Case Study: AlphaGo
Outline for Today:

1. Setting

2. The imitation learning component

3. The policy Gradient Component

4. The combination of policy, value, and tree search
Setting: Two player Markov Games:

\[ \mathcal{M} = \{S, A, f, r, H, s_0\} \]
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We have two players \( \pi_1 \) and \( \pi_2 \), they take turn to play:

\[ \begin{align*}
  s_0, & \quad a_0 \sim \pi_1(s_0), & s_1 = f(s_0, a_0), & \quad a_1 \sim \pi_2(s_1), & s_2 = f(s_1, a_1), & \ldots, & s_H
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Sparse reward at the termination state: \( r(s_H) = 1 \) if wins, -1 otherwise
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Min-max formulation:

\[ \max_{\pi_1} \min_{\pi_2} \mathbb{E} \left[ r(s_H) \mid \pi_1, \pi_2 \right] \]
Setting: Two player Markov Games:

Denote optimal value function $V^*$ as:

$$V^*(s) = \max_{\pi_1, \pi_2} \min \mathbb{E}[r(s_H) \mid s_0 = s, \pi_1, \pi_2]$$
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The optimal game value if we start at $s$, and both player plays optimally...
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Player 2 tries to minimize the expected win rate of player 1, which is equivalent to maximizes its own win rate
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Go has known and deterministic dynamic, i.e., $s' = f(s, a)$ is known and simple, in theory we can do **Dynamic Programming** to solve the max-min formulation.
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But...

For Go, $H \approx 150$, $|A| \approx 250$, and $|S| \approx |A|^H$
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Thus, we cannot enumerate, we must **generalize via function approximation**.
Setting: Function Approximation

1. Policy Network $\approx \pi^*$

\[ \pi(s|\cdot) \]
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$$\pi(\cdot \mid s)$$

2. Value Network $\approx V^*(s')$

$$V(s')$$
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Warm start our policy net via Imitation Learning
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\min_{\pi} \sum_{s,a} - \ln \pi(a | s)
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3. Optimize via Stochastic Gradient Descent:

$$\theta_{t+1} = \theta_t - \eta \sum_{(s,a) \in B} \nabla_{\theta} \left( -\ln \pi_{\theta_t}(a | s) \right) / |B|$$
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How well can it predict expert moves on a hold out test dataset?

It achieves 57% accuracy on expert test dataset.
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How well does this BC policy perform?
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Test it against the open-source Go program: Pachi (ranked 2 amateur dan on KGS)
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How well does this BC policy perform?

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Win rate: 11%
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Further Improving Policy via PG on Self-playing
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For \( t = 0 \rightarrow T - 1 \)
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Play \( \pi_{\theta_t} \) against \( \pi_{\theta_\tau} \), get a trajectory \((s_0, a_0, s_1, a_1', s_2, a_2, s_3, a_3', \ldots s_H)\)
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   **PG update:** \( \theta_{t+1} = \theta_t + \eta \sum_{h:a_h \sim \pi_{\theta_t}} \nabla_{\theta} \ln \pi_{\theta_t}(a_h | s_h)r(s_H) \)  

   (\# fictitious play to avoid catastrophic forgetting..)
How does the performance improved after PG optimization?
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Test it against the open-source Go program: Pachi (ranked 2 amateur dan on KGS)

RL policy has win rate 85%
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4. The combination of policy, value, and tree search
Final stage of training: Learn a value function $\hat{V}(s) \approx V^*$

Denote the PG policy as $\hat{\pi}$, we will approximate $V^{\hat{\pi}}$ instead:

$$V^{\hat{\pi}}(s) = \mathbb{E} \left[ r(s_H) \mid s_0 = s, \hat{\pi}, \hat{\pi} \right]$$
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i.e., the value of the game when both players play $\hat{\pi}$, starting at $s$
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We use simple least square regression here:

$$\min_\beta \sum_{s,z} (V_\beta(s) - z)^2$$
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Where $s$ is a random state in one game play, and $z$ is the outcome of the play.
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Where $s$ is a **random state in one game play**, and $z$ is the outcome of the play.. (We only keep one sample per game play, i.e., we are really sampling $s \sim d^{\hat{\pi}}$ i.i.d)
Final stage of training: Learn a value function $V(s) \approx V^*$

Self-play 30m games, and get 30m $(s, z)$ pairs
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Self-play 30m games, and get 30m $(s, z)$ pairs

Optimize least square via SGD again:

$$\beta_{t+1} = \beta_t - \eta \sum_{(s, z) \in B} (V_\beta(s) - z) \nabla_\beta V_\beta(s)$$
Summary so far

We have learned a policy $\hat{\pi}$ (BC+PG) and $\hat{V} \approx V^{\hat{\pi}}$

To make the program even more powerful, we combine them with a **Search Tree**
Combine with Tree Search (a naive version)

Imagine that we are at state $s$ right now, let’s simulate all possible moves into the future
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\[
\hat{V}(s') \quad \hat{V}(s'')
\]

$\hat{V}(s')$: win rate of red player starting at $s'$
Combine with Tree Search (a naive version)

Imagine that we are at state $s$ right now, let’s simulate all possible moves into the future.

$v_L = \min\{\hat{V}(s'), \hat{V}(s'')\}$

$\hat{V}(s')$: win rate of red player starting at $s'$

\[ \hat{V}(s') = \min\{\hat{V}(s'), \hat{V}(s'')\} \]

\[ \hat{V}(s'') \]
Combine with Tree Search (a naive version)

Imagine that we are at state $s$ right now, let’s simulate all possible moves into the future.

\[ v_L = \min \{ \hat{V}(s'), \hat{V}(s'') \} \]
\[ v_R = \min \{ \hat{V}(\tilde{s}'), \hat{V}(\tilde{s}'') \} \]

$\hat{V}(s')$: win rate of red player starting at $s'$
Combine with Tree Search (a naive version)

Imagine that we are at state $s$ right now, let’s simulate all possible moves into the future

$$v_{root} = \max\{v_L, v_R\}$$

$$v_L = \min\{\hat{V}(s'), \hat{V}(s'')\}$$

$$v_R = \min\{\hat{V}(\tilde{s}'), \hat{V}(\tilde{s}'')\}$$

$\hat{V}(s')$: win rate of red player starting at $s'$
AlphaGo uses Monte-Carlo Tree Search algorithm:
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(a) Selection

\[ Q + u(P) \text{ max } Q + u(P) \]

(b) Expansion

\[ p_\sigma \left( \bullet \right) \]

\[ P \]

\[ P \]

(c) Evaluation

\[ \hat{V} \]

\[ \sim p_\pi \]

\[ r \]
AlphaGo uses Monte-Carlo Tree Search algorithm:

Combination of value network output and a roll out value from policy.
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i.e., we enumerate and plan for several steps into the future, and bottom up by a predicted outcome

Combination of value network output and a roll out value from policy
Summary of the AlphaGo Program

1. Behavior cloning on 30m expert data samples

2. Classic Policy gradient on self-play games

3. Train a value network $\hat{V}$ to predict PG policy’s outcome (on 30m self-played games)

4. Build search tree and use $\hat{V}$ to significantly reduce the search tree depth