Exploration in RL: Contextual Bandit
Recap: MAB

Interactive learning process:

For $t = 0 \rightarrow T - 1$

1. Learner pulls arm $I_t \in \{1, \ldots, K\}$

2. Learner observes an i.i.d reward $r_t \sim \nu_{I_t}$ of arm $I_t$
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Learning metric:

$$\text{Regret}_T = T\mu^* - \sum_{t=0}^{T-1} \mu_{I_t}$$
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Arm distributions are fixed across learning.
Question for Today:

Incorporate contexts into the interactive learning framework
Outline for today:

1. Introduction of the model

2. Algorithm

3. Theory and some practical considerations
Make the framework Context Dependent:

Interactive learning process:

For $t = 0 \rightarrow T - 1$

1. A new context $x_t \in \mathcal{X}$ appears
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Interactive learning process:

For $t = 0 \rightarrow T - 1$

1. A new context $x_t \in \mathcal{X}$ appears

2. Learner picks action $a_t \in \mathcal{A}$ (# based on context $x_t$ and historical information)

3. Learner observes an reward $r_t := r(x_t, a_t)$

Reward is context and arm dependent now!
Make the framework Context Dependent:

Interactive learning process:

\[ x_t \sim \mu \]

\[ a_t \]

\[ r_t := r(x_t, a_t) \]
Examples:

Personalize recommendation system
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Context: user’s information (e.g., history health conditions, age, height, weight, job type, etc)
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Decisions (arms): news articles
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**Goal:** learn to maximize user click rate
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**Personalize recommendation system**

**Context:** user’s information (e.g., history health conditions, age, height, weight, job type, etc)

**Decisions** (arms): news articles

**Goal:** learn to maximizes user click rate

Different users have different preferences on news, so need to personalize
Equivalently, it is an MDP with $H = 1$

Finite horizon MDP with $H = 1$

$\mathcal{M} = \{ \mathcal{X}, \mathcal{A}, r, H = 1, \mu \}$
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Objective function:

$$\max_{\pi \in \Pi} \mathbb{E}_{x \sim \mu} \left[ r(x, \pi(x)) \right]$$
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Finite horizon MDP with $H = 1$

$$\mathcal{M} = \{\mathcal{X}, \mathcal{A}, r, H = 1, \mu\}$$

Objective function:

$$\max_{\pi \in \Pi} \mathbb{E}_{x \sim \mu} \left[ r(x, \pi(x)) \right]$$

For simplicity, we assume reward is deterministic; The challenge is really from randomness in contexts
The Regret Metric

Fix a policy class $\Pi$ (think about $\pi$ as a classifier from $x \to a$)

Denote optimal policy $\pi^* = \arg\max_{\pi \in \Pi} \mathbb{E}_{x \sim \mu} r(x, \pi(x))$
The Regret Metric

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Every iteration, learner has a policy \( \pi^t \in \Pi \)
(Recommends \( a_t = \pi^t(x_t) \), receives reward \( r_t := r(x_t, \pi(x_t)) \))
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$$\text{Regret}_T = T \mathbb{E}_{x \sim \mu} [r(x, \pi^*(x))] - \sum_{t=0}^{T-1} \mathbb{E}_{x \sim \mu} [r(x, \pi^t(x))]$$
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Total expected reward if we always uses $\pi^*$ to recommend
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Total expected reward if we always uses $\pi^*$ to recommend
Total expected reward of our learned sequence of policies
Outline for today:

1. Introduction of the model

2. Algorithm

3. Theory and some practical considerations
Ingredient 1: Importance Weighting

The key challenging here is that we observe $r_t := r(x_t, a_t)$, but we do not know $r(x_t, a)$ for $a \neq a_t$.
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Importance weighting actually allows us to get unbiased estimate for ALL actions!
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Importance weighting actually allows us to get unbiased estimate for ALL actions!

Assume $a_t \sim p \ (p \in \Delta(\mathcal{A}))$, and we log $p(a_t)$, receive $r_t = r(x_t, a_t)$,
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Assume $a_t \sim p (p \in \Delta(A))$, and we log $p(a_t)$, receive $r_t = r(x_t, a_t)$,

For all $a \in A$, define $\hat{r}[a] = \frac{r(x_t, a) \mathbf{1}[a = a_t]}{p(a_t)}$,
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For all $a \in \mathcal{A}$, define $\hat{r}[a] = \frac{r(x_t, a)1[a = a_t]}{p(a_t)}$

$$\hat{r} := \begin{bmatrix} 0 \\ 0 \\ \vdots \\ r_t/p(a_t) \\ \vdots \\ 0 \\ 0 \\ \vdots \\ a_t \end{bmatrix}$$
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Assume \( a_t \sim p (p \in \Delta(\mathcal{A})) \), and we log \( p(a_t) \), receive \( r_t = r(x_t, a_t) \),

For all \( a \in \mathcal{A} \), define \( \hat{r}[a] = \frac{r(x_t, a)\mathbf{1}[a = a_t]}{p(a_t)} \),

\[
\mathbb{E}_{a_t \sim p} \hat{r}[a] = r(x_t, a), \quad \forall a \in \mathcal{A}
\]

\[
\hat{r} := \begin{bmatrix} 0 \\ 0 \\ \vdots \\ r_t/p(a_t) \\ 0, \ldots \\ 0 \end{bmatrix}
\]

\[
\mathbb{E} \left[ \hat{r} \right] = r(x_t^*, 0)
\]
Proving Importance Weighting

Assume \( a_t \sim p (p \in \Delta(\mathcal{A})) \), and we log \( p_t(a) \), receive \( r_t = r(x_t, a_t) \).

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Consider any $a \in \mathcal{A}$:
Proving Importance Weighting

Assume \( a_t \sim p \) (\( p \in \Delta(\mathcal{A}) \)), and we log \( p_t(a) \), receive \( r_t = r(x_t, a_t) \).

For all \( a \in \mathcal{A} \), define \( \hat{r}[a] = \frac{r(x_t, a)\mathbf{1}[a = a_t]}{p(a_t)} \), we have: \( E_{a_t \sim p} \hat{r}[a] = r(x_t, a) \), \( \forall a \in \mathcal{A} \)

Consider any \( a \in \mathcal{A} : \)

\[
E_{a_t \sim p} \frac{r(x_t, a)\mathbf{1}[a = a_t]}{p(a_t)} = \sum_{a_t \in \mathcal{A}} p(a_t) \frac{r(x_t, a)\mathbf{1}[a = a_t]}{p(a_t)}
= \sum_{a_t \in \mathcal{A}} r(x_t, a)\mathbf{1}[a = a_t]
= E_{a_t \sim p} r(x_t, a)
\]
Proving Importance Weighting

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Consider any $a \in \mathcal{A}$:

$$
\mathbb{E}_{a_t \sim p} \frac{r(x_t, a)1[a = a_t]}{p(a_t)} = \sum_{a_t \in \mathcal{A}} p(a_t) \frac{r(x_t, a)1[a = a_t]}{p(a_t)}
= p(a) \frac{r(x_t, a)}{p(a)} = r(x_t, a)
$$

$\mathbb{E}_{a_t \sim p}[\hat{r}[a]] = r(x_t, a)$ defines when $a_t = a$. 

\[ \mathbb{E}_{a_t \sim p}[\hat{r}[a]] \geq r(x_t, a) \]
Ingredient 2: Reward-sensitive Classification Oracle

Recall classic classification:
Ingredient 2: Reward-sensitive Classification Oracle

Recall classic classification:

\[ \{x_i, y_i\}_{i=1}^N, \quad x_i \in \mathcal{X}, y_i \in \{1, \ldots, |\mathcal{A}|\} \]
Ingredient 2: Reward-sensitive Classification Oracle

Recall classic classification:

\[
\{x_i, y_i\}_{i=1}^N, \quad x_i \in \mathcal{X}, y_i \in \{1, \ldots, |\mathcal{Y}|\}
\]

\[
\arg\max_{\pi \in \Pi} \sum_{i=1}^N 1\{\pi(x_i) = y_i\}
\]

\[
\max_{\pi} \sum_{i=1}^N \mathbb{1}(\pi(x_i) = y_i)
\]

\[
\max_{\pi} \sum_{i=2}^N \mathbb{1}(\pi(x_i) = y_i)
\]

\[
\Rightarrow \max_{\pi} \sum_{i=1}^N \mathbb{1}(\pi(x_i) = y_i)
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Let’s generalize it to **Reward-Sensitive Classification**:
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Let’s generalize it to Reward-Sensitive Classification:

\[ \{x_i, r_i\}_{i=1}^{N}, \quad x_i \in \mathcal{X}, r_i \in [0,1]^{|\mathcal{A}|} \]
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\]

\[
x, \begin{pmatrix} r[1] \\ \vdots \\ r[|\mathcal{A}|] \end{pmatrix}
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\[ \arg \max_{\pi \in \Pi} \sum_{i=1}^N r_i[\pi(x_i)] \]
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We will do reduction to RSC
Summary so far:

1. Importance Weighting: we “magically” get unbiased estimate for all actions!

Assume $a_t \sim p$ ($p \in \Delta(\mathcal{A})$), For all $a \in \mathcal{A}$, define $\hat{r}[a] = \frac{r(x_t, a)1[a = a_t]}{p(a_t)}$, we have:

$$\mathbb{E}_{a_t \sim p} \hat{r}[a] = r(x_t, a), \forall a \in \mathcal{A}$$
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Assume $a_t \sim p$ ($p \in \Delta(\mathcal{A})$), For all $a \in \mathcal{A}$, define $\hat{r}[a] = \frac{r(x_t, a) 1[a = a_t]}{p(a_t)}$, we have:

$$\mathbb{E}_{a_t \sim p} \hat{r}[a] = r(x_t, a), \forall a \in \mathcal{A}$$

$$\hat{r} := \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ r_t/p(a_t) \\ 0, \ldots, 0 \end{bmatrix}$$

2. Reward-Sensitive Classification:

$$\{x_i, r_i\}_{i=1}^{N}, \quad x_i \in \mathcal{X}, r_i \in [0,1]|\mathcal{A}|$$

$$\arg \max_{\pi \in \Pi} \sum_{i=1}^{N} r_i[\pi(x_i)]$$

$$x, \begin{bmatrix} r[1] \\ r[2] \\ \vdots \\ r[|\mathcal{A}|] \end{bmatrix}$$
Put things together: Explore and Commit

For $t = 0 \rightarrow N - 1$ :  (# exploration phase)
Put things together: Explore and Commit

For $t = 0 \rightarrow N - 1$ : (exploration phase)

1. Observe $x_t \sim \mu$
Put things together: Explore and Commit

For $t = 0 \rightarrow N - 1$:

1. Observe $x_t \sim \mu$

2. **Uniform-randomly** sample $a_t \sim \text{Unif}(\mathcal{A})$, receive reward $r_t = r(x_t, a_t)$
Put things together: Explore and Commit

For \( t = 0 \rightarrow N - 1 \):  (# exploration phase)

1. Observe \( x_t \sim \mu \)

2. **Uniform-randomly** sample \( a_t \sim \text{Unif}(\mathcal{A}) \), receive reward \( r_t = r(x_t, a_t) \)

3. Use IW, form unbiased estimate \( \hat{r}_t[a] = \begin{cases} 
0 & \text{if } a \neq a_t \\
\frac{r_t}{1/|\mathcal{A}|} & \text{if } a = a_t 
\end{cases} \)
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For \( t = 0 \rightarrow N - 1 \) :  (# exploration phase)

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\[
\{ X_i, R_i \}_{i=0}^{N-1} \leftarrow \text{Dataset for RSC}
\]

\( \hat{r}_t \in \mathbb{R}^{|\mathcal{A}|} \)

\( p(a_t) = \frac{1}{|\mathcal{A}|} \)
Put things together: Explore and Commit

For $t = 0 \rightarrow N - 1$:

1. Observe $x_t \sim \mu$

2. **Uniform-randomly** sample $a_t \sim \text{Unif}(\mathcal{A})$, receive reward $r_t = r(x_t, a_t)$

3. Use **IW**, form unbiased estimate $\hat{r}_t[a] = \begin{cases} 0 & a \neq a_t \\ \frac{r_t}{1/|\mathcal{A}|} & a = a_t \end{cases}$

**Call RSC oracle:** $\hat{\pi} = \arg\max_{\pi \in \Pi} \sum_{i=0}^{N-1} \hat{r}_t[\pi(x_i)]$

For $\pi$:

$$\frac{1}{N} \sum_{i=1}^{N} \pi (x_i)$$

is unbiased est $+ \frac{E}{\mathcal{X} \sim \mu} \left[ r(x, \pi(x)) \right] \xrightarrow{\text{law}} E_{\mathcal{X} \sim \mu} \left[ r(x, \pi(x)) \right]$
Put things together: Explore and Commit

For $t = 0 \rightarrow N - 1$ :  (# exploration phase)

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2. **Uniform-randomly** sample $a_t \sim \text{Unif}(\mathcal{A})$, receive reward $r_t = r(x_t, a_t)$

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Call RSC oracle: $\hat{\pi} = \arg\max_{\pi \in \Pi} \sum_{i=0}^{N-1} \hat{r}_t[\pi(x_t)]$

$E[\hat{F}_\pi] = r(x_k, \cdot)$

For $t = N \rightarrow T - 1$ :  (# exploitation phase)
Put things together: Explore and Commit

For $t = 0 \rightarrow N - 1$:  (# exploration phase)

1. Observe $x_t \sim \mu$

2. **Uniform-randomly** sample $a_t \sim \text{Unif}(\mathcal{A})$, receive reward $r_t = r(x_t, a_t)$

3. Use IW, form unbiased estimate $\hat{r}_t[a] = \begin{cases} 0 & a \neq a_t \\ \frac{r_t}{1/|\mathcal{A}|} & a = a_t \end{cases}$

Call RSC oracle: $\hat{\pi} = \arg \max_{\pi \in \Pi} \sum_{i=0}^{N-1} \hat{r}_t[\pi(x_t)]$

For $t = N \rightarrow T - 1$:  (# exploitation phase)

1. Observe $x_t \sim \mu$, and play $a_t = \hat{\pi}(x_t)$
Outline for today:

1. Introduction of the model
2. Algorithm
3. Theory and some practical considerations
Theory of the Explore and Commit Algorithm

For simplicity, assume $\Pi$ is discrete (but could be exponential large)
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For simplicity, assume $\Pi$ is discrete (but could be exponential large)

[Theorem—in informal] W/ high probability, properly setting the hyper-parameter $N$, Explore-and-Commit has the following regret:

$$\text{Regret}_T = T \mathbb{E}_{x \sim \mu}[r(x, \pi^*(x))] - \sum_{t=0}^{T-1} \mathbb{E}_{x \sim \mu}[r(x, \pi^t(x))] = O \left( T^{2/3} K^{1/3} \cdot \ln(|\Pi|)^{1/3} \right)$$

$$\stackrel{\text{Regret}}{\sim} \frac{1}{K^{\frac{2}{3}}} T^{-\frac{1}{3}}$$
Practical Consideration

Instead of setting a hard threshold for explore and commit, we often interleave explore and exploitation.
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\[ \epsilon \in (0, 1) \]

\( \epsilon \)-greedy:

Every iteration \( t \):

With probability \( 1 - \epsilon \), we play \( a_t = \pi^t(x_t) \),
and with probability \( \epsilon \), we play \( a_t \sim \text{Unif}(\mathcal{A}) \)
Practical Consideration

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Every iteration \( t \):

With probability \( 1 - \epsilon \), we play \( a_t = \pi^t(x_t) \),

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Q: What’s the action distribution induced by \( \epsilon \)-greedy at iteration \( t \)?
Practical Consideration

Instead of setting a hard threshold for explore and commit, we often interleave explore and exploitation

$\epsilon$-greedy:

Every iteration $t$:
- With probability $1 - \epsilon$, we play $a_t = \pi^t(x_t)$,
- and with probability $\epsilon$, we play $a_t \sim \text{Unif}(\mathcal{A})$

Q: What’s the action distribution induced by $\epsilon$-greedy at iteration $t$?

$$a \sim p_t, \quad p_t = (1 - \epsilon)\delta(\pi^t(x_t)) + \epsilon\text{Unif}(\mathcal{A})$$
Put things together: $\epsilon-$greedy

For $t = 0 \to \infty$ (# interleve exploration & exploitation)

1. Observe $x_t \sim \mu$
Put things together: $\epsilon-$greedy

For $t = 0 \rightarrow \infty$ (# interleve exploration & exploitation)

1. Observe $x_t \sim \mu$

2. Use $\epsilon$-greedy to form action distribution $p_t = (1 - \epsilon)\delta(\pi^t(x_t)) + \epsilon \text{Unif}(\mathcal{A})$
Put things together: $\epsilon$—greedy

For $t = 0 \to \infty$ (# interleve exploration & exploitation)

1. Observe $x_t \sim \mu$

2. Use $\epsilon$-greedy to form action distribution $p_t = (1 - \epsilon)\delta(\pi^t(x_t)) + \epsilon\text{Unif}(\mathcal{A})$

3. Play $a_t \sim p_t$, and observe $r_t := r(x_t, a_t)$
Put things together: $\epsilon-$greedy

For $t = 0 \to \infty$ (# interleave exploration & exploitation)

1. Observe $x_t \sim \mu$

2. Use $\epsilon$-greedy to form action distribution $p_t = (1 - \epsilon)\delta(\pi^t(x_t)) + \epsilon \text{Unif}(\mathcal{A})$

3. Play $a_t \sim p_t$, and observe $r_t := r(x_t, a_t)$

4. Via IW, form unbiased estimate $\hat{r}_t$
Put things together: $\varepsilon-$greedy

For $t = 0 \rightarrow \infty$ (# interleave exploration & exploitation)

1. Observe $x_t \sim \mu$

2. Use $\varepsilon$-greedy to form action distribution $p_t = (1 - \varepsilon)\delta(\pi^t(x_t)) + \varepsilon \text{Unif}(\mathcal{A})$

3. Play $a_t \sim p_t$, and observe $r_t := r(x_t, a_t)$

4. Via IW, form unbiased estimate $\hat{r}_t$

5. Update via RSC oracle: $\pi^{t+1} = \arg \max_{\pi \in \Pi} \sum_{i=1}^t \hat{r}_i[\pi(x_i)]$
Put things together: $\epsilon -$greedy

For $t = 0 \rightarrow \infty$ (#interleave exploration & exploitation)

1. Observe $x_t \sim \mu$

2. Use $\epsilon$-greedy to form action distribution
   $p_t = (1 - \epsilon)\delta(\pi^t(x_t)) + \epsilon\text{Unif}(\mathcal{A})$

3. Play $a_t \sim p_t$, and observe $r_t := r(x_t, a_t)$

4. Via IW, form unbiased estimate $r_t$

5. Update via RSC oracle: $\pi^{t+1} = \arg \max_{\pi \in \Pi} \sum_{i=1}^{t} r_i[\pi(x_i)]$
   (Additionally 6. Gradually decay $\epsilon$…)

CB algorithm is being used in real world application at Microsoft:
