Make the framework Context Dependent:

Interactive learning process:



Examples:

Personalize
recommendation system



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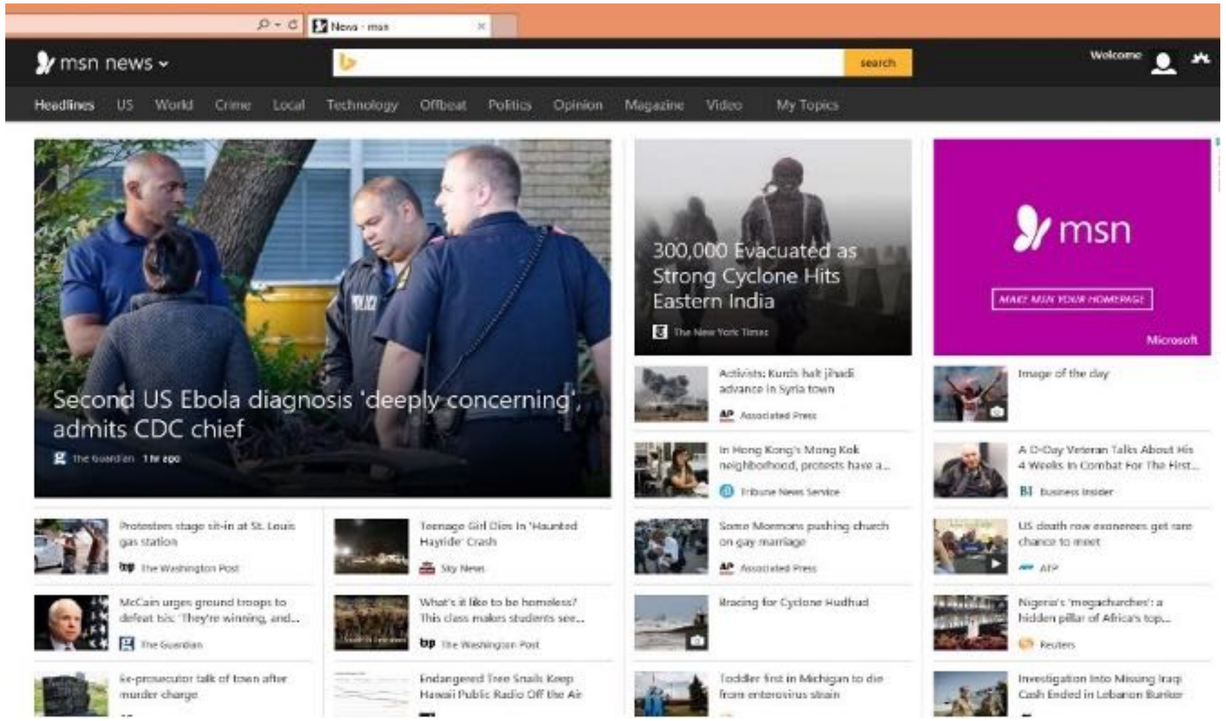

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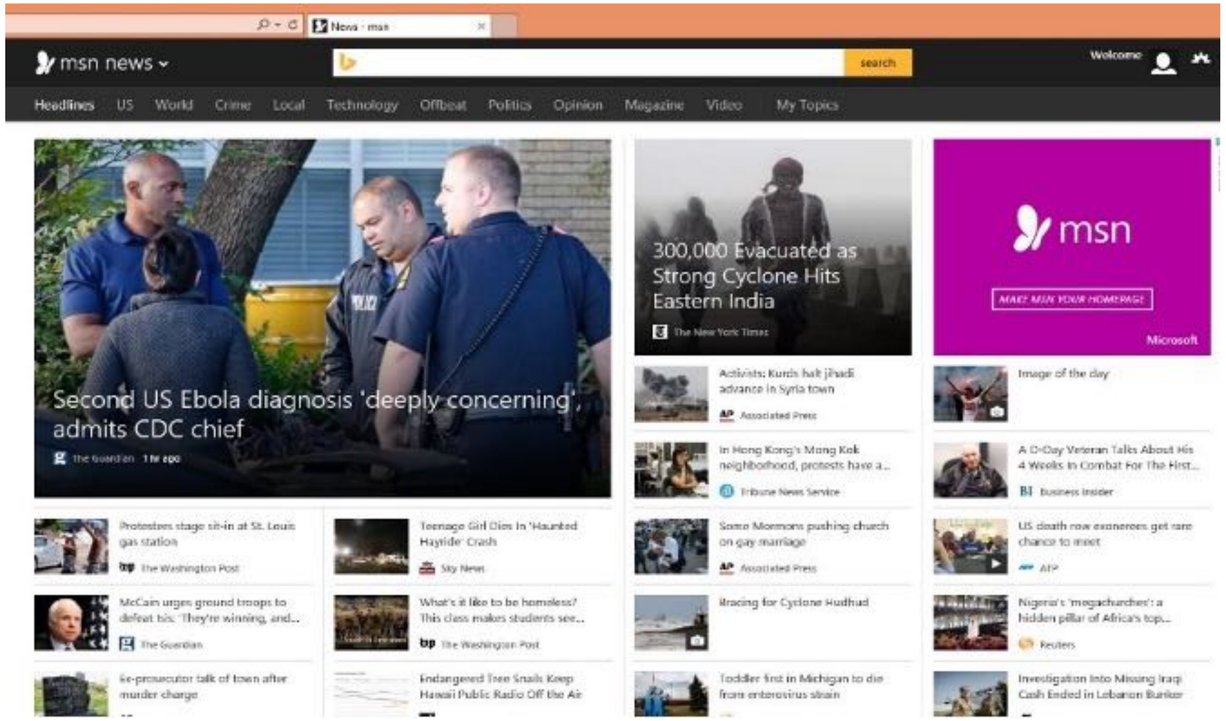

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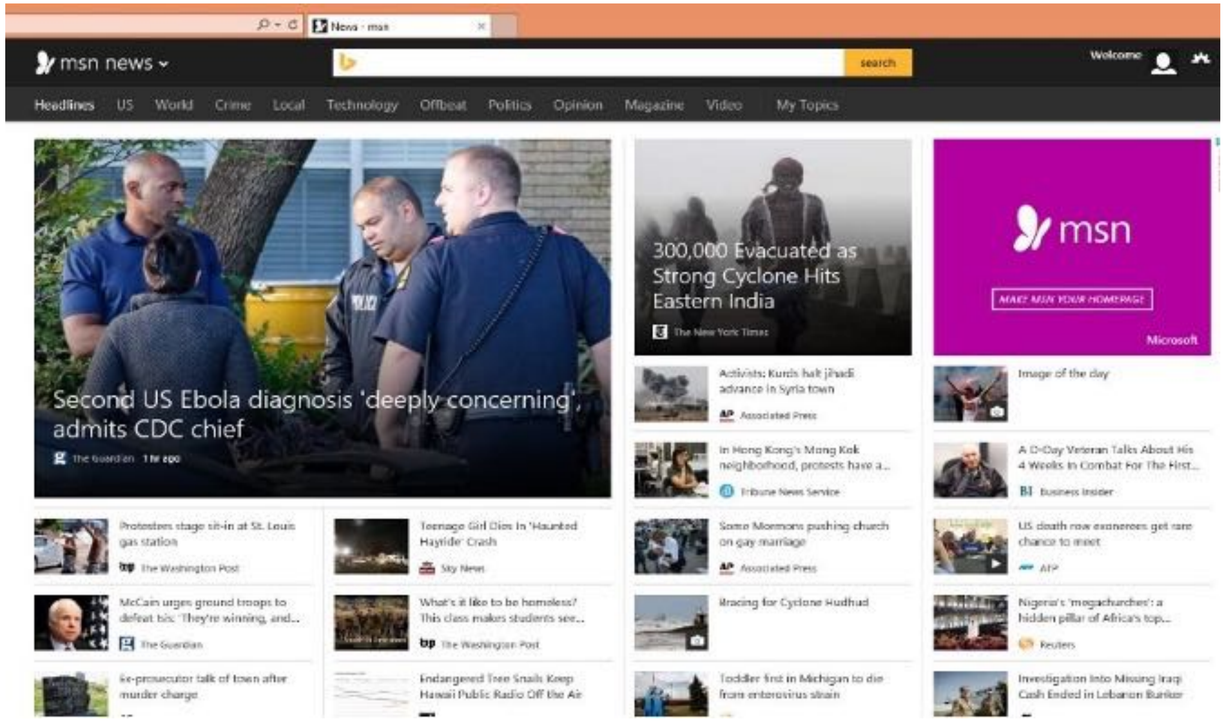

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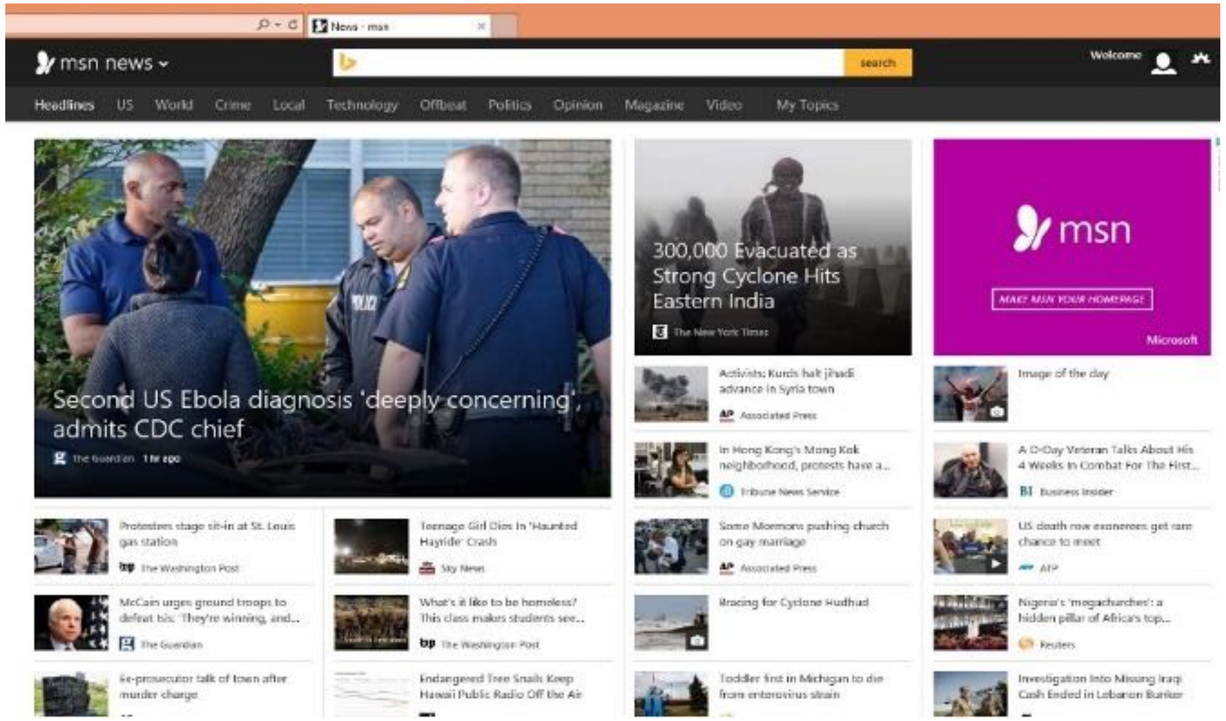

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Decisions (arms): news articles

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Different users have different preferences on news, so need to personalize

Equivalently, it is an MDP with H = 1

Finite horizon MDP with $H = 1$

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For simplicity, we assume reward is deterministic;
The challenge is really from randomness in contexts

The Regret Metric

Fix a policy class Π (think about π as a classier from $x \to a$)

Denote optimal policy $\pi^\star = \arg\max_{\pi \in \Pi} \mathbb{E}_{x \sim \mu} r(x, \pi(x))$

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Total expected reward of our learned sequence of policies

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$$= p(a)\frac{r(x_t, a)}{p(a)} = r(x_t, a)$$

Ingredient 2: Reward-sensitive Classification Oracle

Recall classic classification:

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We will do reduction to RSC

Summary so far:

1. Importance Weighting: we “magically” get unbiased estimate for all actions!

Assume $a_t \sim p$ ($p \in \Delta(\mathscr{A})$), For all $a \in \mathscr{A}$, define $\hat{\mathbf{r}}[a] = \dfrac{r(x_t, a)\mathbf{1}[a = a_t]}{p(a_t)}$, we have:

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✓ 2. Algorithm

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Theory of the Explore and Commit Algorithm

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[Theorem—informal] W/ high probability, properly setting the hyper-parameter N, Explore-and-Commit has the following regret:

$$\text{Regret}_T = T\mathbb{E}_{x\sim\mu}[r(x,\pi^\star(x))] - \sum_{t=0}^{T-1}\mathbb{E}_{x\sim\mu}[r(x,\pi^t(x))] = O\left(T^{2/3}K^{1/3}\cdot\ln(|\Pi|)^{1/3}\right)$$

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Q: What's the action distribution induced by ϵ-greedy at iteration t?

Practical Consideration

Instead of setting a hard threshold for explore and commit, we often interleave explore and exploitation

ϵ-greedy:

Every iteration t:

With probability $1 - \epsilon$, we play $a_t = \pi^t(x_t)$,

and w/ probability ϵ, we play $a_t \sim \text{Unif}(\mathscr{A})$

Q: What's the action distribution induced by ϵ-greedy at iteration t?

$$a \sim p_t, \quad p_t = (1 - \epsilon)\delta(\pi^t(x_t)) + \epsilon \text{Unif}(\mathscr{A})$$

Put things together: $\epsilon-$greedy

For $t = 0 \rightarrow \infty$ (# interleve exploration & exploitation)

1. Observe $x_t \sim \mu$

Put things together: $\epsilon-$greedy

For $t = 0 \to \infty$ (# interleve exploration & exploitation)

1. Observe $x_t \sim \mu$

2. Use ϵ-greedy to form action distribution $p_t = (1-\epsilon)\delta(\pi^t(x_t)) + \epsilon \text{Unif}(\mathscr{A})$

Put things together: $\epsilon-$greedy

For $t = 0 \to \infty$ (# interleve exploration & exploitation)

1. Observe $x_t \sim \mu$
2. Use ϵ-greedy to form action distribution $p_t = (1-\epsilon)\delta(\pi^t(x_t)) + \epsilon \text{Unif}(\mathscr{A})$
3. Play $a_t \sim p_t$, and observe $r_t := r(x_t, a_t)$

Put things together: $\epsilon-$greedy

For $t = 0 \to \infty$ (# interleve exploration & exploitation)

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Put things together: $\epsilon-$greedy

For $t = 0 \to \infty$ (# interleve exploration & exploitation)

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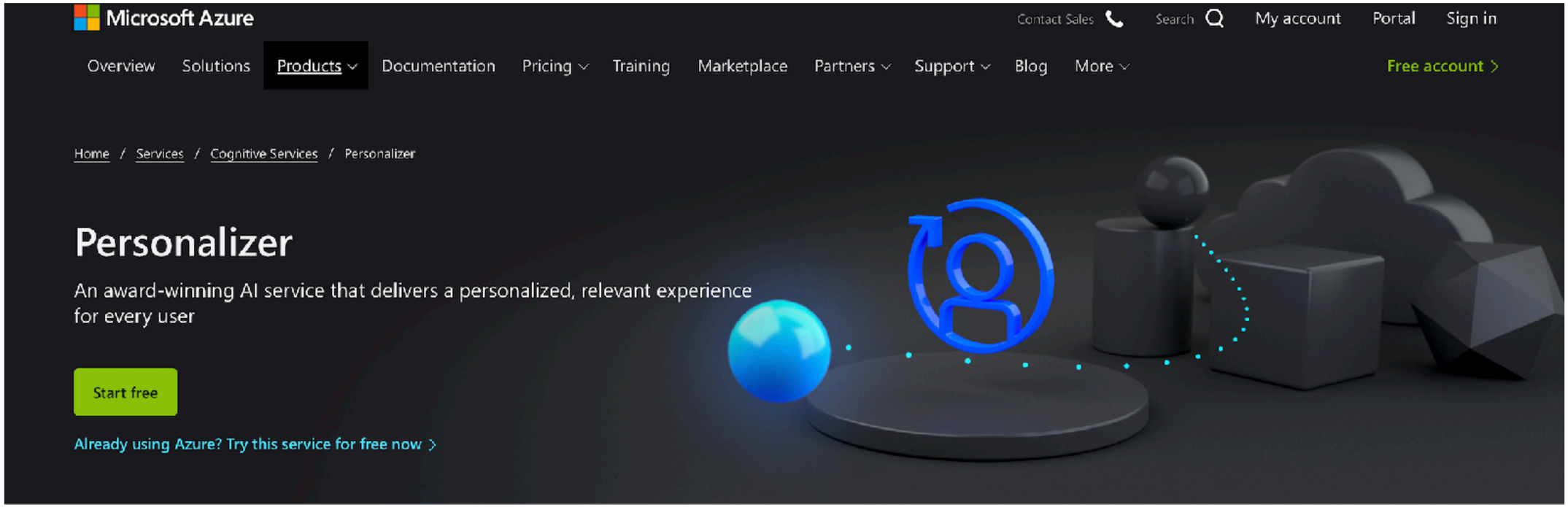
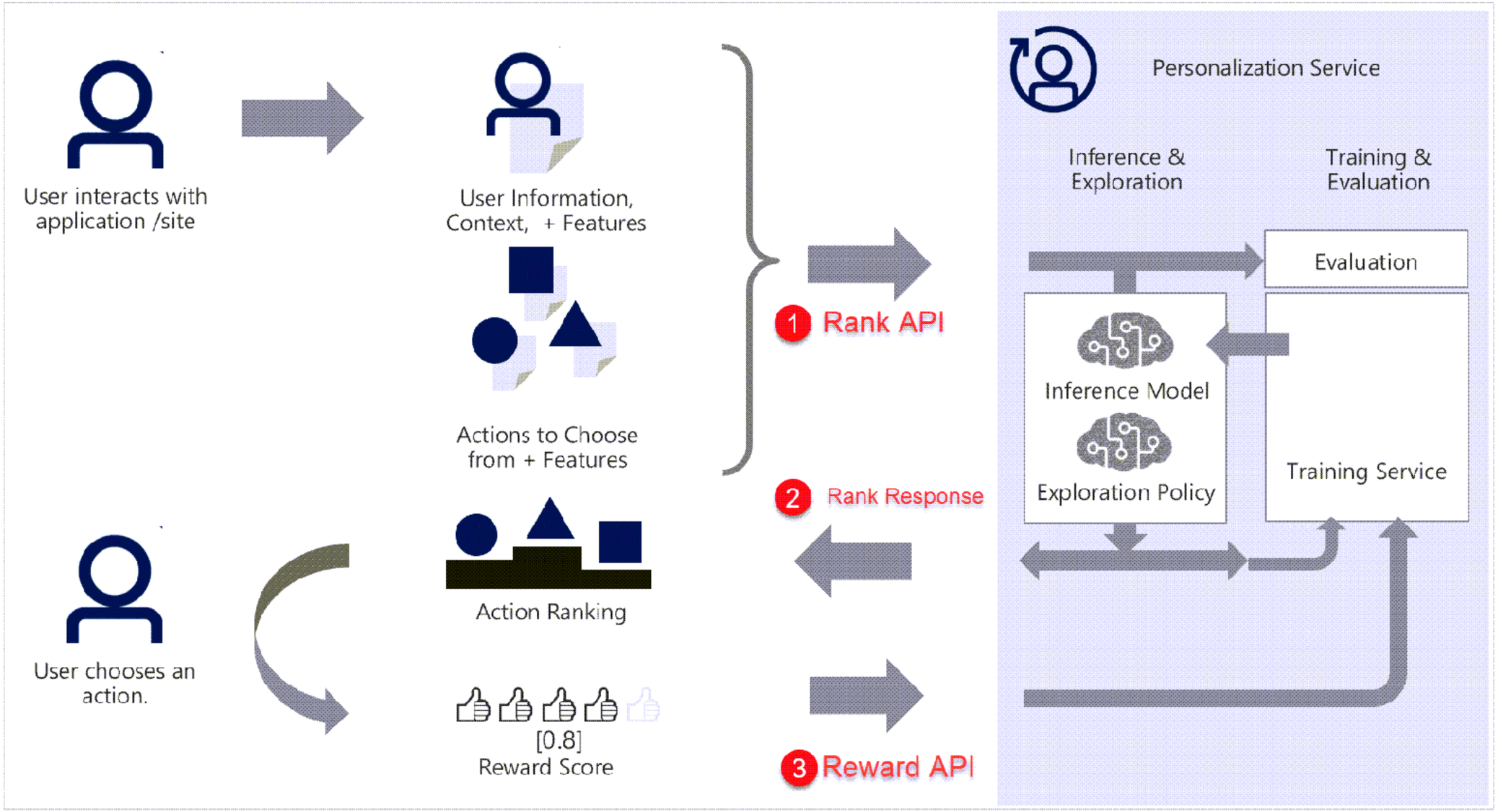
(Additionally 6. Gradually decay ϵ…)

CB algorithm is being used in real world application at Microsoft:

https://azure.microsoft.com/en-us/services/cognitive-services/personalizer/



Framework


User interacts with application /site
User Information, Context, + Features
Actions to Choose from + Features
1 Rank API
2 Rank Response
Action Ranking
User chooses an action.
[0.8]
Reward Score
3 Reward API
Personalization Service
Inference & Exploration
Training & Evaluation
Evaluation
Inference Model
Exploration Policy
Training Service