Approximate Policy Iteration

Recap: Policy Iteration

Recall Policy Iteration (PI) for the setting where P and r are known:

We compute $Q^\pi(s,a)$ exactly for all s, a, PI updates policy as:

$$\pi'(s) = \arg\max_a Q^\pi(s,a)$$

i.e., be greedy with respect to π at every state s,

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What if P & r are unknown, and MDP is large (e.g., infinitely many states)?

Recap: Model-based RL

Simulation Lemma:

$$\widehat{V}^{\pi}(s_0) - V^{\pi}(s_0) = \frac{\gamma}{1-\gamma}\mathbb{E}_{s,a\sim d^{\pi}_{s_0}}\left[\mathbb{E}_{s'\sim \widehat{P}(s,a)}\widehat{V}^{\pi}(s') - \mathbb{E}_{s'\sim P(s,a)}\widehat{V}^{\pi}(s')\right]$$

$$\leq \frac{\gamma}{(1-\gamma)^2}\mathbb{E}_{s,a\sim d^{\pi}_{s_0}}\left\| \widehat{P}(\cdot \,|\, s,a) - P(\cdot \,|\, s,a) \right\|_1$$

Recap: Model-based RL

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Recap: Model-based RL

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1. Model fitting:

$\forall s, a$: collect N next states, $s_i' \sim P(\cdot \mid s, a), i \in [N]$;

set $\widehat{P}(s' \mid s, a) = \dfrac{\sum_{i=1}^{N} \mathbf{1}\{s_i' = s'\}}{N}$;

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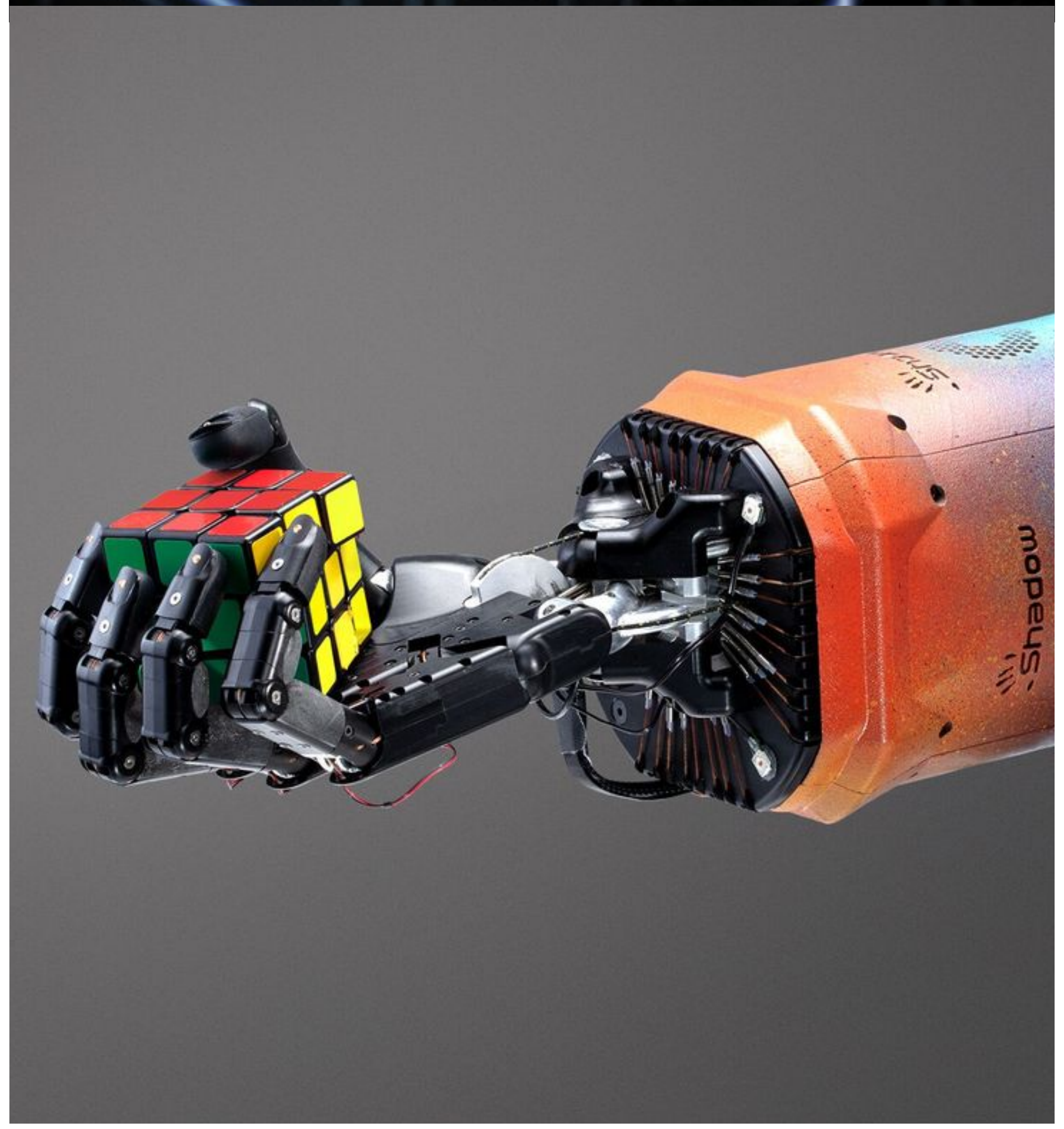
2. Planning w/ the learned model:

$\widehat{\pi}^\star = \textbf{PI}\left(\widehat{P}, r\right)$

We are moving on to large scale MDPs



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Enumerate over all state-action pairs is not possible in both computation, space, and statistics;

What should we do?

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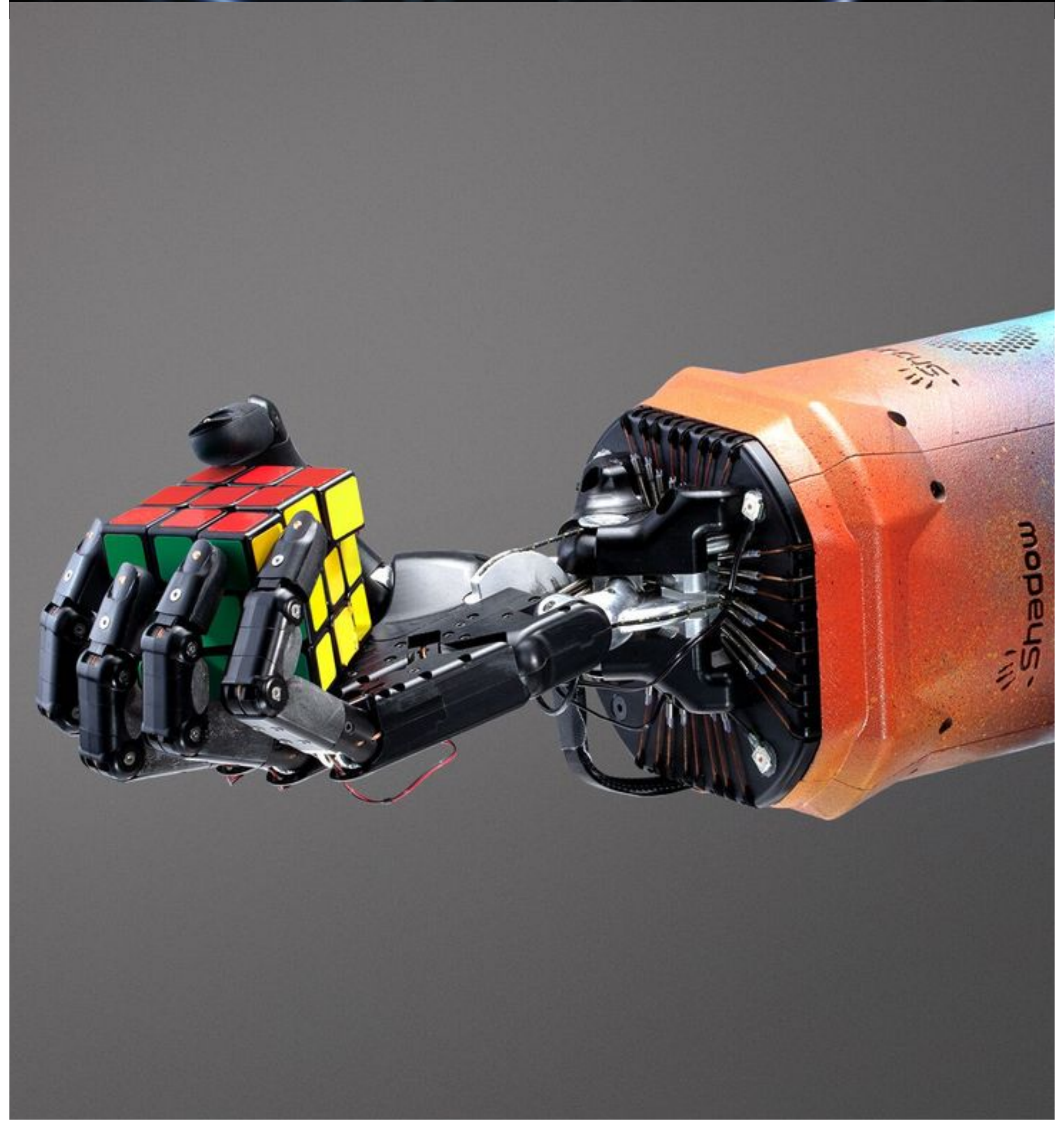

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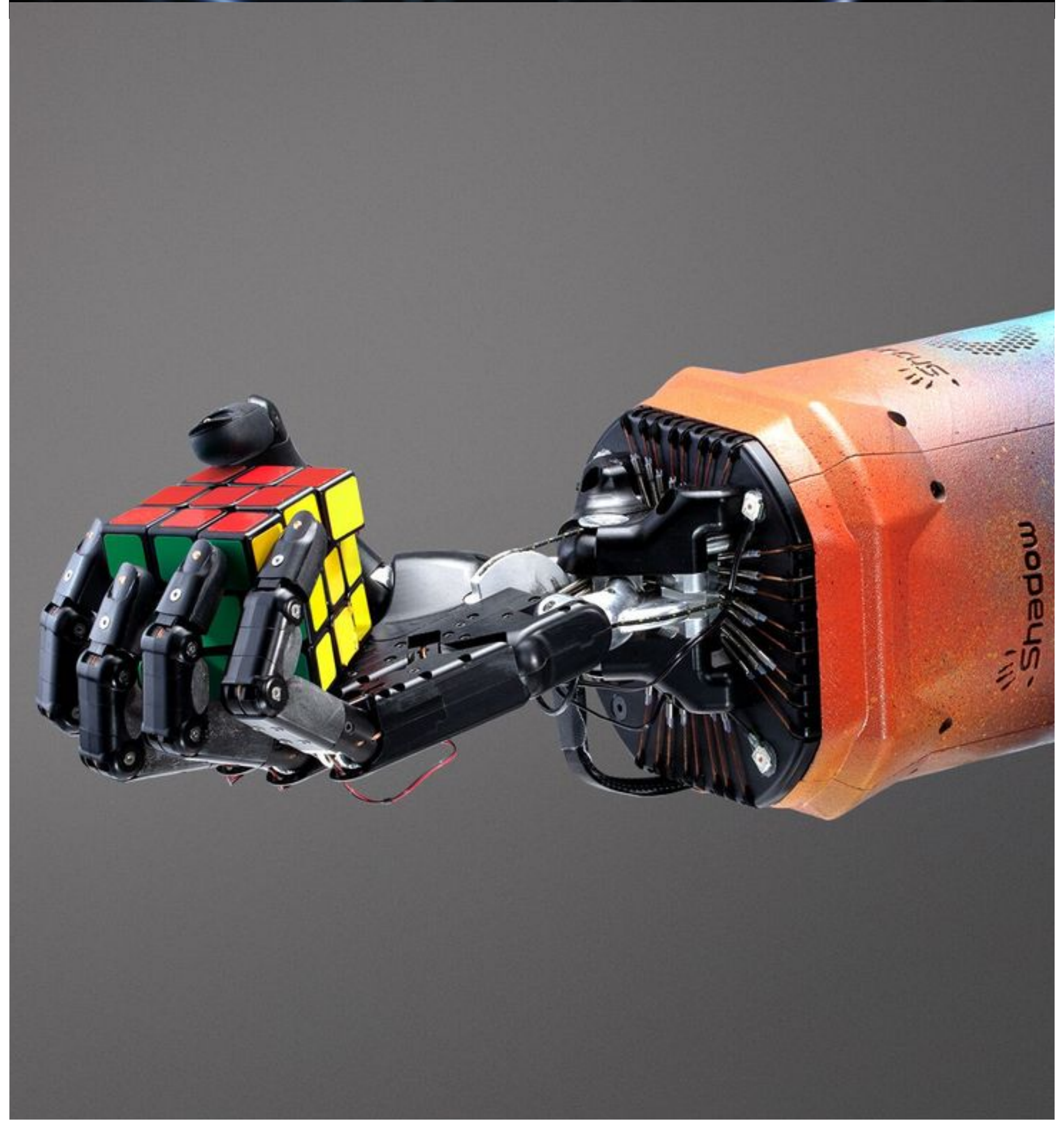

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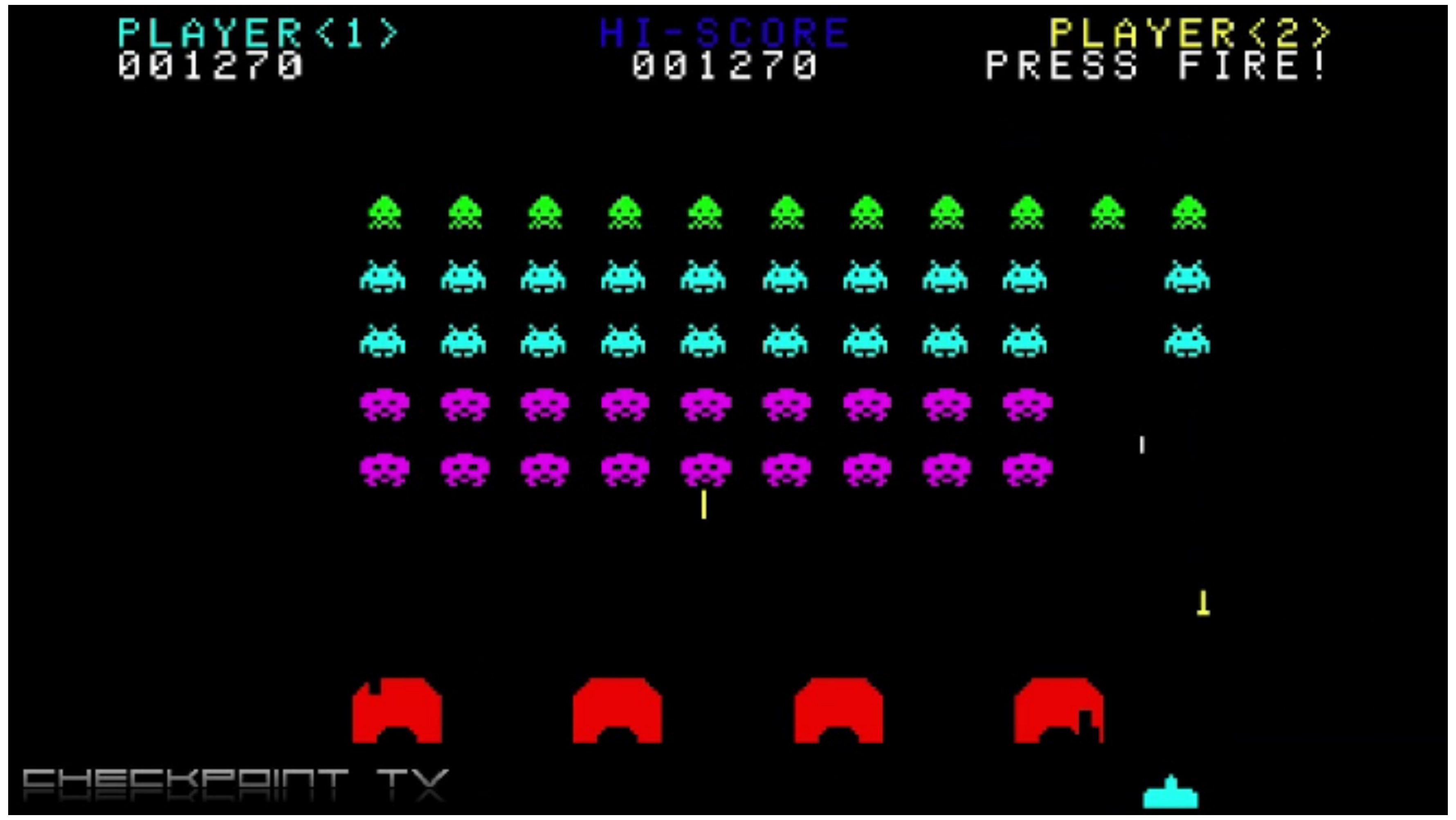
Indeed, in LQR, we are using quadratic function to represent Q & V

Another example: Video games

State s: RGB image



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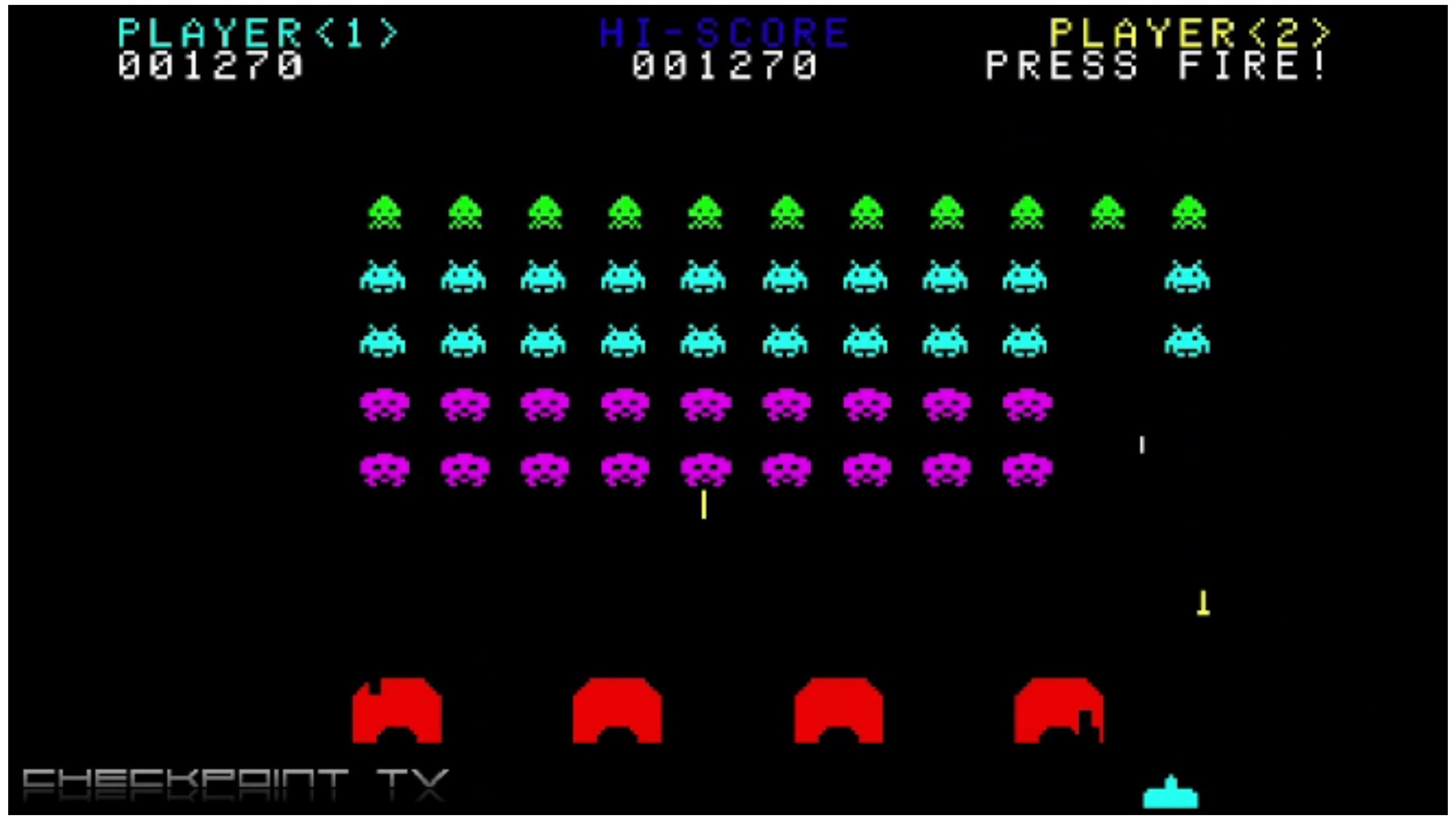

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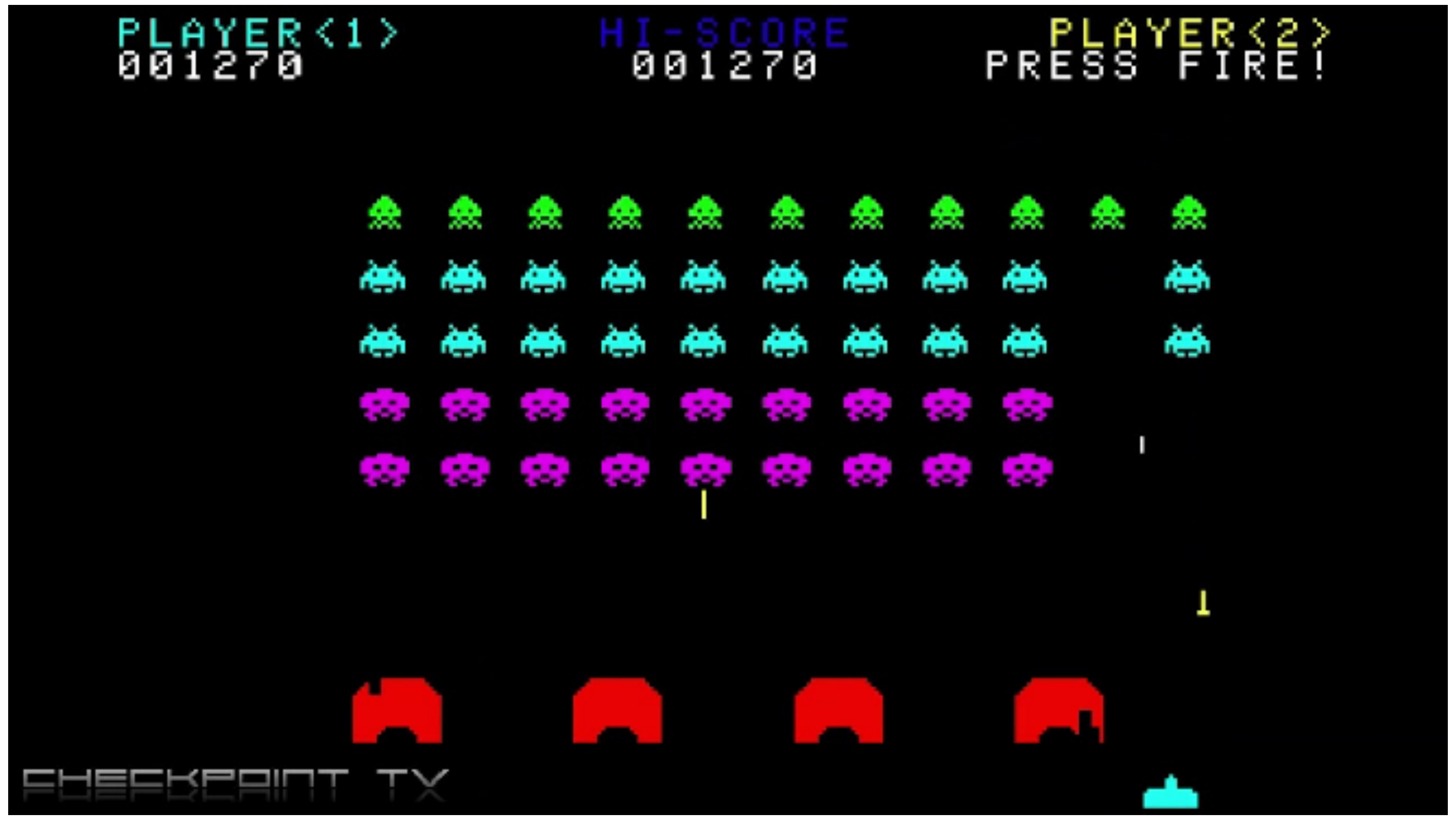

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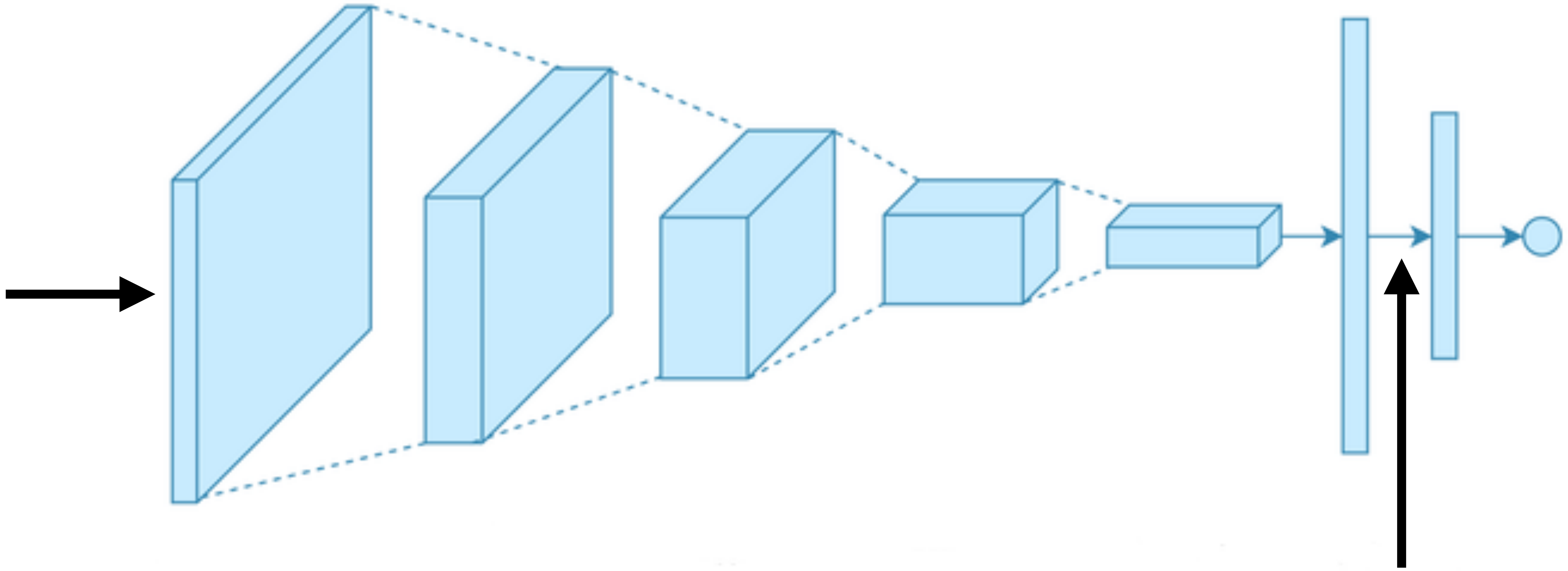

Game action a

Question for Today (and the next a few lectures):

How to (approximately) learn $\pi^\star$ using function approximation for large scale MDPs?
(i.e., numeration over state-action is not feasible)

Outline:

1. Quick recap on supervised learning's performance guarantee (classification & regression)

2. Approximate Policy Iteration (relies regression oracle)

Recap on Supervised Learning: Classification

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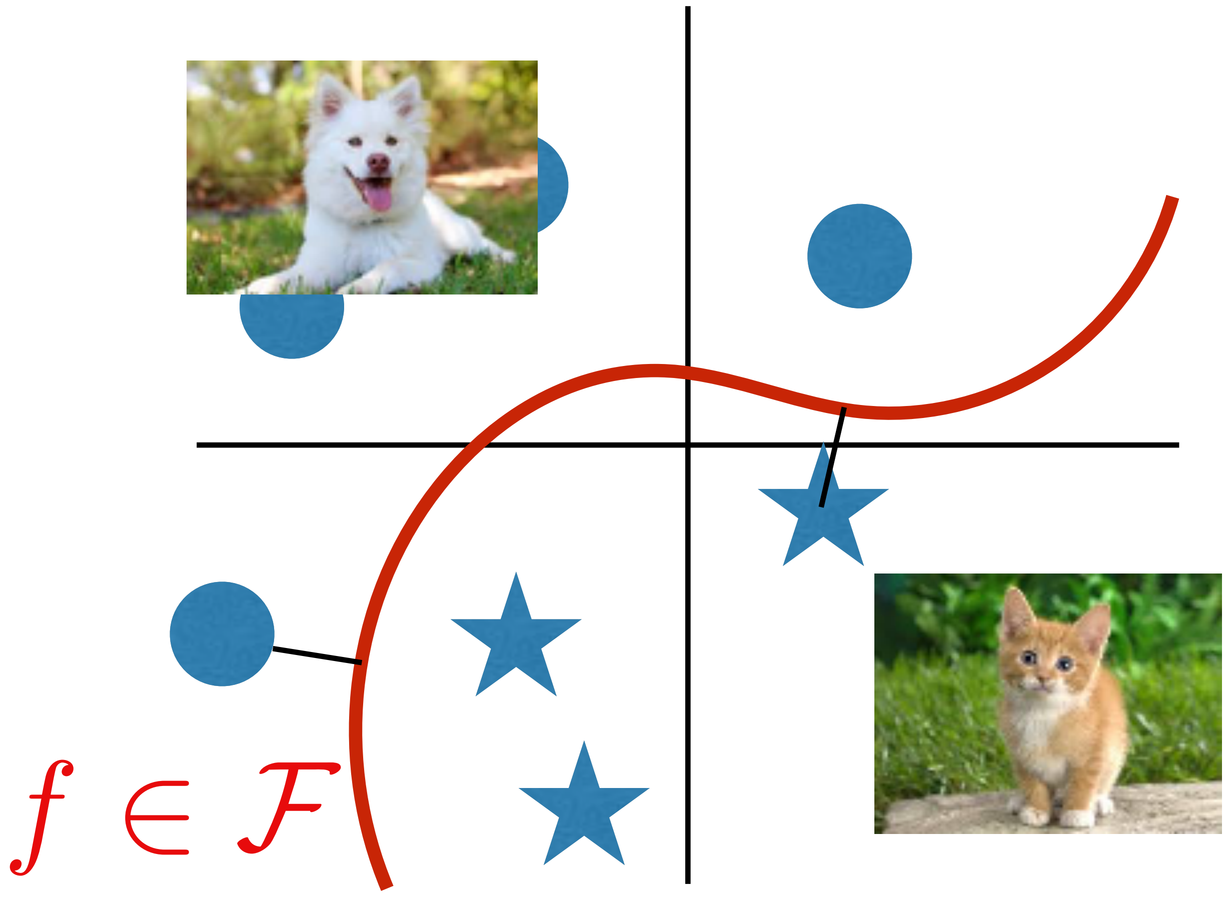

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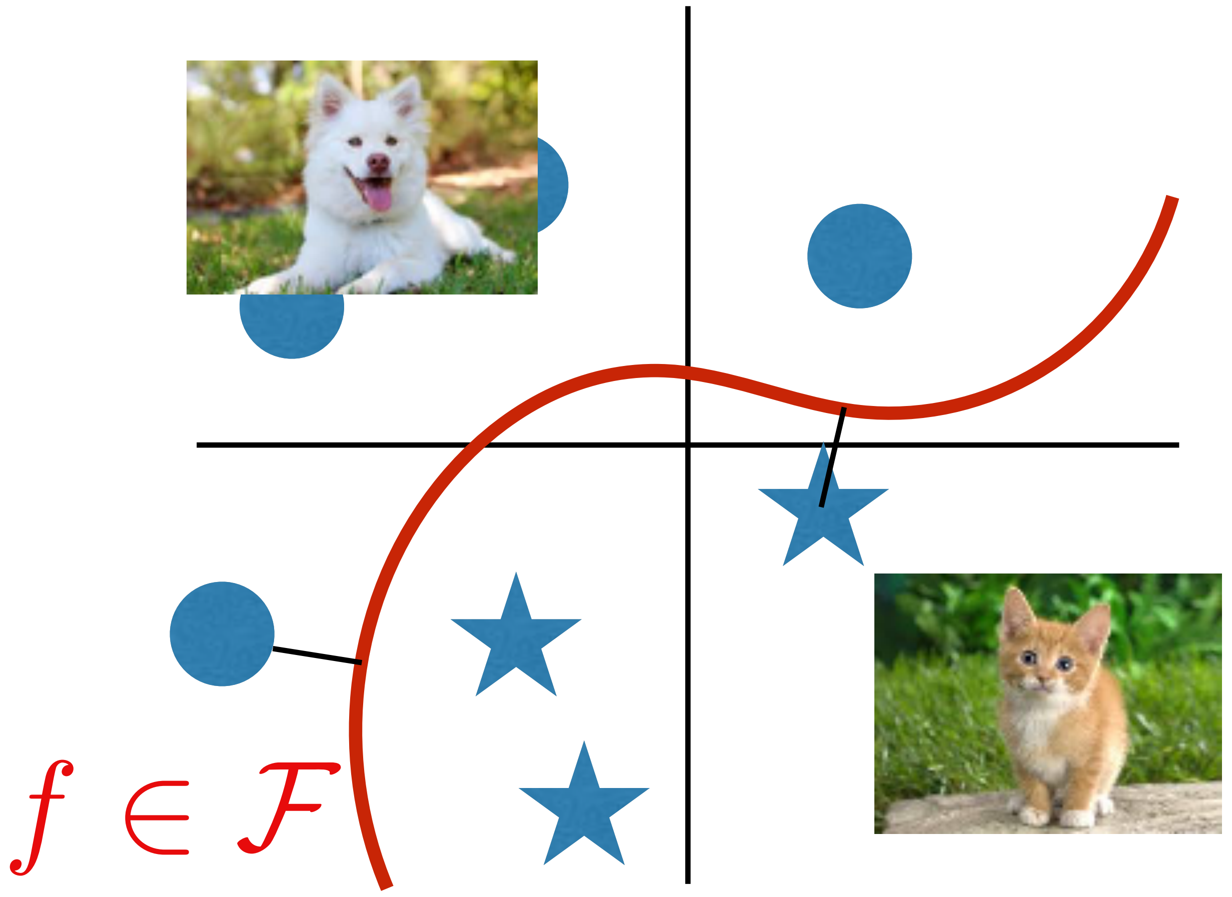
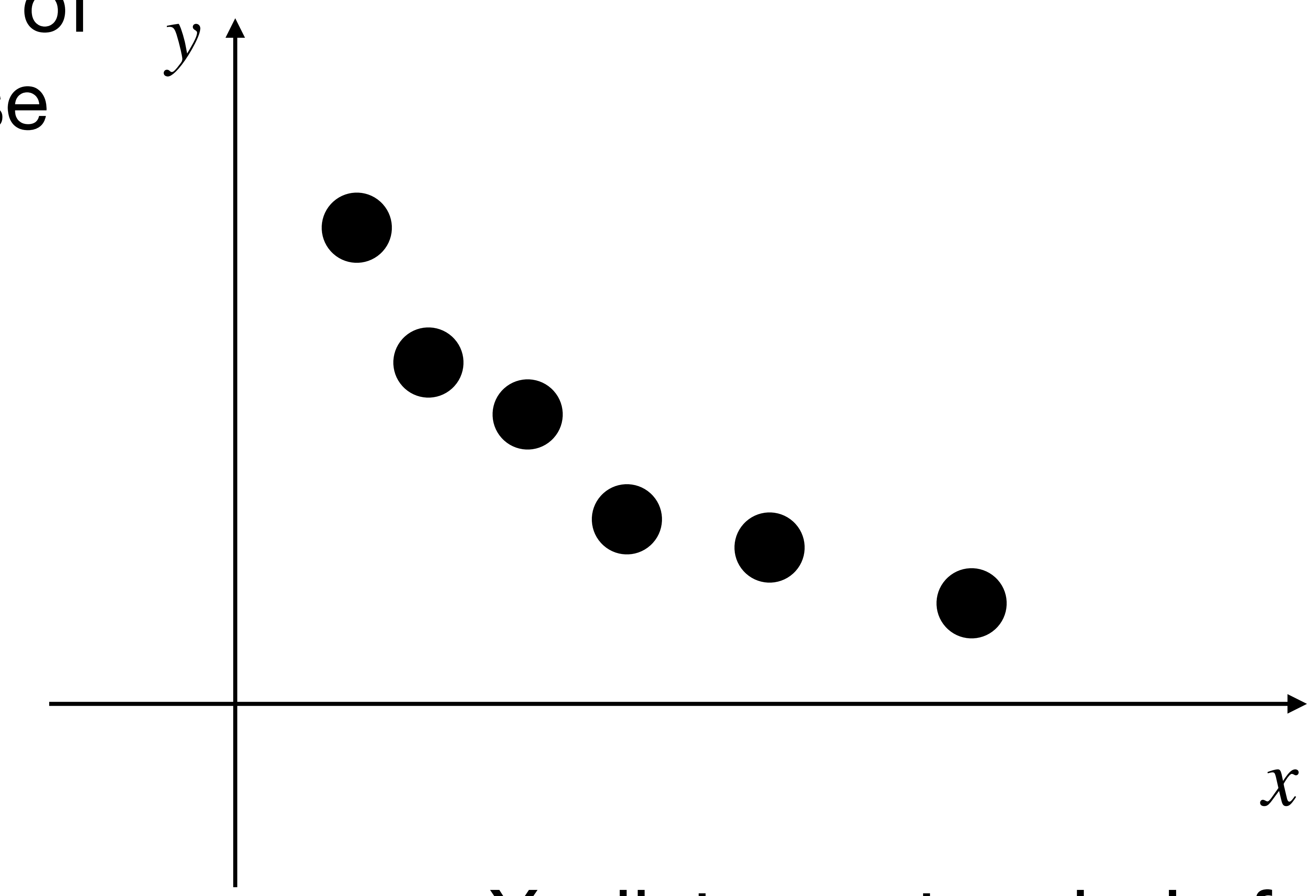


Using function approximator, we are able to predict on cats/dogs that we **never see before** (i.e., we **generalize**)

Recap on Supervised Learning: Regression

Y: value of
a house



X: distance to whole foods

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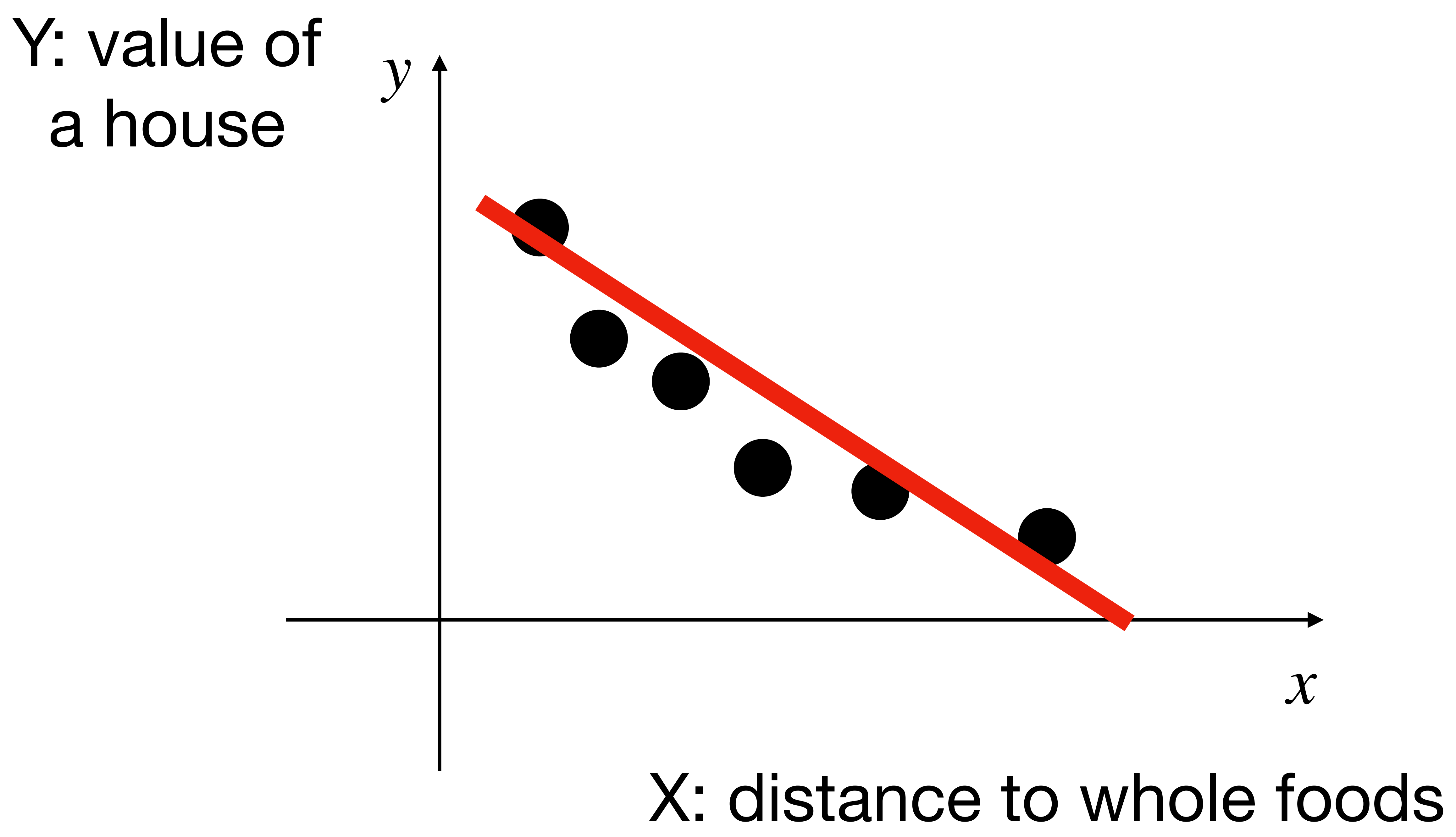

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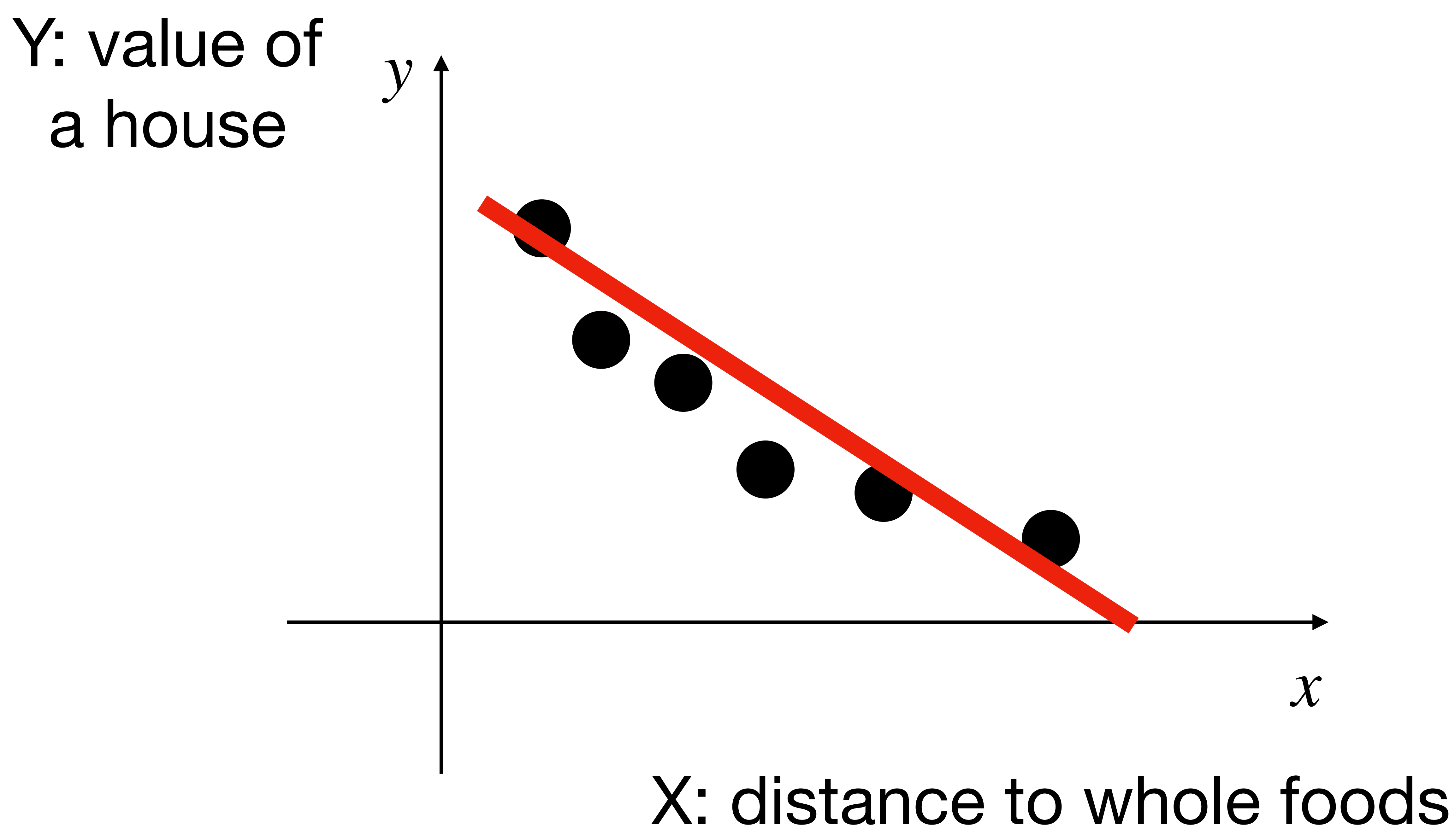

Using function approximator, we are able to predict on the value of some house not from the training data

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Q: quality of ERM $\hat{f}$?

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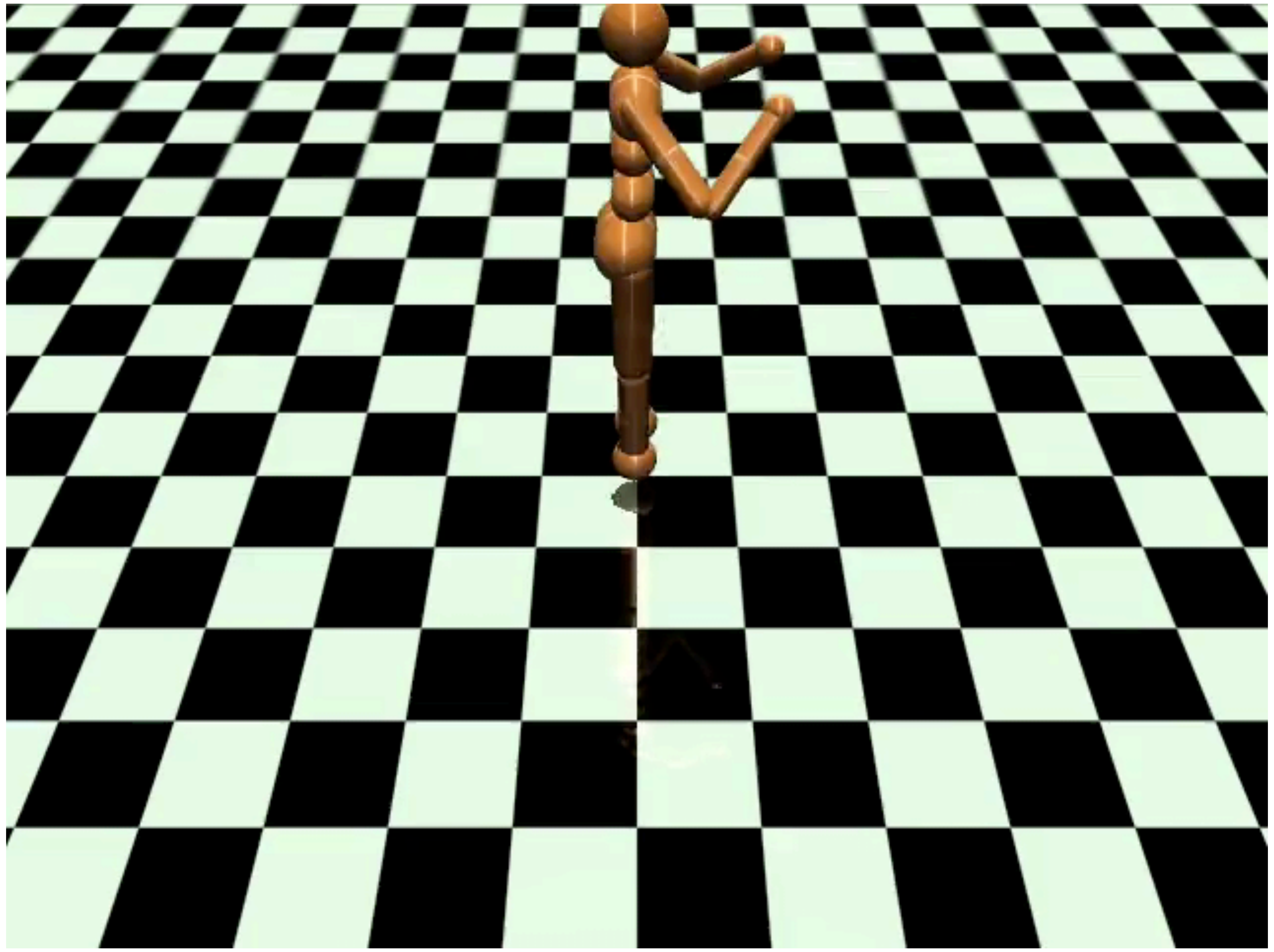
However, for some $\mathscr{D}' \neq \mathscr{D}$, $\mathbb{E}_{x\sim\mathscr{D}'}(f(x) - f^\star(x))^2$ might be arbitrarily large



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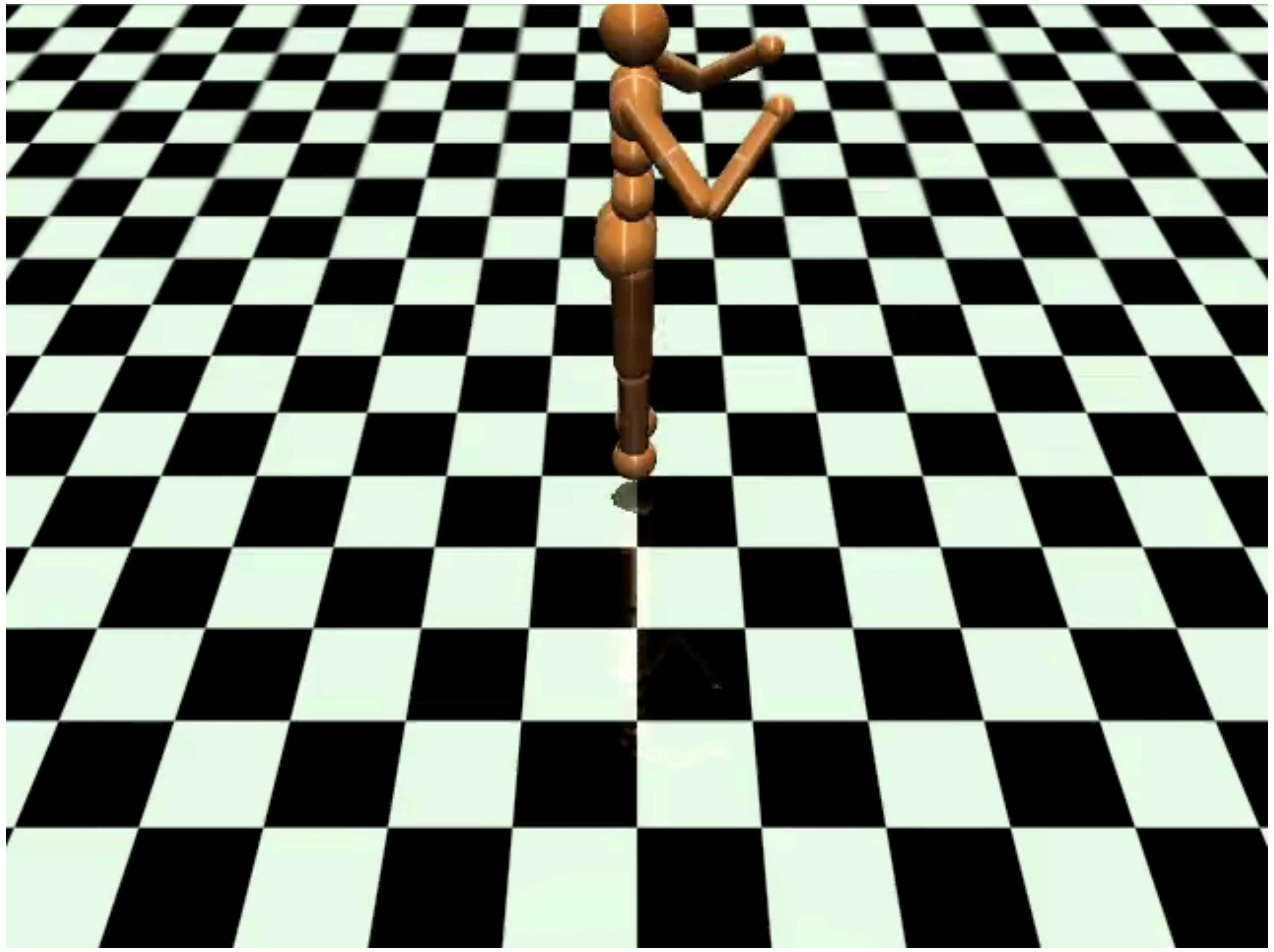

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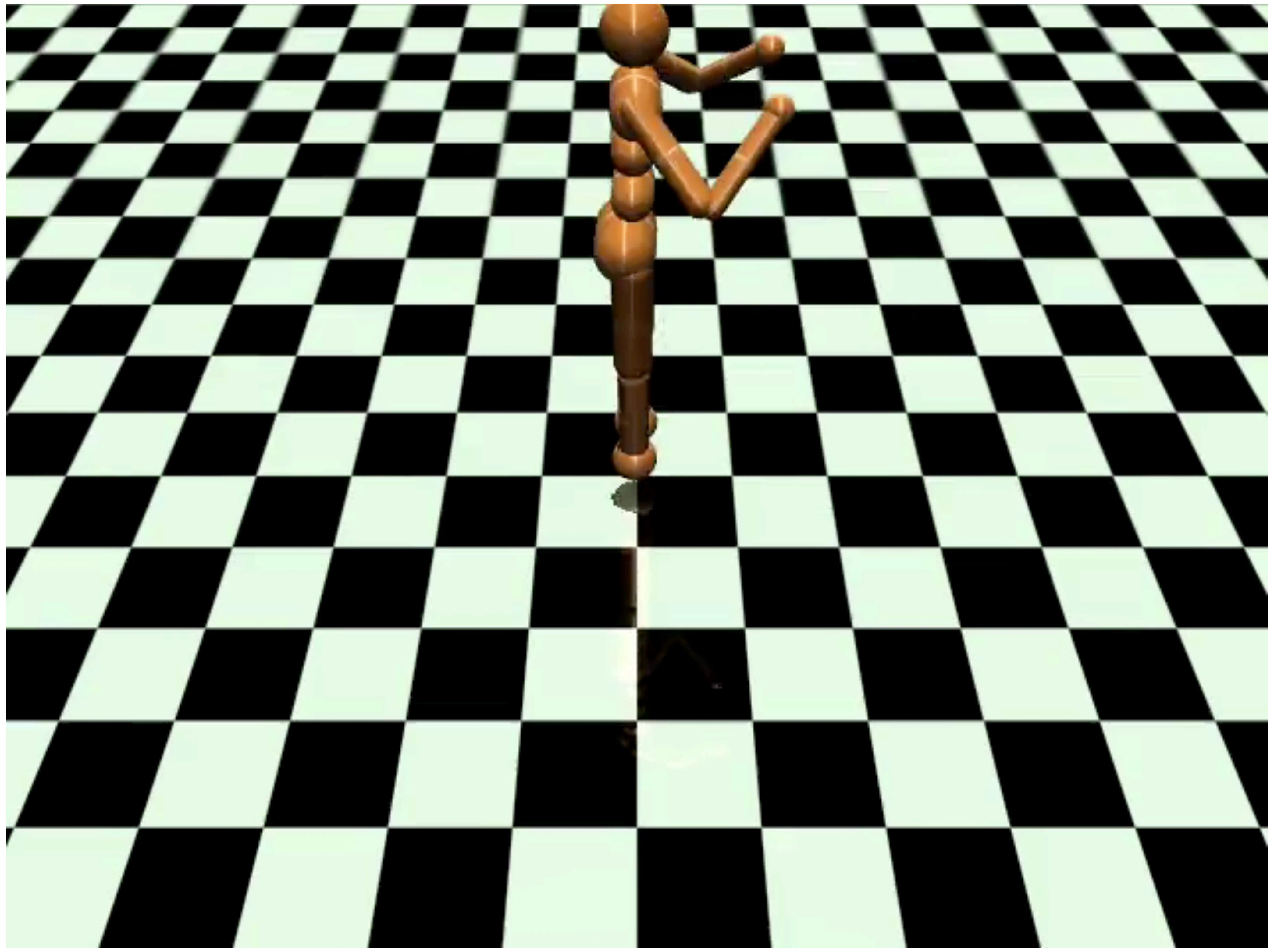

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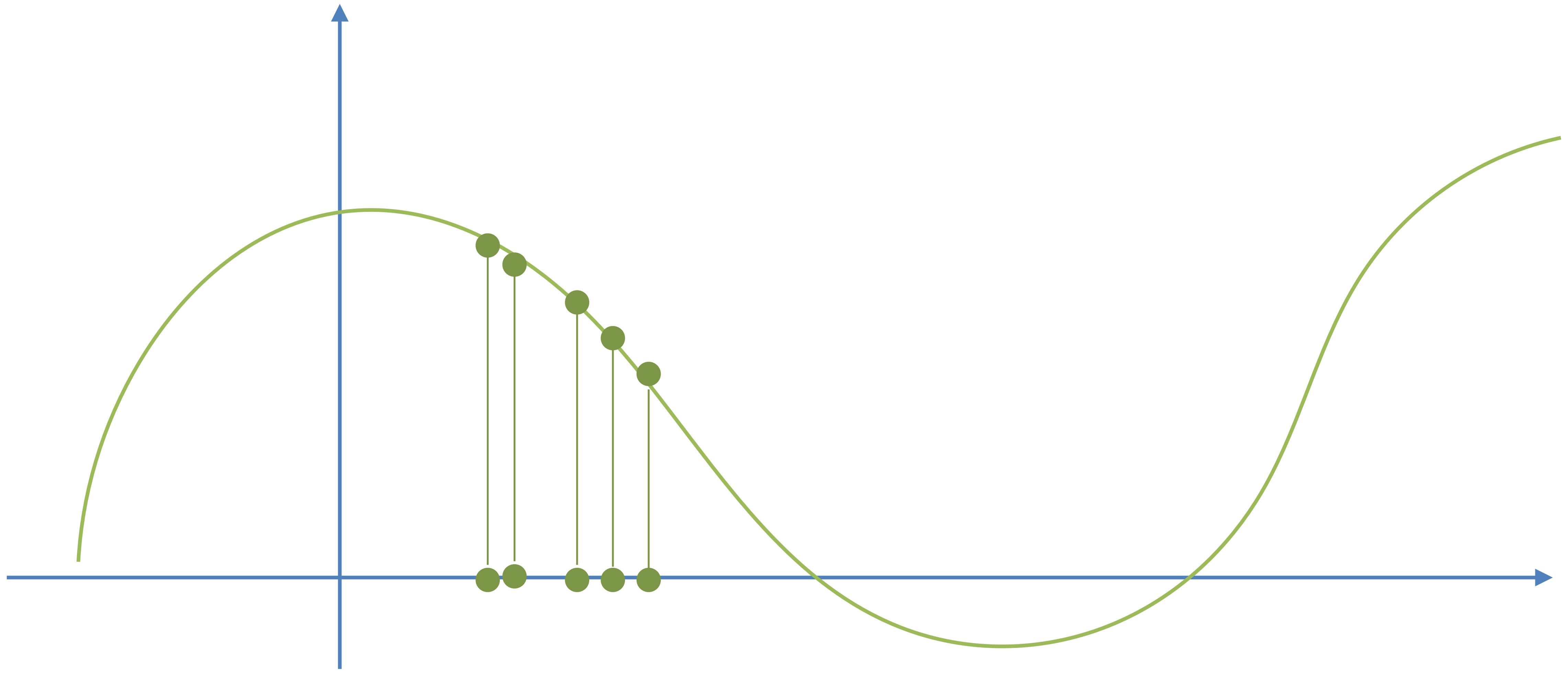

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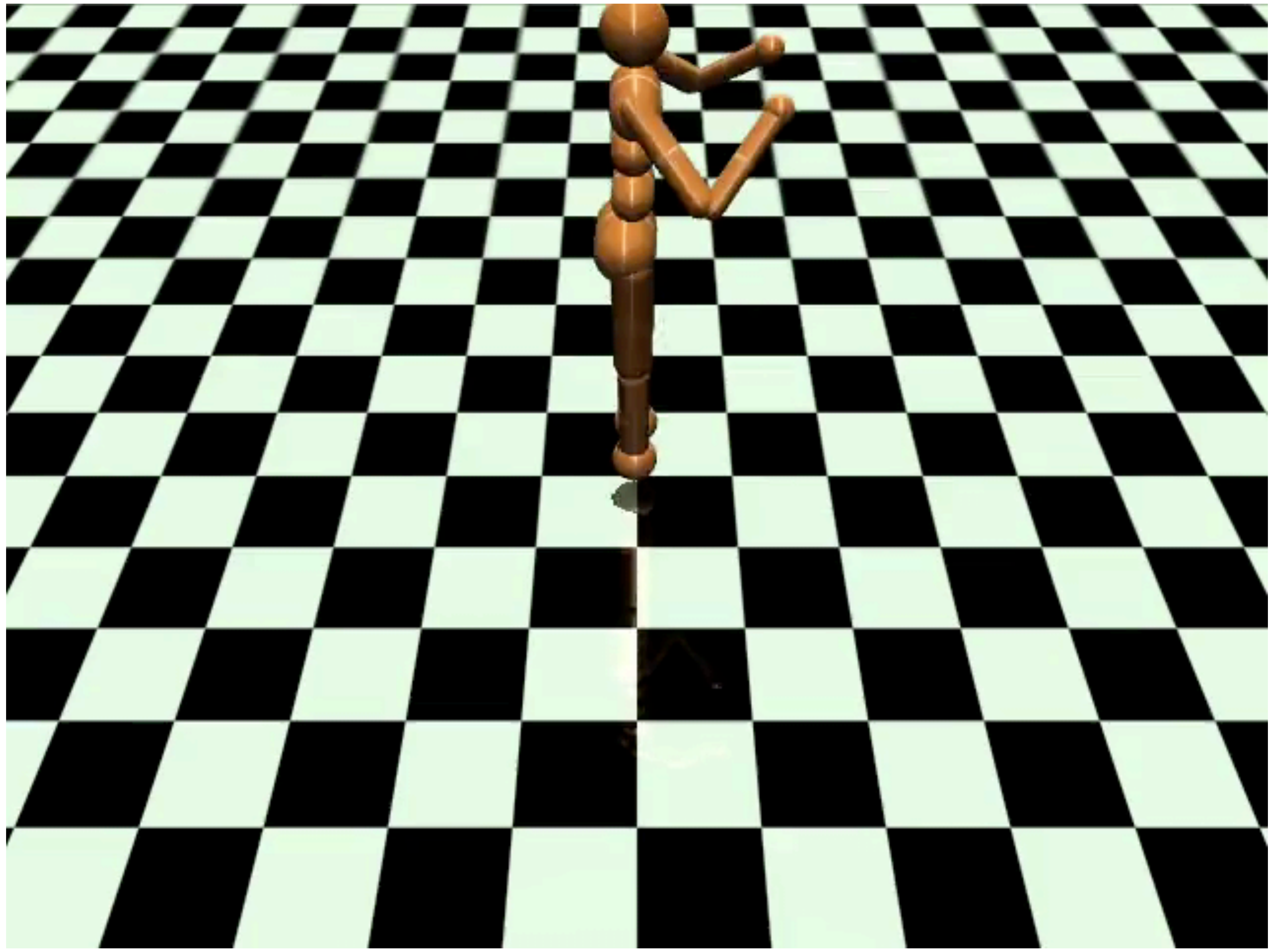

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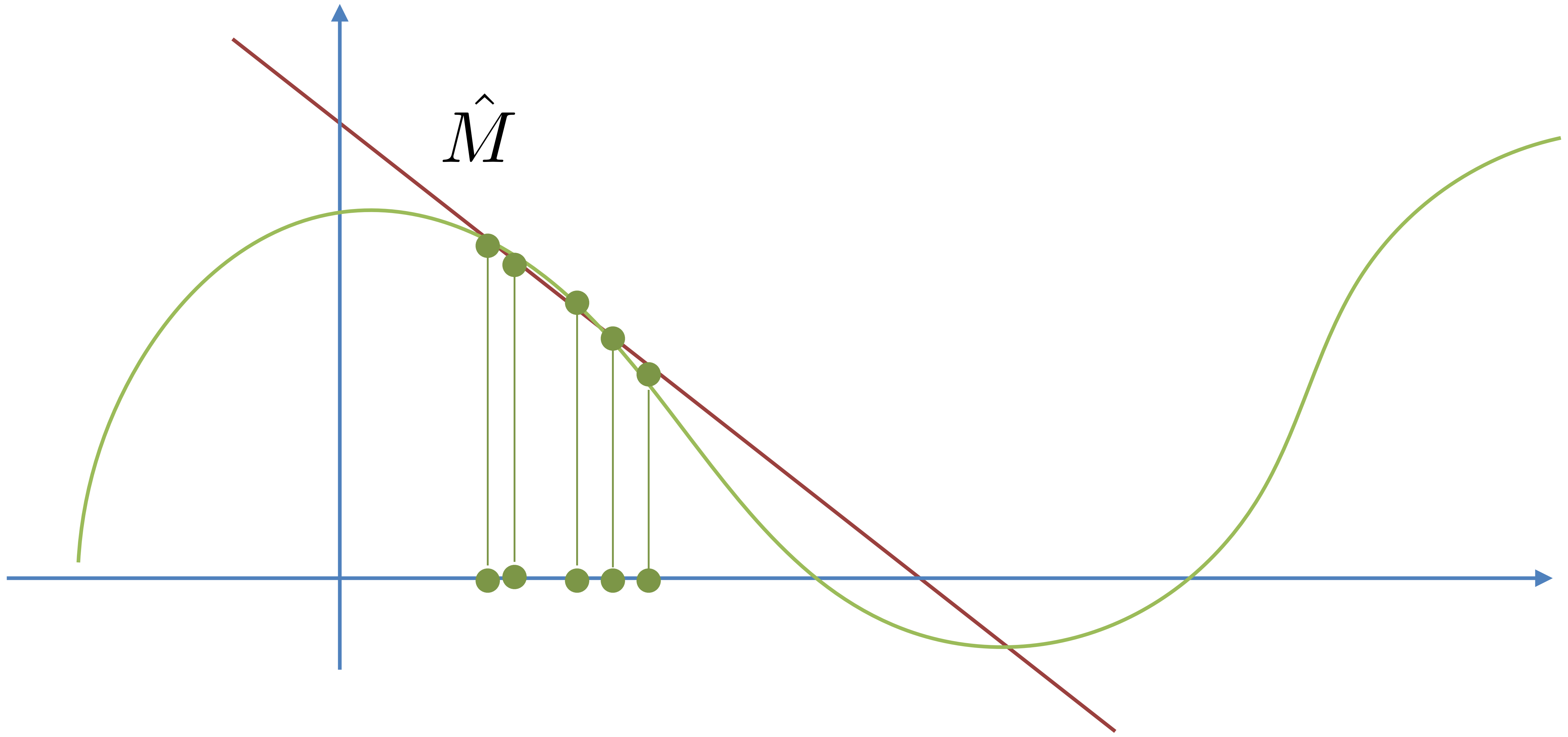

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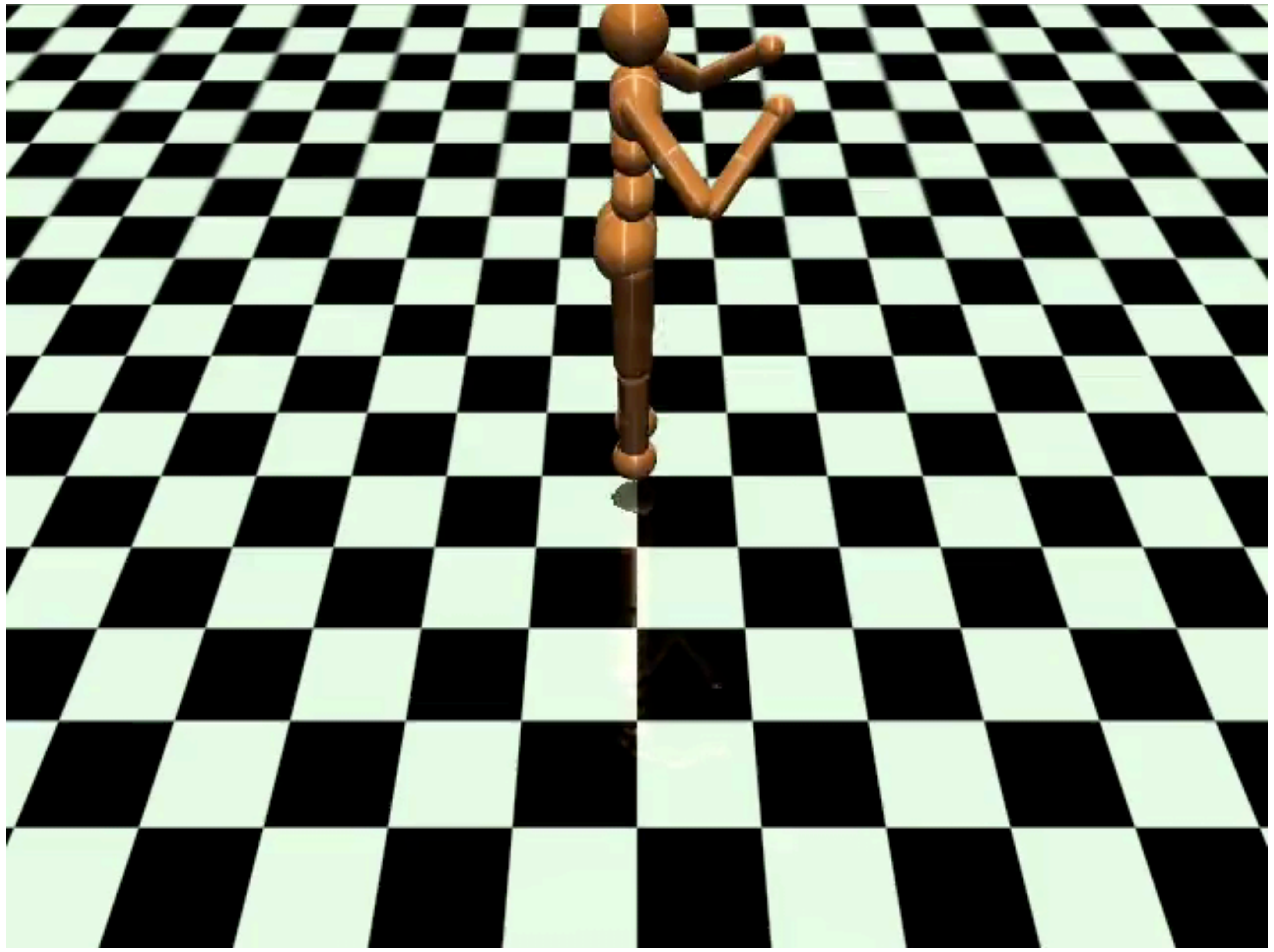

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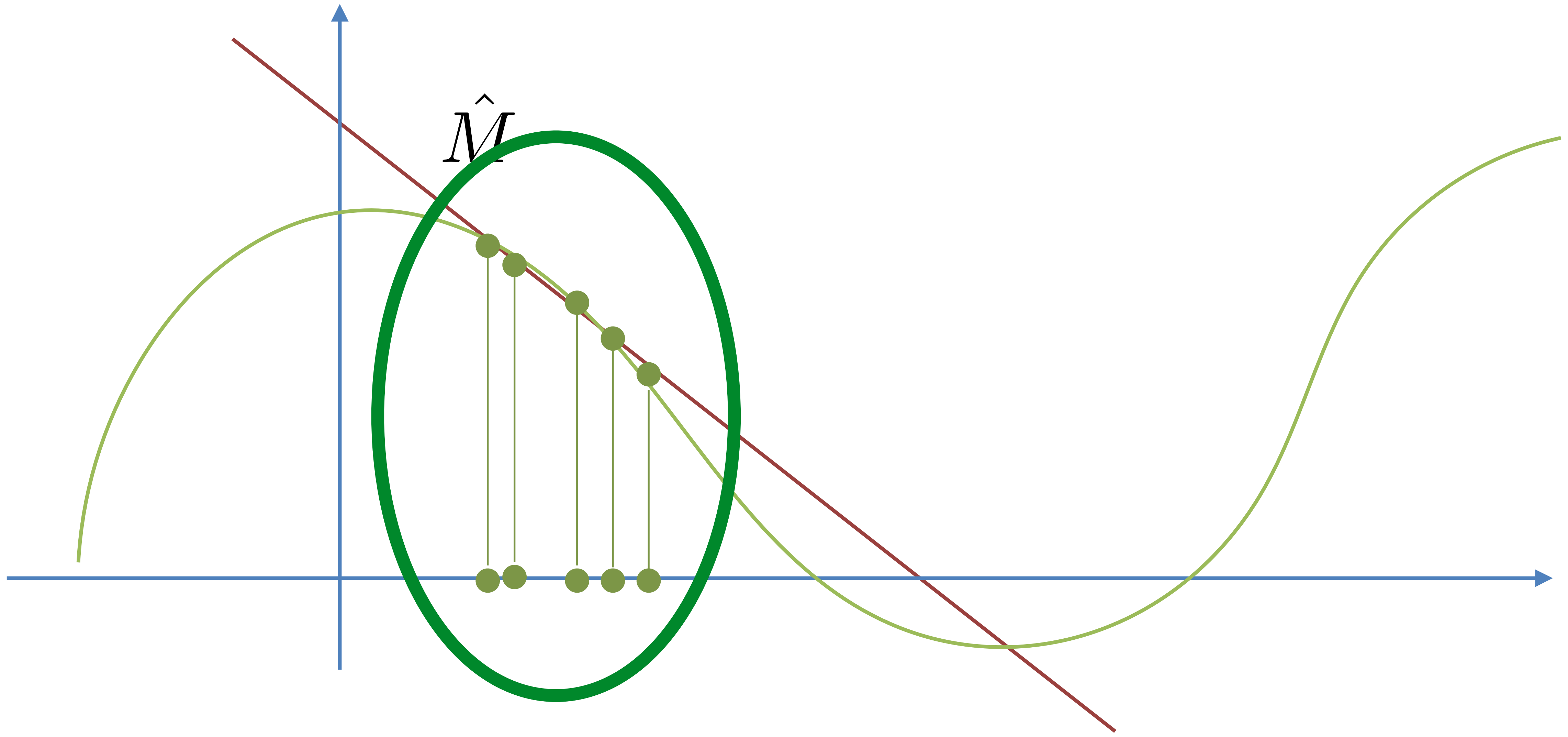

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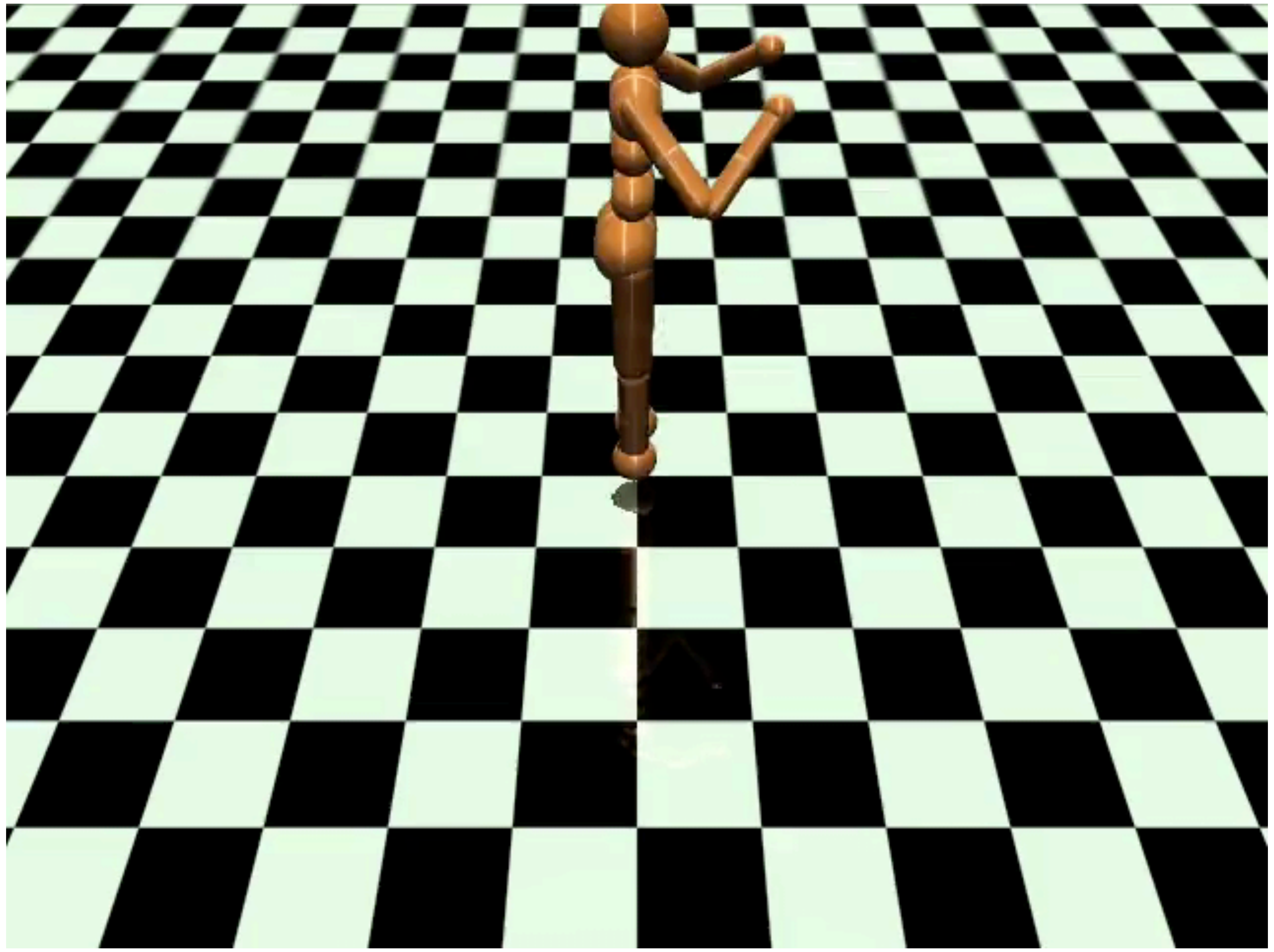

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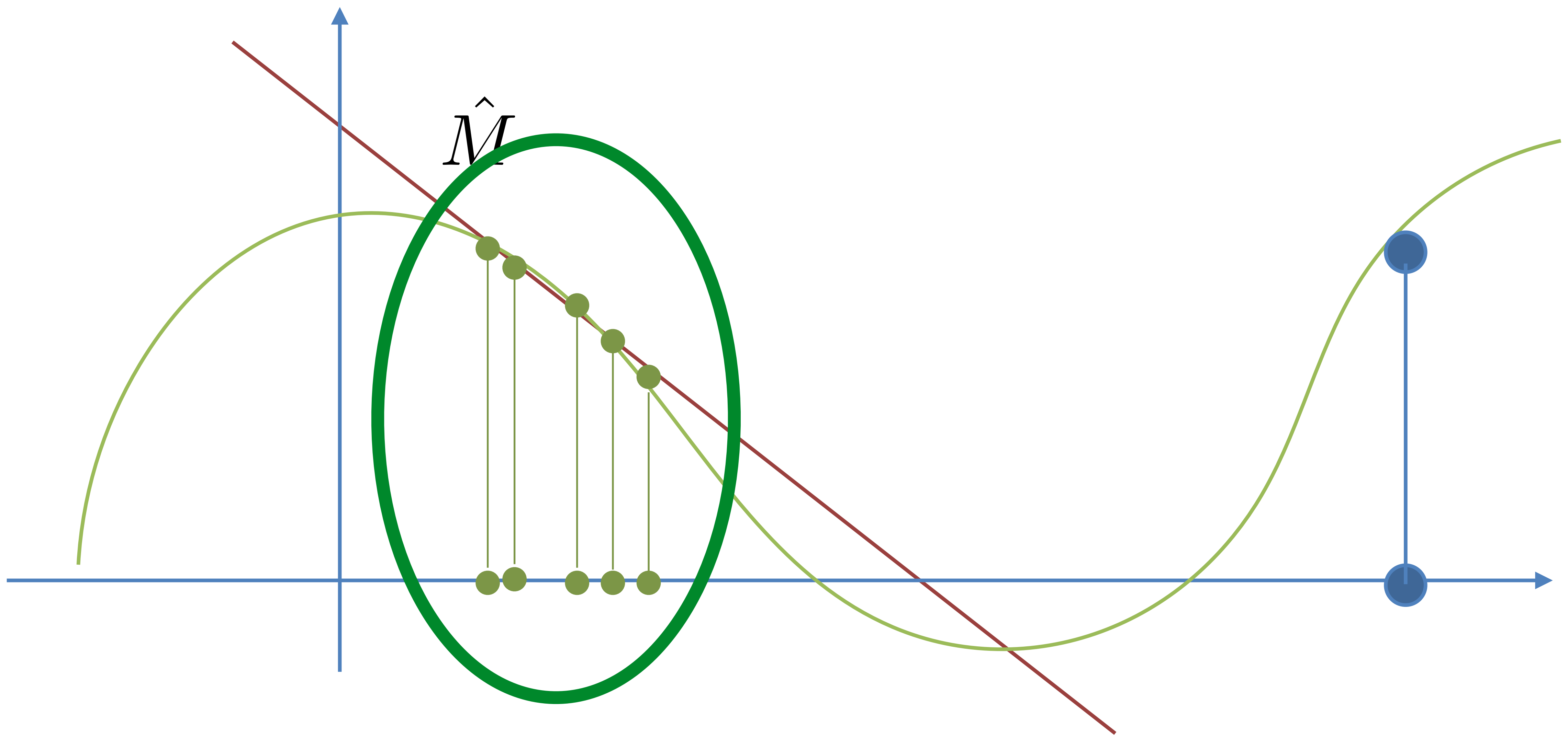

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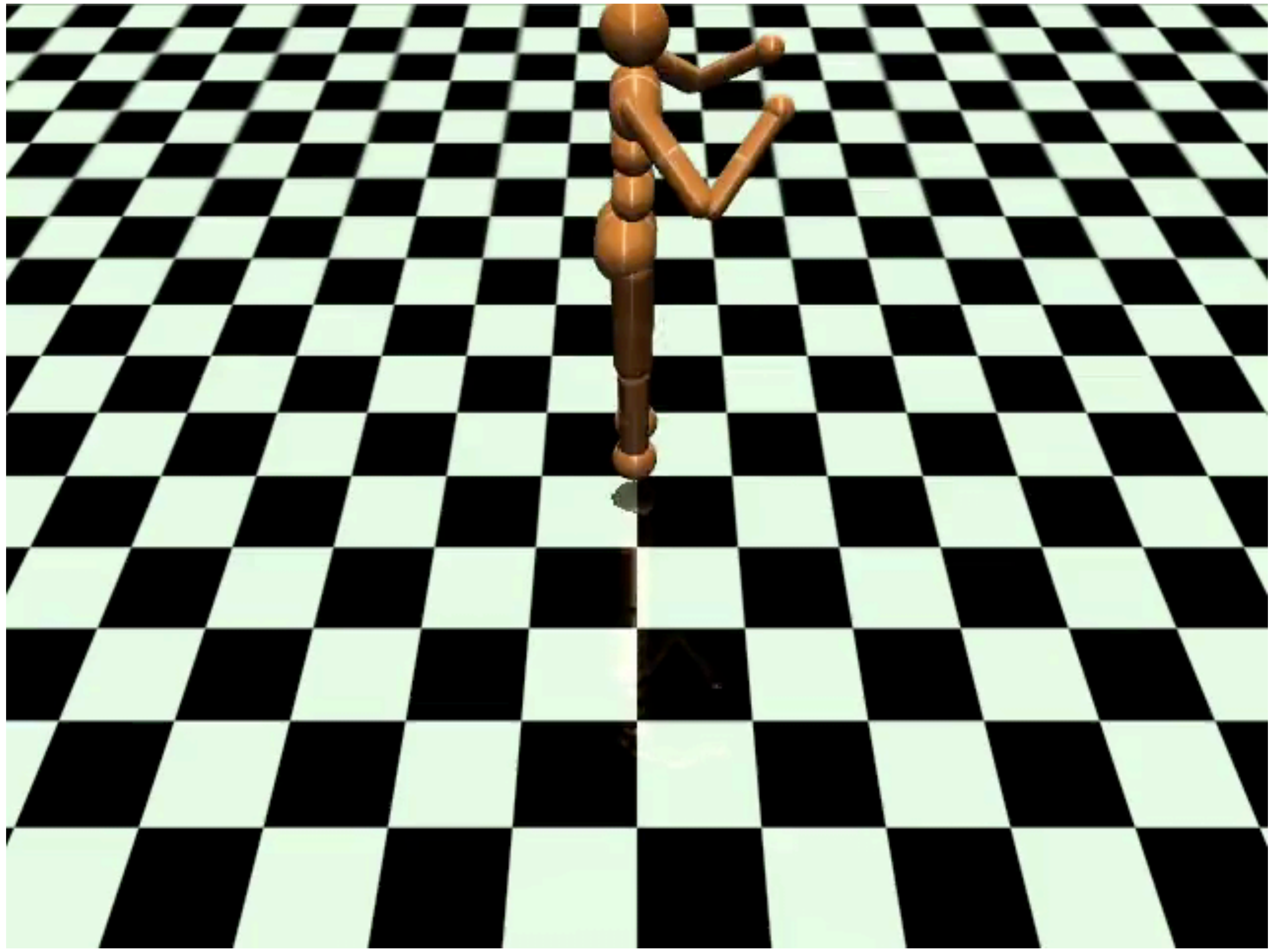

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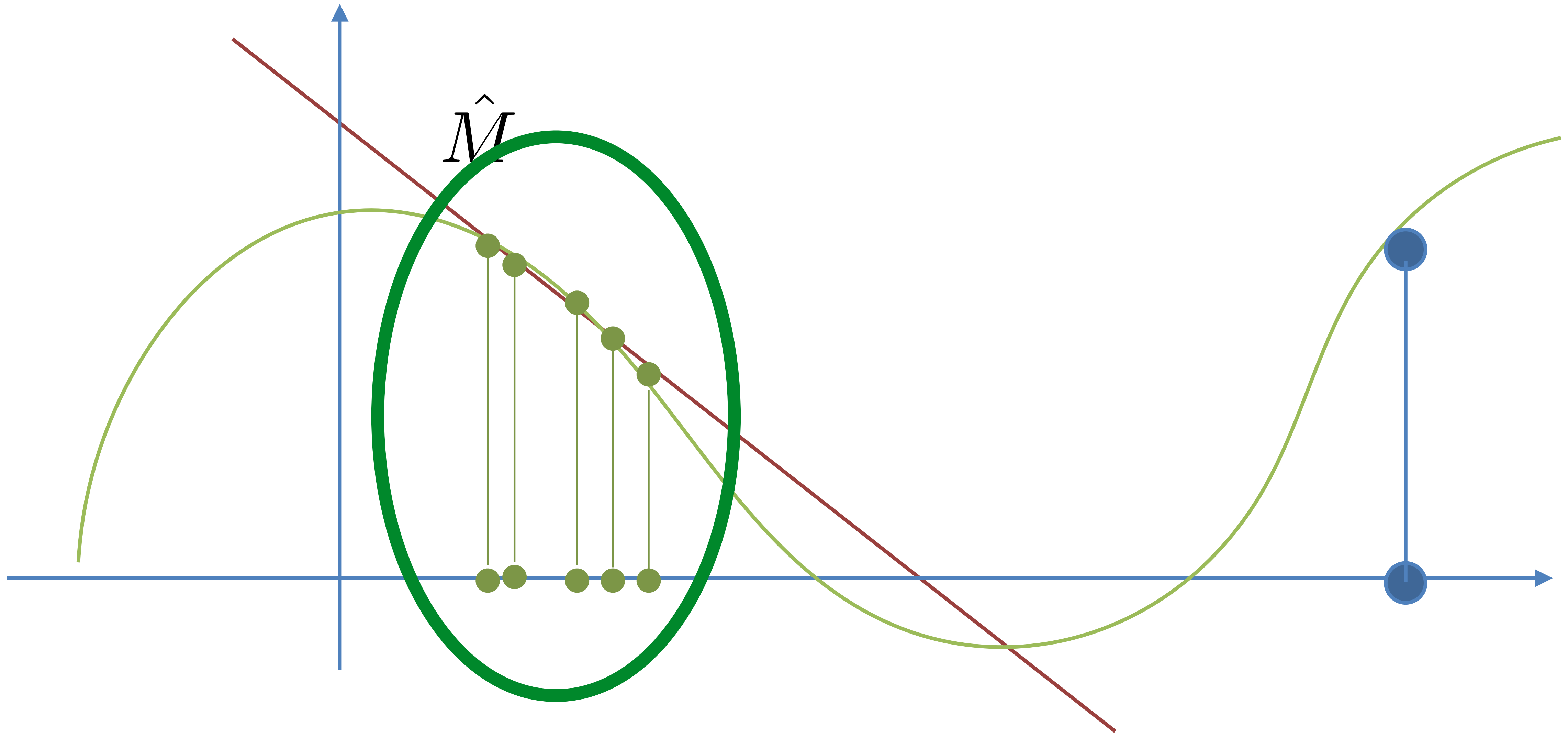

Deeper neural nets or larger dataset do not help if there is distribution shift;
(ML is not black magic)

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(where $\delta \approx \sqrt{1/N}$ (sometime it could be $1/N$))

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2. Approximate Policy Iteration (relies regression oracle)

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$$\mathscr{M} = \{S, A, \gamma, r, P, \mu_0\}$$

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We can only reset according to $s_0 \sim \mu_0$

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a. How to get training data?

b. Quality of the learned $\widehat{Q}^t$?

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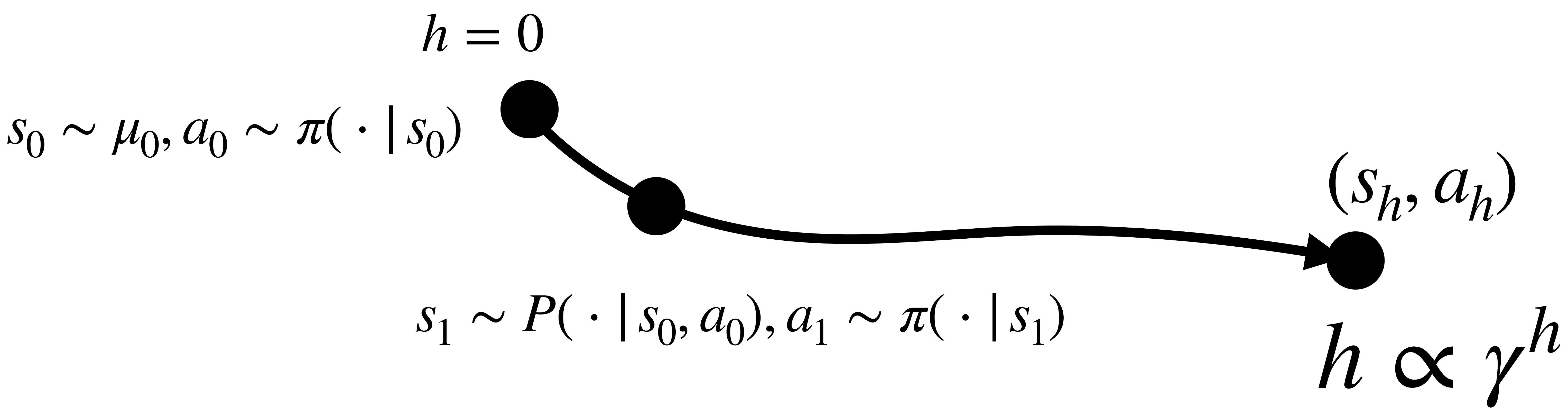

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$$(1-\gamma)r_0 + \gamma(1-\gamma)(r_0 + r_1) + \gamma^2(1-\gamma)(r_0 + r_1 + r_2) + \ldots = \sum_{h=0}^{\infty} \gamma^h r_h$$

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