Case Study: AlphaGo

ALPHAGO
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LEE SEDOL
00:00:27
BBC NEWS

Outline for Today:

1. Setting

2. The imitation learning component

3. The policy Gradient Component

4. The combination of policy, value, and tree search

Setting: Two player Markov Games:

$$\mathscr{M} = \{S, A, f, r, H, s_0\}$$

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Min-max formulation:

$$\max_{\pi_1} \min_{\pi_2} \mathbb{E}\left[r(s_H) \,|\, \pi_1, \pi_2\right]$$

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Denote optimal value function $V^\star$ as:

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Player 2 tries to minimize the expected win rate of player 1,
which is equivalent to maximizes its own win rate

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Thus, we cannot enumerate, we must **generalize via function approximation..**

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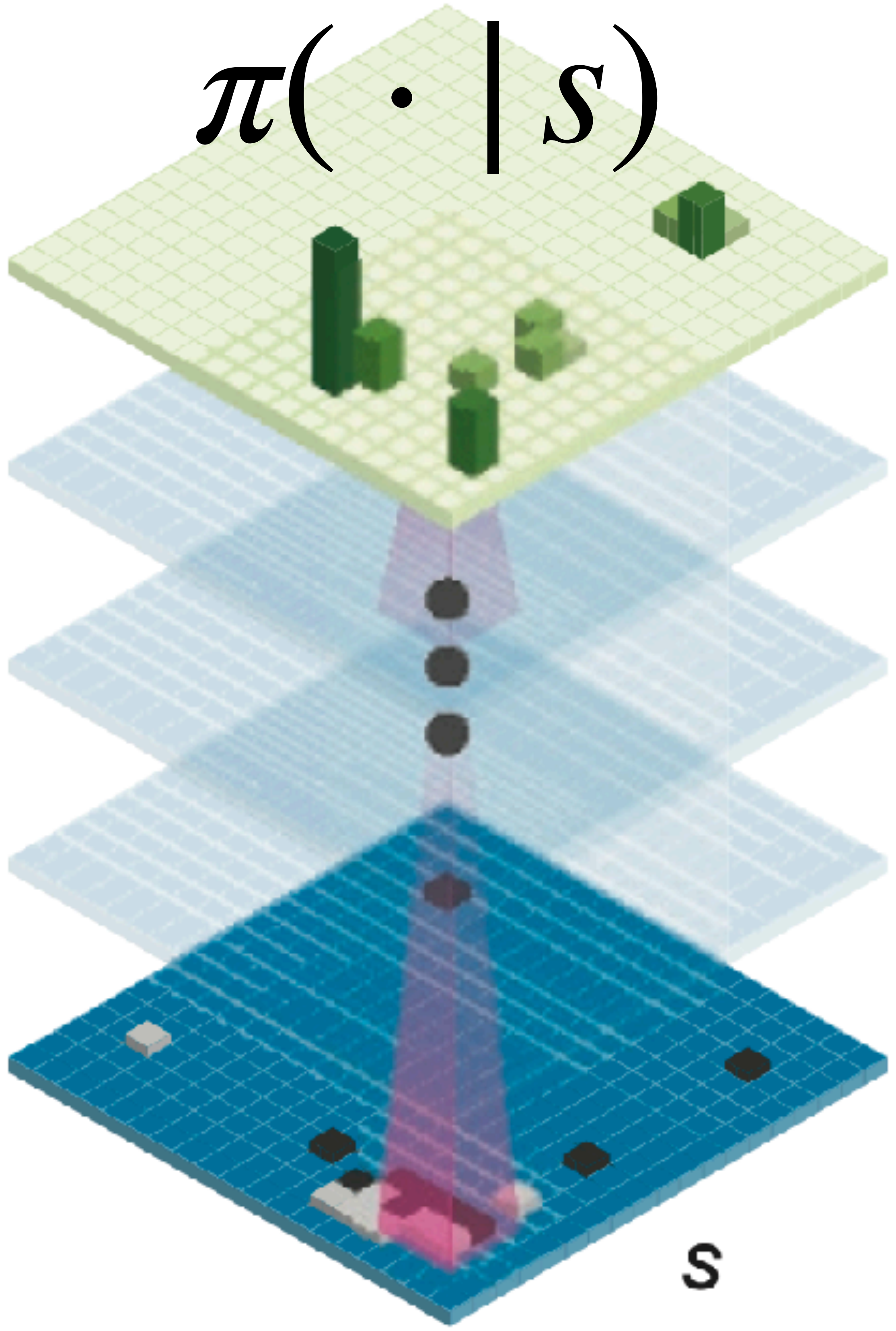
1. Policy Network $\approx \pi^{\star}$



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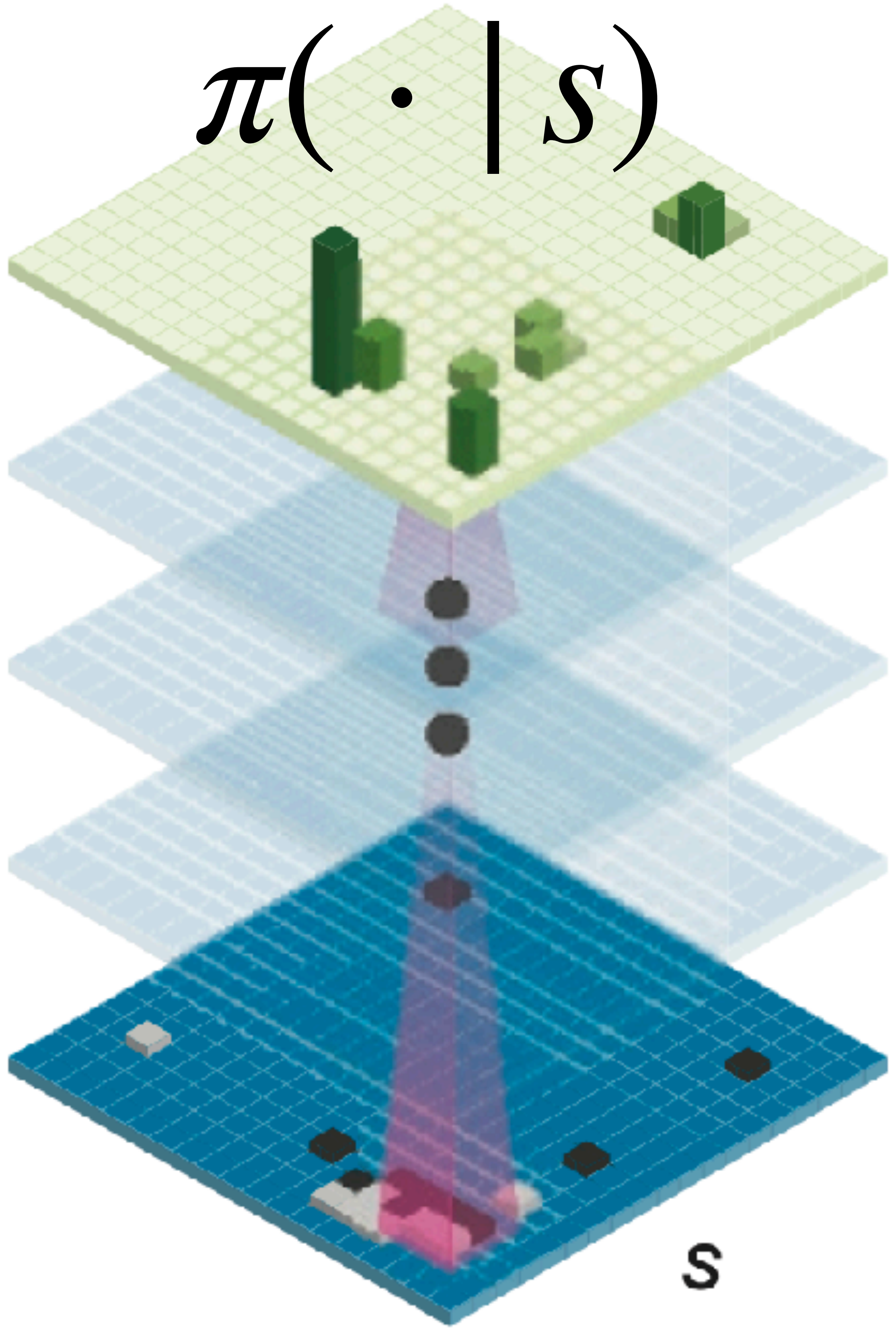
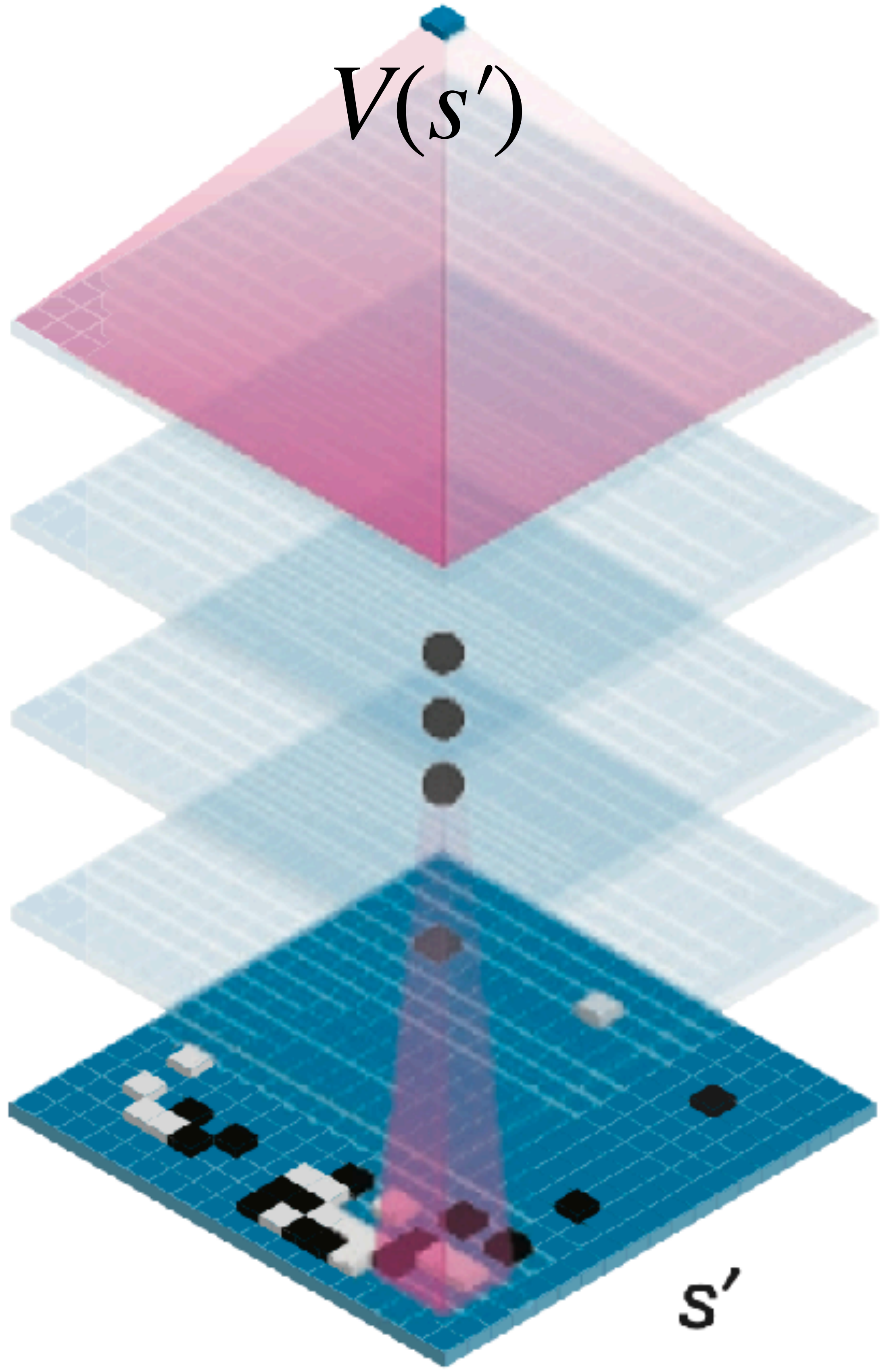


2. Value Network $\approx V^\star(s')$



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Behavior Cloning!

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Win rate: 11%

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PG update: $\theta_{t+1} = \theta_t + \eta \sum_{h: a_h \sim \pi_{\theta_t}} \nabla_\theta \ln \pi_{\theta_t}(a_h \mid s_h) r(s_H)$

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RL policy has win rate 85%

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Denote the PG policy as $\hat{\pi}$, we will approximate $V^{\hat{\pi}}$ instead:

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(We only keep one sample per game play, i.e., we are really sampling $s \sim d^{\hat{\pi}}$ i.i.d)

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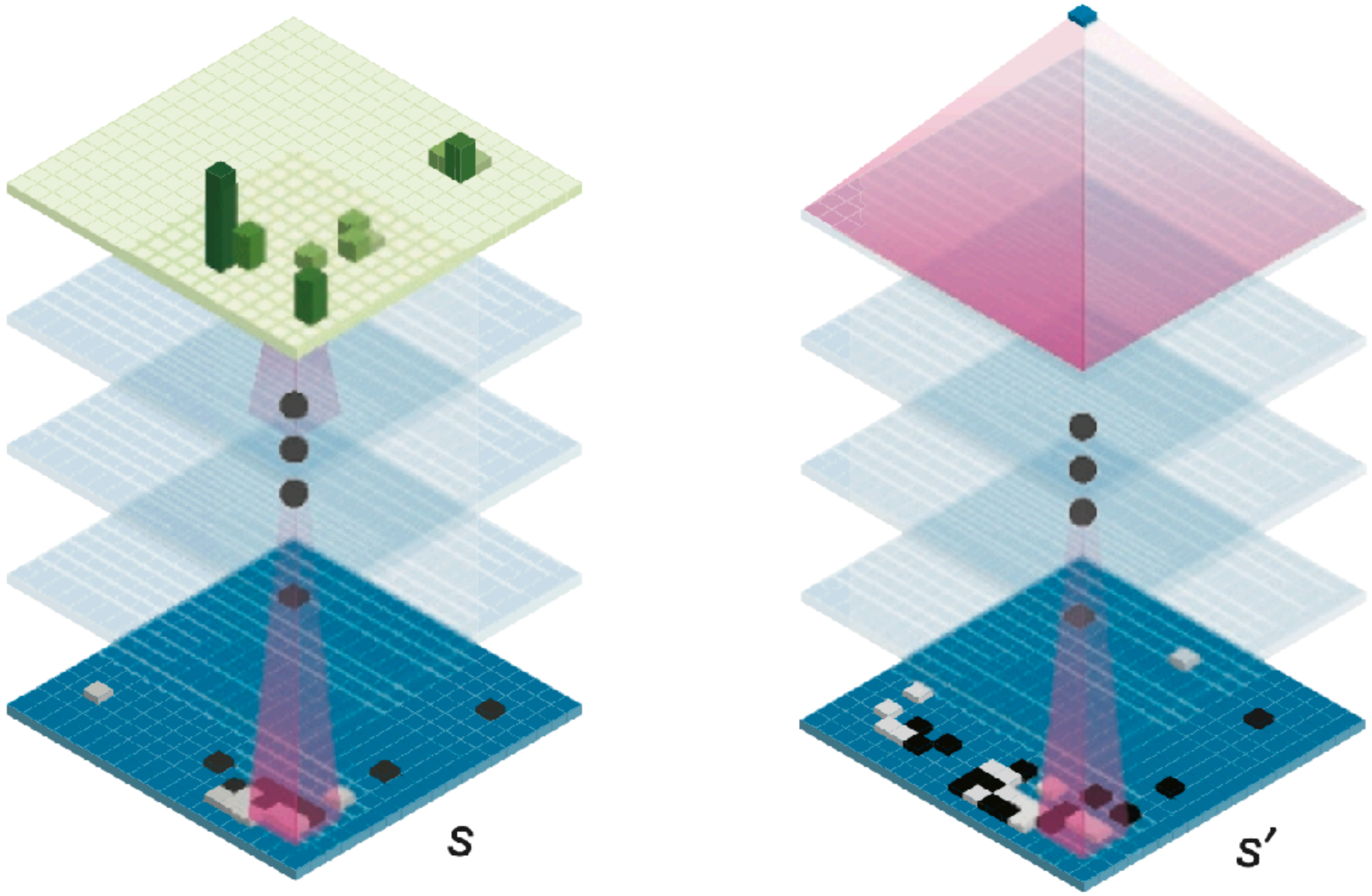
Optimize least square via SGD again:

$$\beta_{t+1} = \beta_t - \eta \sum_{(s,z)\in B} (V_\beta(s) - z)\, \nabla_\beta V_\beta(s)$$

Summary so far

We have learned a policy $\hat{\pi}$ (BC+PG) and $\hat{V} \approx V^{\hat{\pi}}$



To make the program even more powerful, we combine them with a **Search Tree**

Combine with Tree Search (a naive version)

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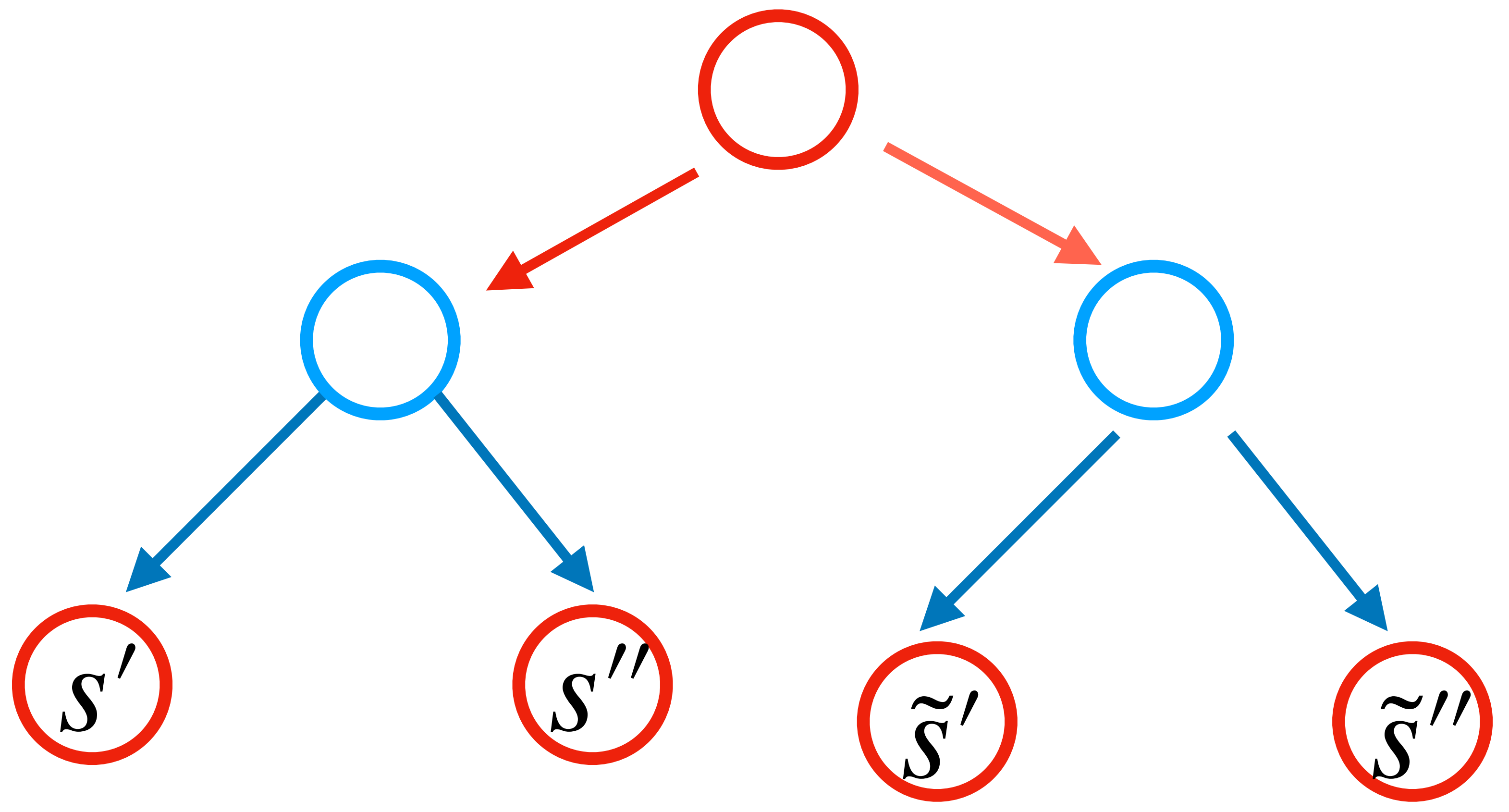

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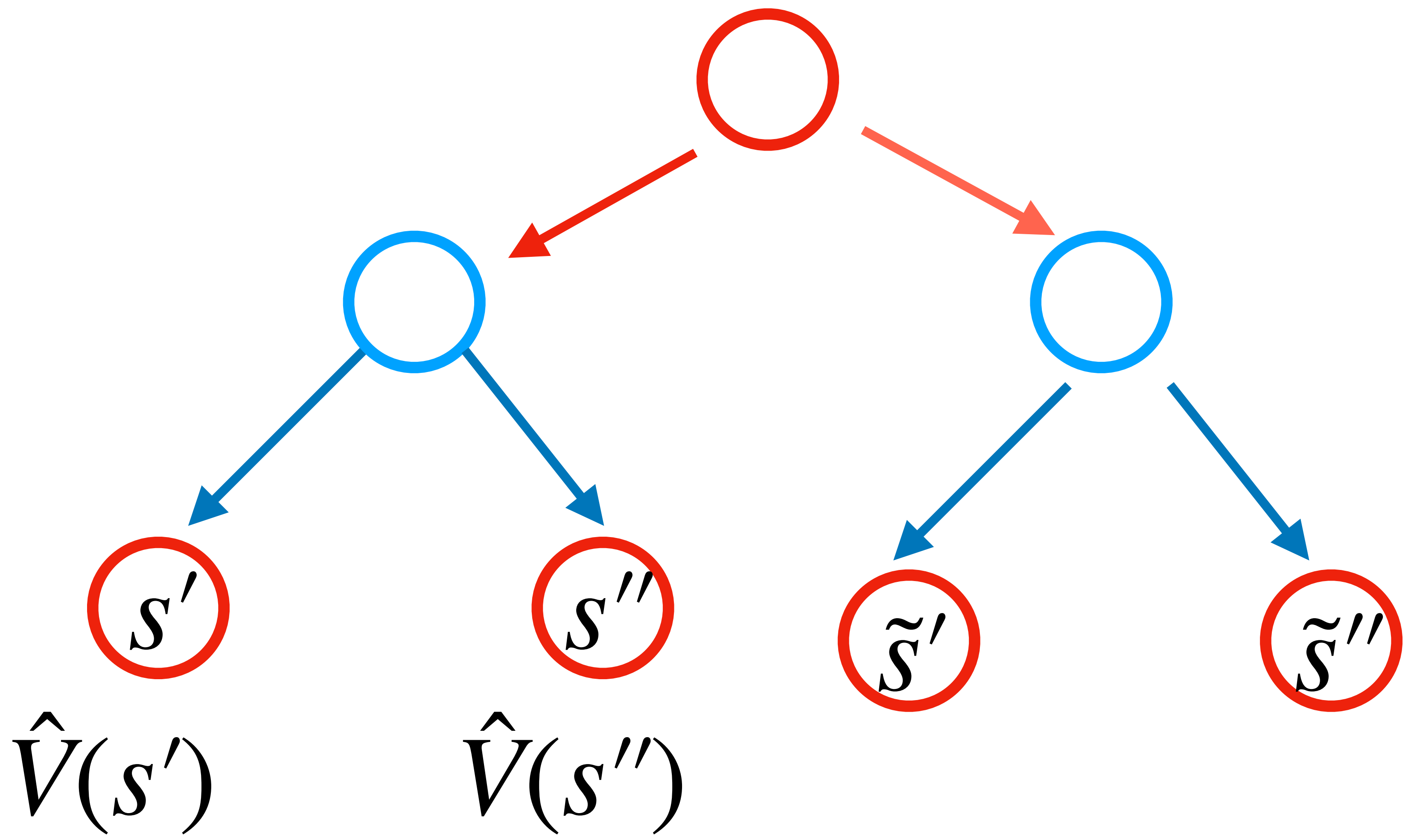

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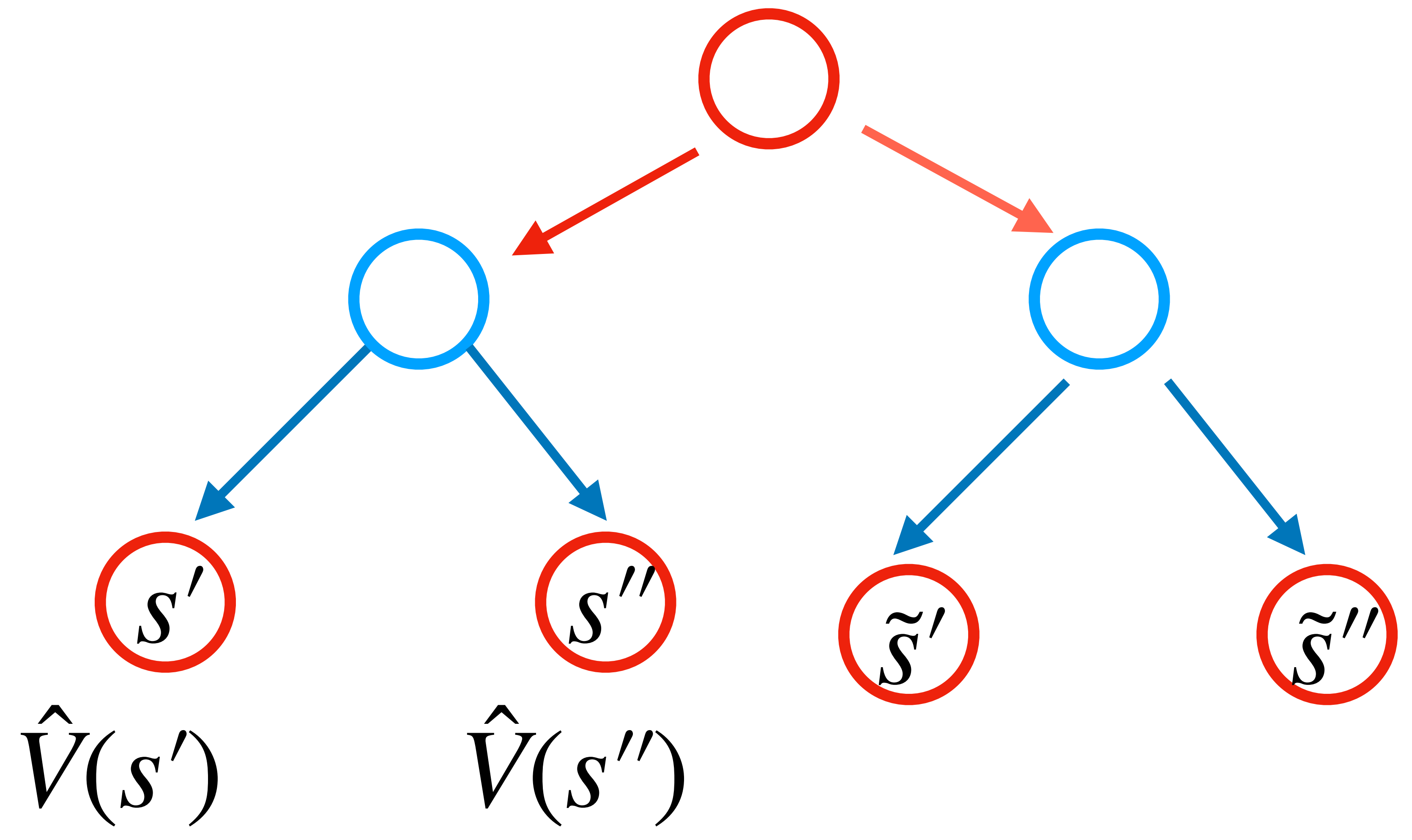

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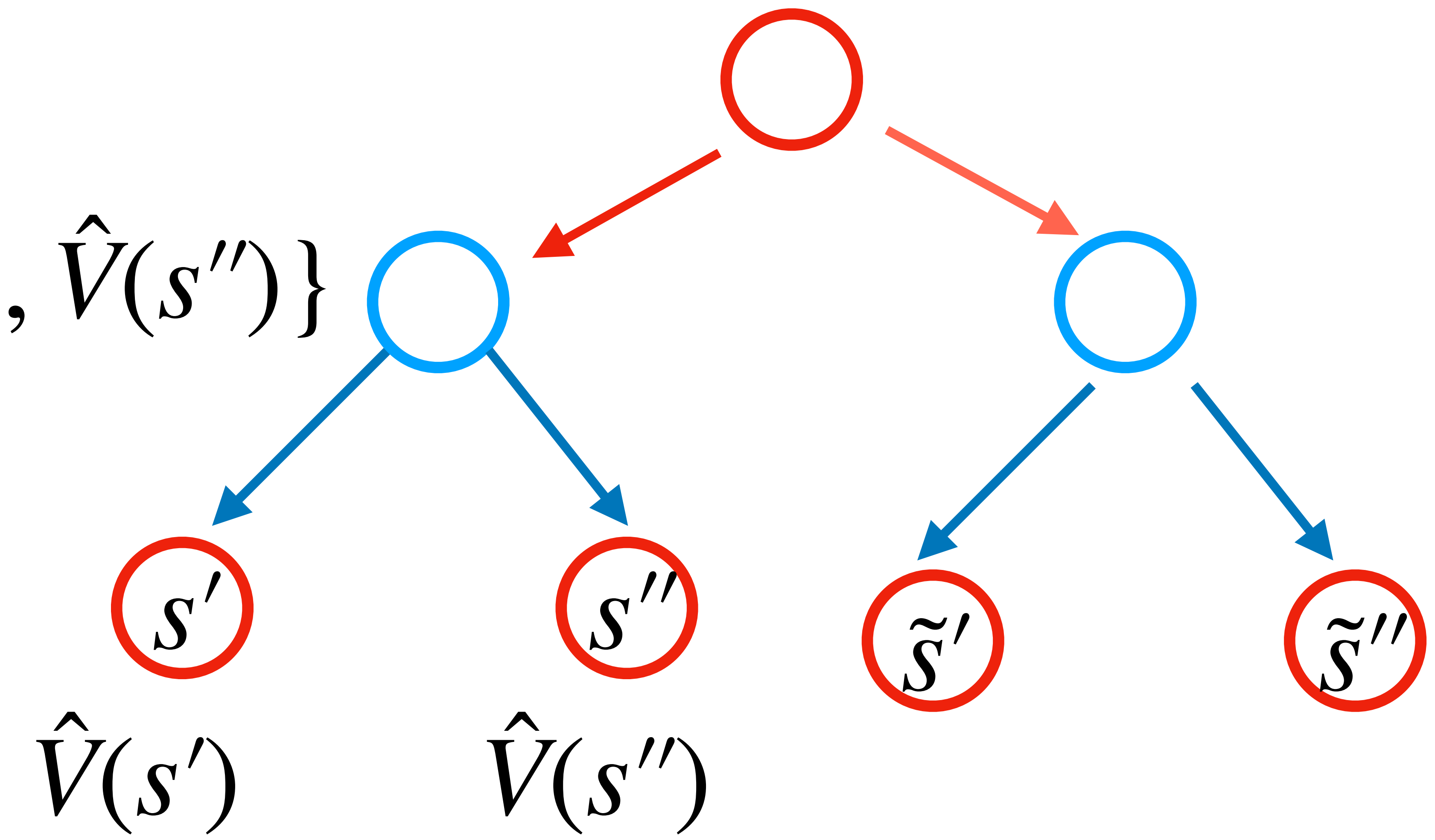
$$v_L = \min\{\hat{V}(s'), \hat{V}(s'')\}$$



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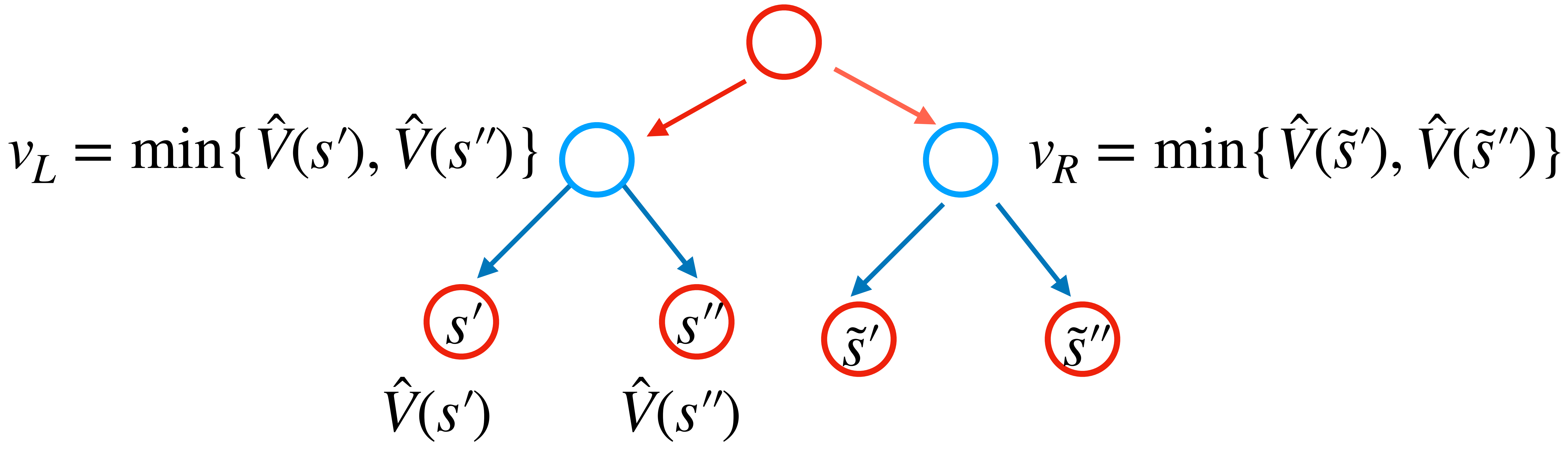

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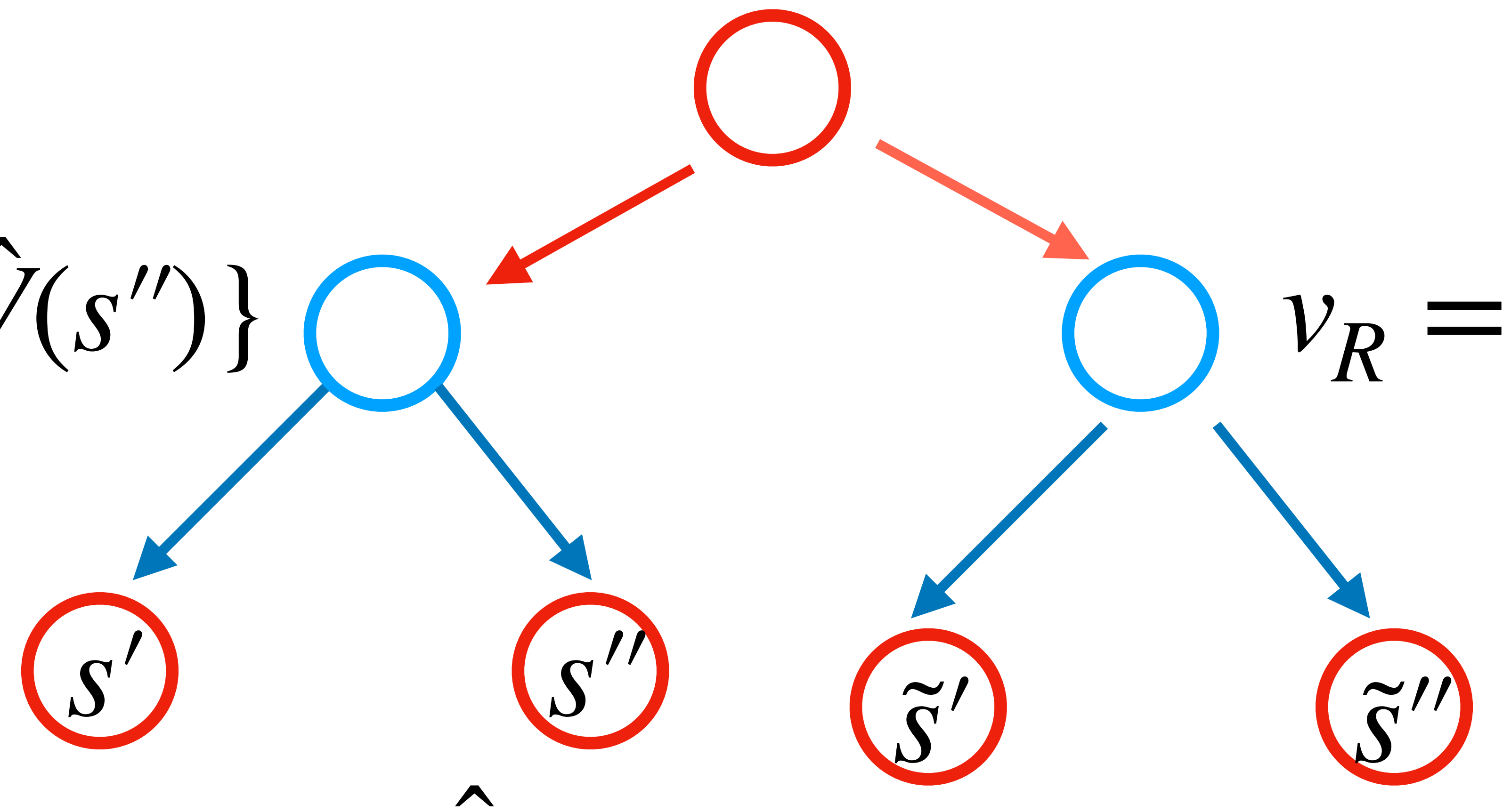
$$v_{root} = \max\{v_L, v_R\}$$

$$v_L = \min\{\hat{V}(s'), \hat{V}(s'')\} \qquad v_R = \min\{\hat{V}(\tilde{s}'), \hat{V}(\tilde{s}'')\}$$



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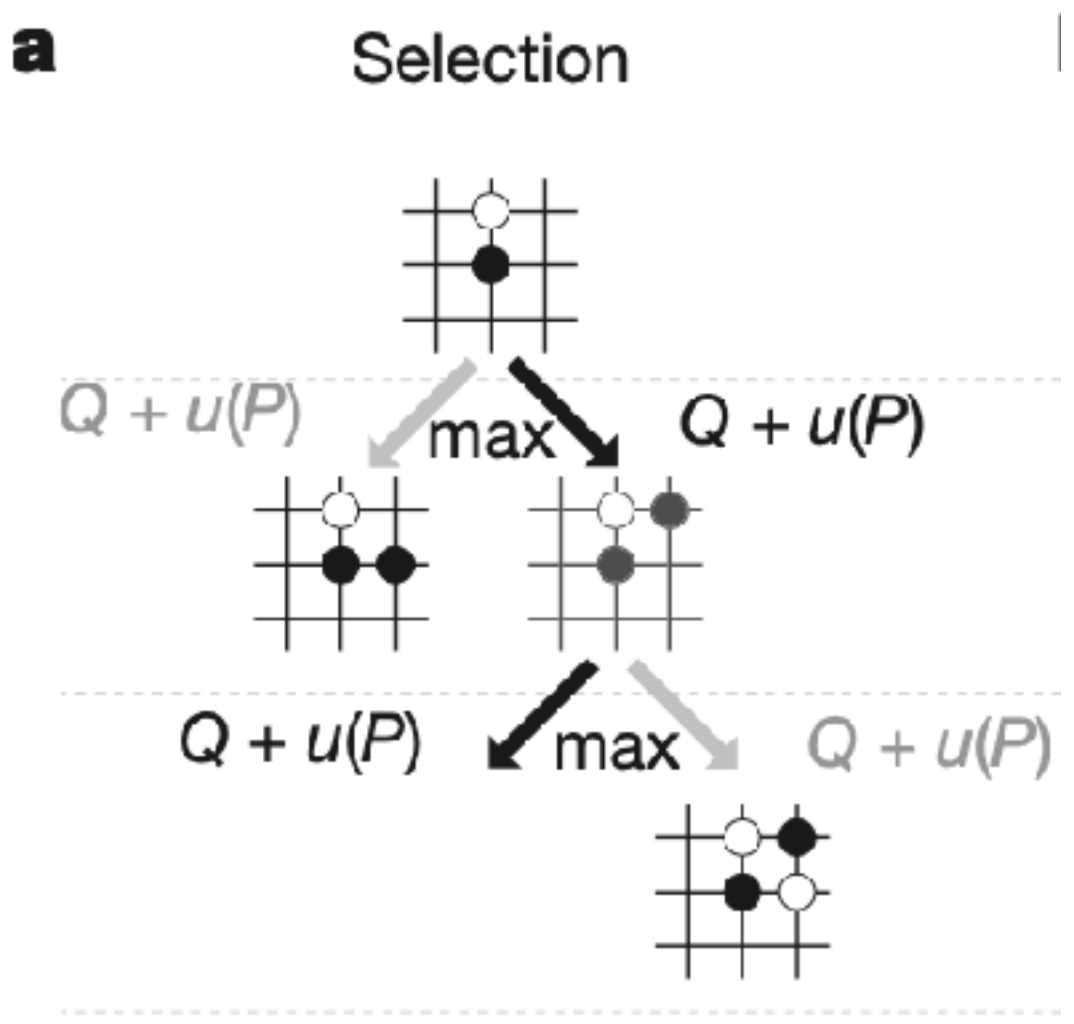

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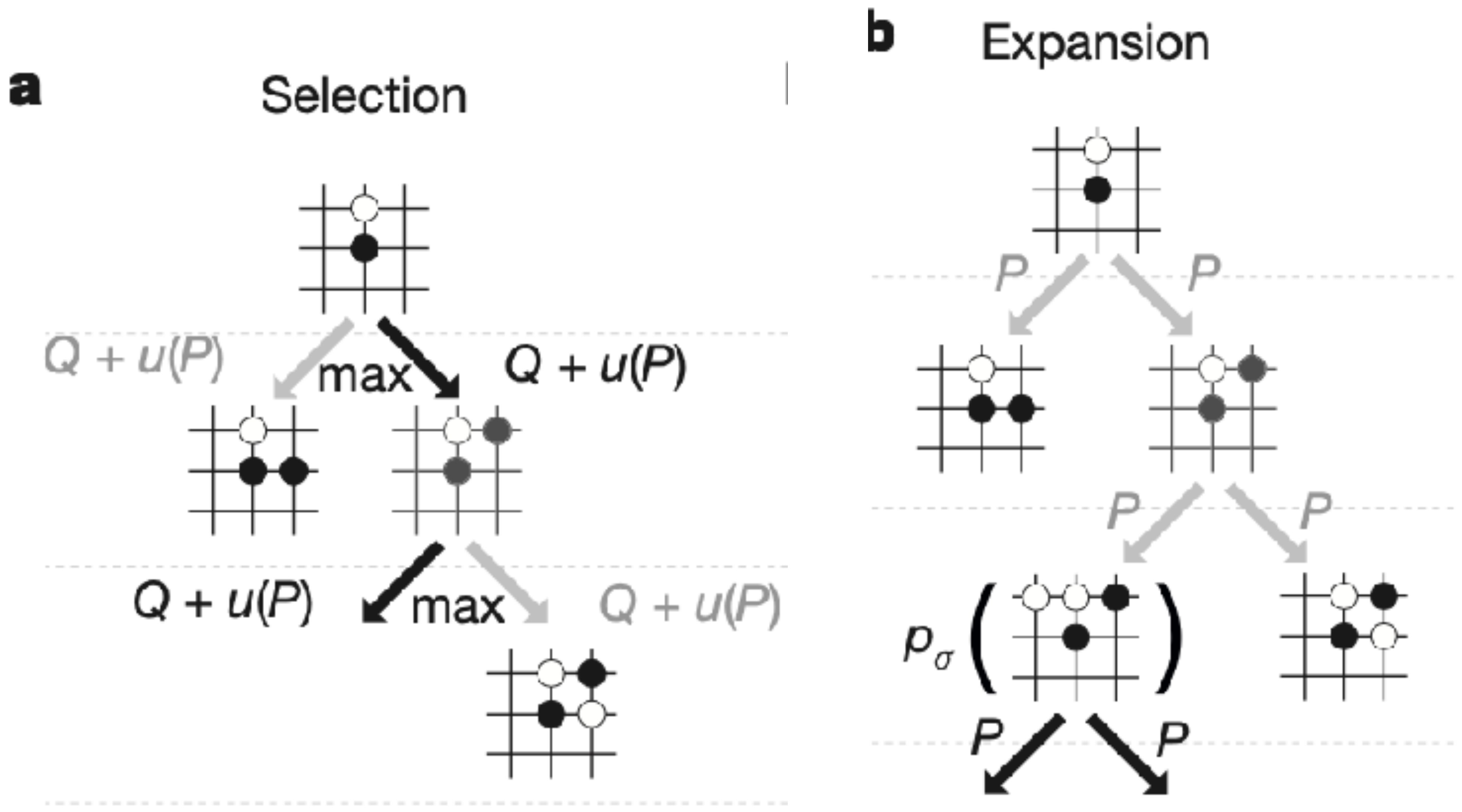

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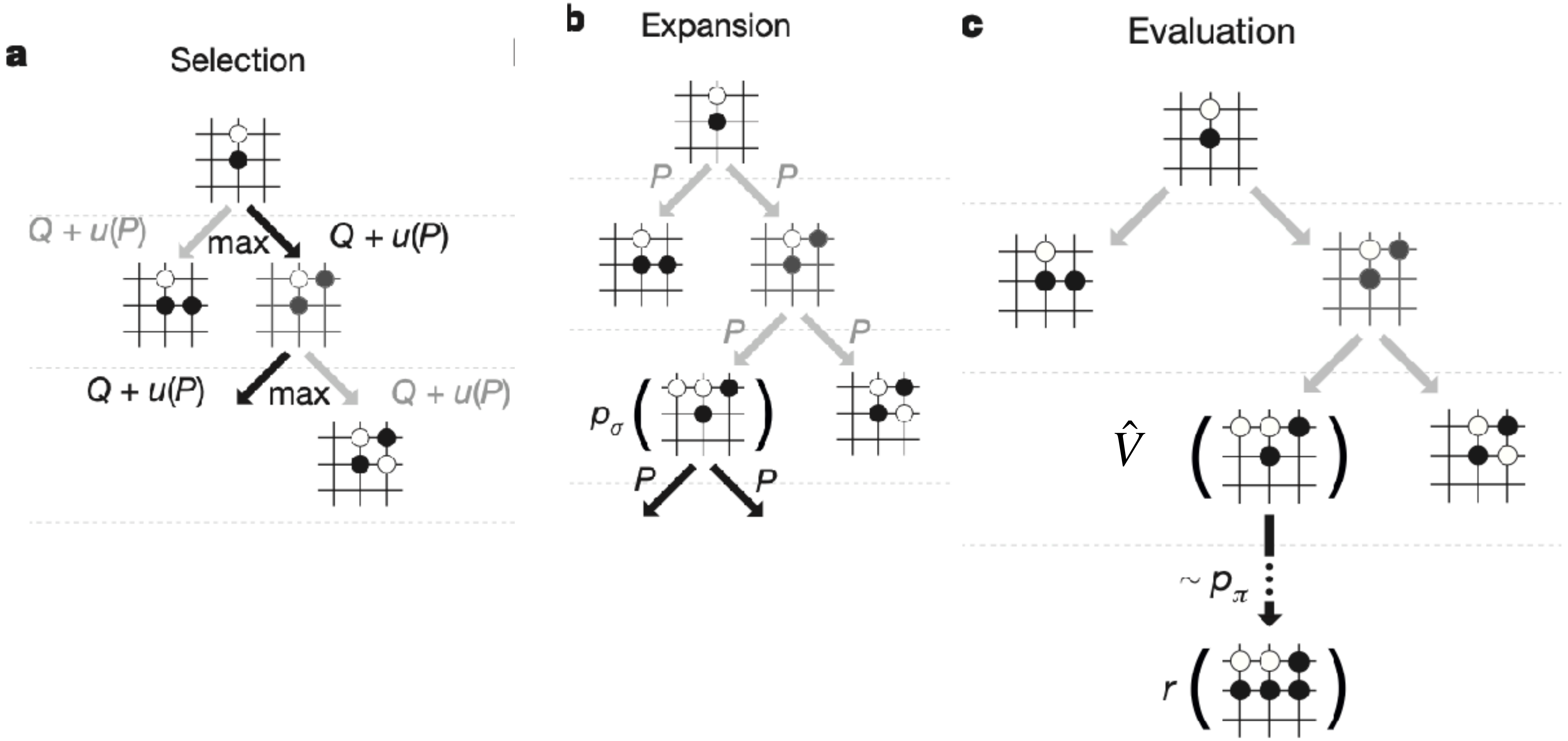

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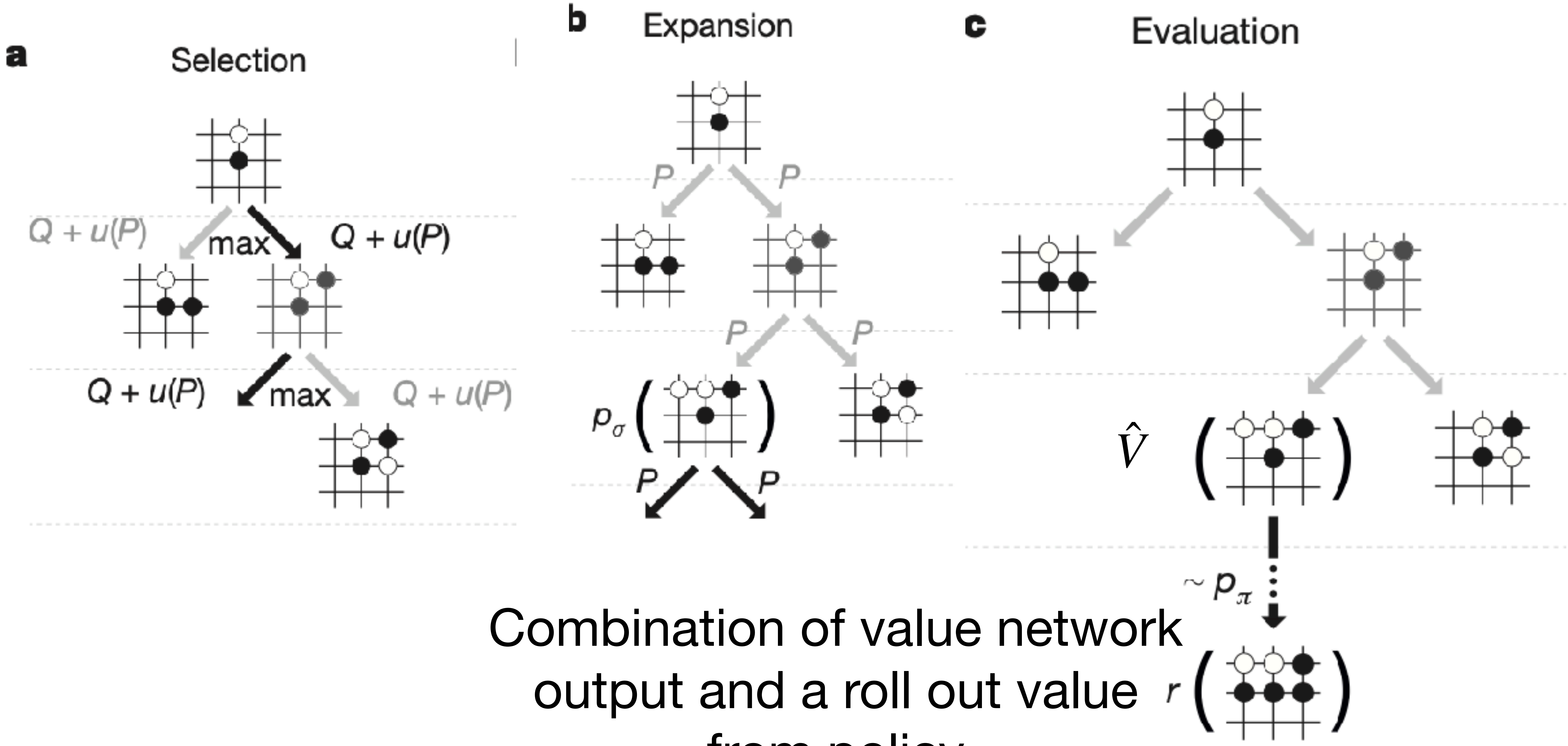

Combination of value network output and a roll out value from policy

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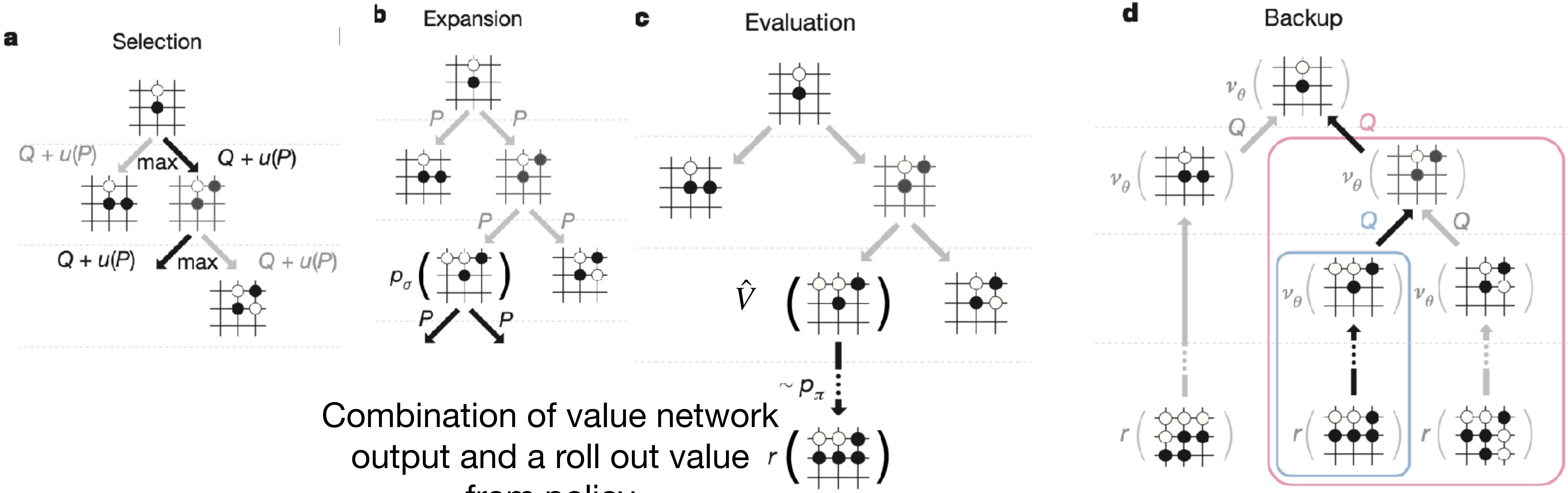

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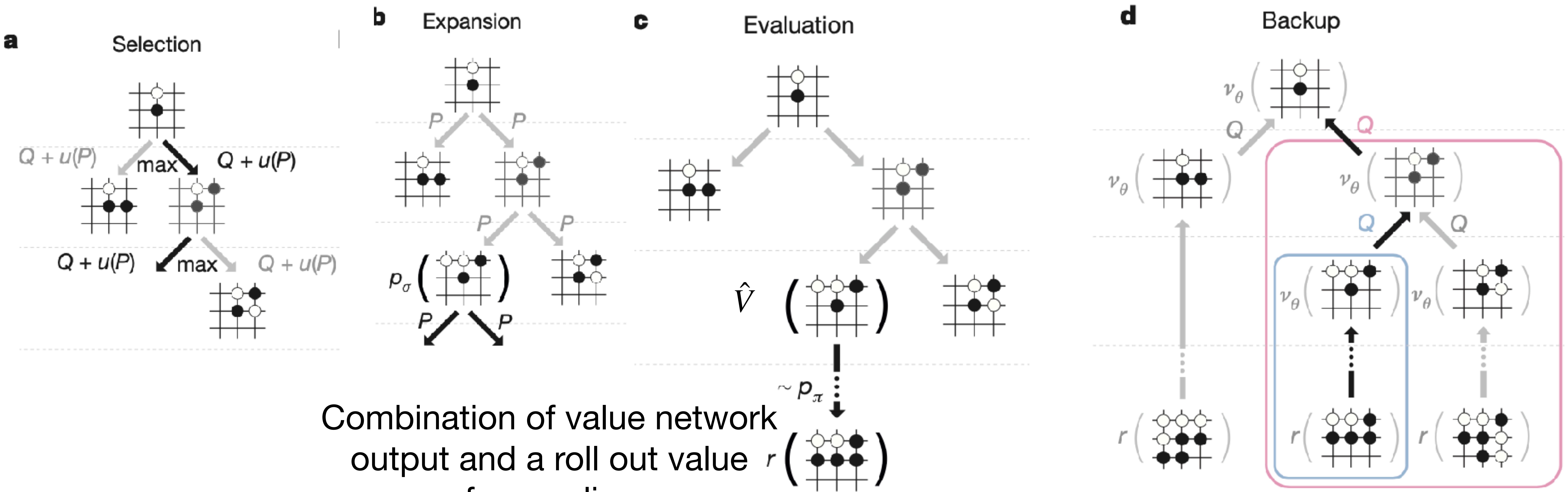

Combination of value network output and a roll out value from policy

i.e., we enumerate and plan for several steps into the future, and bottom up by a predicted outcome

Summary of the AlphaGo Program

1. Behavior cloning on 30m expert data samples

2. Classic Policy gradient on self-play games

3. Train a value network $\hat{V}$ to predict PG policy's outcome (on 30m self-played games)

4. Build search tree and use $\hat{V}$ to significantly reduce the search tree depth