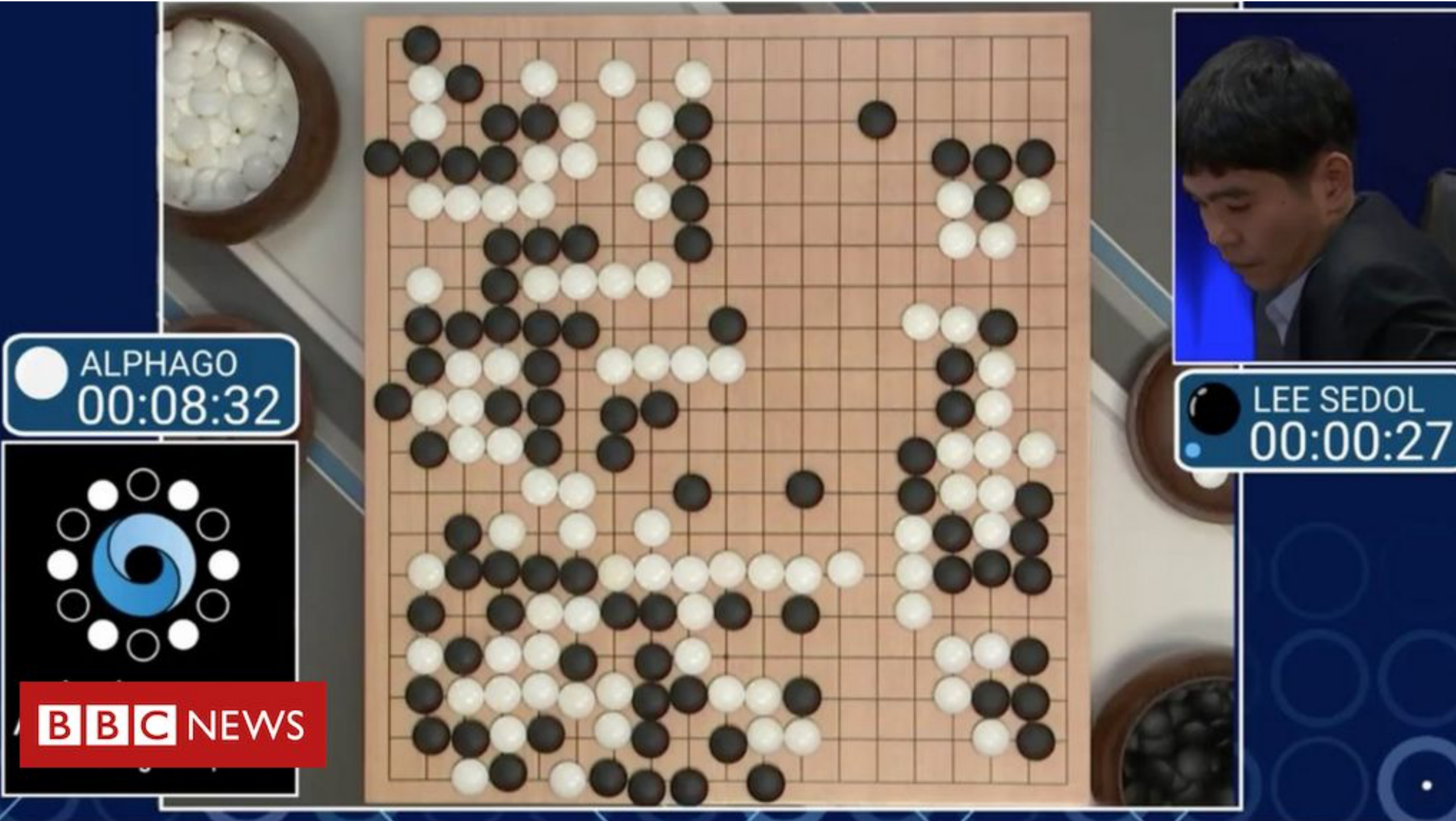
Case Study: AlphaGo


ALPHAGO
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LEE SEDOL
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BBC NEWS

Outline for Today:

1. Setting

2. The imitation learning component

3. The policy Gradient Component

4. The combination of policy, value, and tree search

Setting: Two player Markov Games:

$$\mathscr{M} = \{S, A, f, r, H, s_0\}$$

$s' = f(s, a)$

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player 1

player 2

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Min-max formulation:

$$\max_{\pi_1} \min_{\pi_2} \mathbb{E}\left[r(s_H) \mid \pi_1, \pi_2\right]$$

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Denote optimal value function $V^\star$ as:

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Player 2 tries to minimize the expected win rate of player 1,
which is equivalent to maximizes its own win rate

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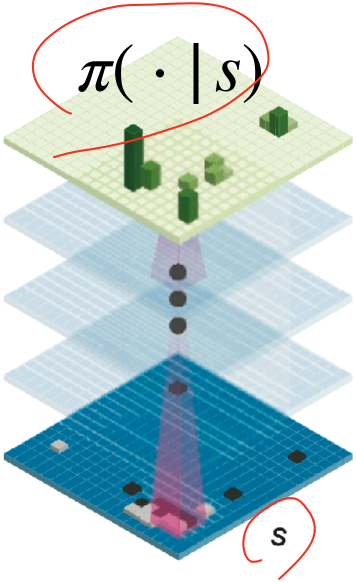
Thus, we cannot enumerate, we must **generalize via function approximation..**

Setting: Function Approximation

1. Policy Network $\approx \pi^\star$



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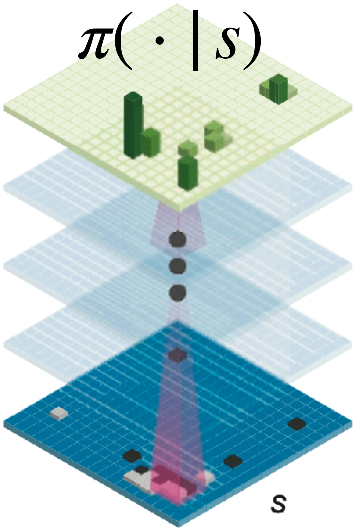
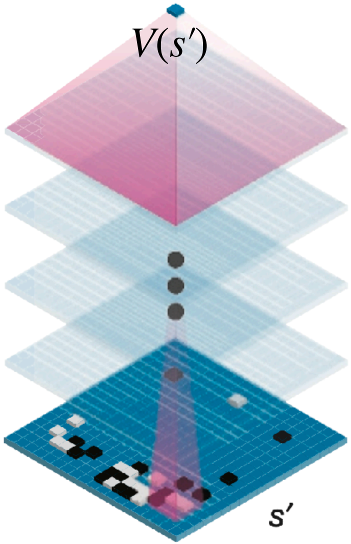


2. Value Network $\approx V^\star(s')$



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3. Optimize via Stochastic Gradient Descent:

$$\theta_{t+1} = \theta_t - \eta \sum_{(s,a)\in B} \nabla_\theta \left(-\ln \pi_{\theta_t}(a \mid s) \right) / |B|$$

unbiased est of Gradient

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Behavior Cloning!

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It achieves 57% accuracy on expert test dataset

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Win rate: 11%

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Terminate state

Play $\underbrace{\pi_{\theta_t}}_{\text{player 1}}$ against $\underbrace{\pi_{\theta_\tau}}_{\text{player 2}}$, get a trajectory $(s_0, \underbrace{a_0}_{\pi_{\theta_t}}, s_1, \underbrace{a_1'}_{\pi_{\theta_\tau}}, s_2, \underbrace{a_2}_{\pi_{\theta_t}}, s_3, a_3' \ldots s_H)$

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Play π_{θ_t} against π_{θ_τ}, get a trajectory $(s_0, a_0, s_1, a_1', s_2, a_2, s_3, a_3' \ldots s_H)$

PG update: $\theta_{t+1} = \theta_t + \eta \sum_{h: a_h \sim \pi_{\theta_t}} \nabla_\theta \ln \pi_{\theta_t}(a_h \mid s_h) r(s_H)$

Reinforce

How does the performance improved after PG optimization?

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Test it against the open-source Go program: Pachi (ranked 2 amateur dan on KGS)

RL policy has win rate 85%

(BC: 11%)

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Final stage of training: Learn a value function $\hat{V}(s) \approx V^\star(s)$

Denote the PG policy as $\hat{\pi}$, we will approximate $V^{\hat{\pi}}$ instead:

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We use simple least square regression here:

$$\min_{\beta} \sum_{s,z} (V_\beta(s) - z)^2$$

parameters of value network

state from the Game

value of the Game

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(We only keep one sample per game play, i.e., we are really sampling $s \sim d^{\hat{\pi}}$ i.i.d)

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$(\hat{\pi}, \hat{\pi})$

Self-play 30m games, and get 30m (s, z) pairs

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Randomly sampled

Self-play 30m games, and get 30m (s, z) pairs

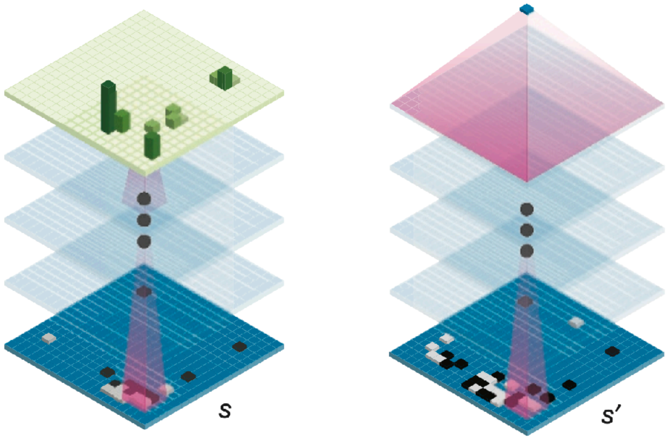
Optimize least square via SGD again:

$$\beta_{t+1} = \beta_t - \eta \sum_{(s,z)\in B} (V_{\beta_t}(s) - z)\,\nabla_\beta V_\beta(s)$$

$$\sum_{s.a} \left(V_\beta(s) - z \right)^2$$

Summary so far

We have learned a policy $\hat{\pi}$ (BC+PG) and $\hat{V} \approx V^{\hat{\pi}}$



To make the program even more powerful, we combine them with a **Search Tree**

Combine with Tree Search (a naive version)

Imagine that we are at state s right now, let's simulate all possible moves into the future

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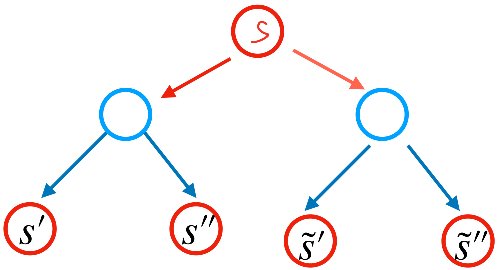
2 actions

$f(s, a) \leftarrow$ known



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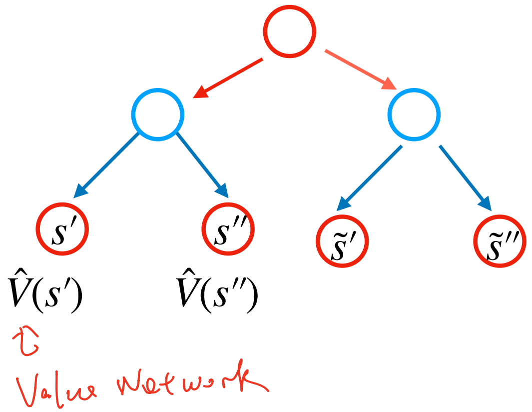

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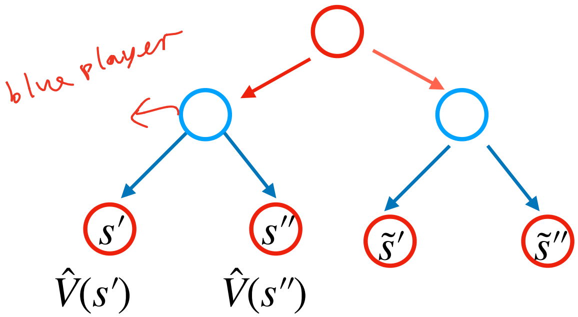

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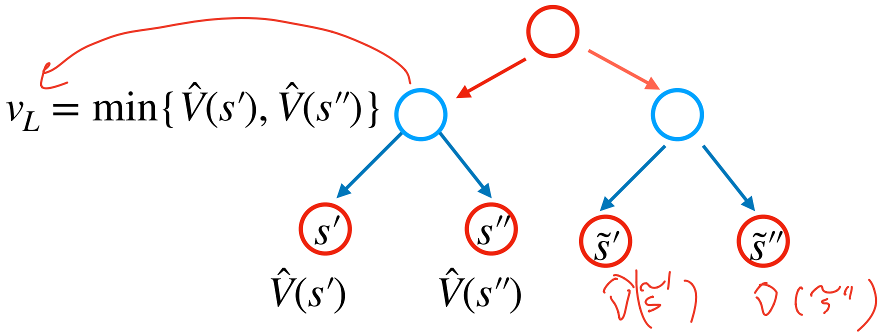

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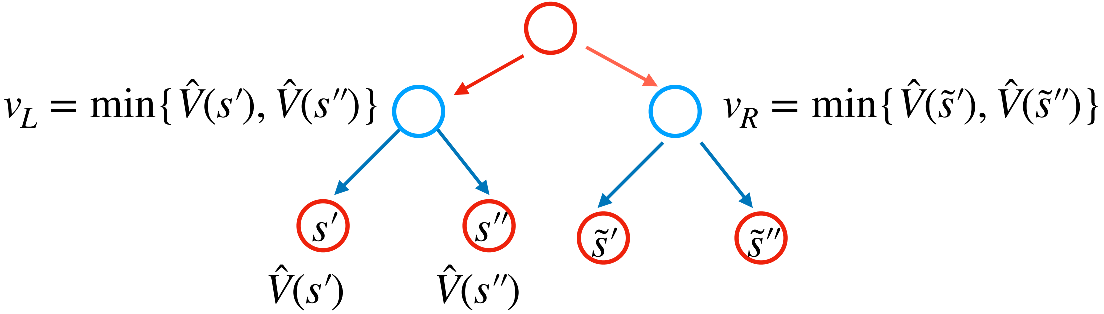

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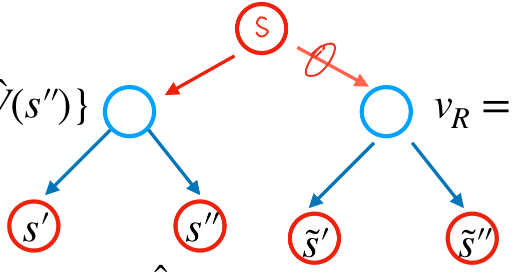
$$v_{root} = \max\{v_L, v_R\}$$

$$v_L = \min\{\hat{V}(s'), \hat{V}(s'')\} \qquad v_R = \min\{\hat{V}(\tilde{s}'), \hat{V}(\tilde{s}'')\}$$



$V^* \approx \hat{V}(s') \qquad \hat{V}(s'')$

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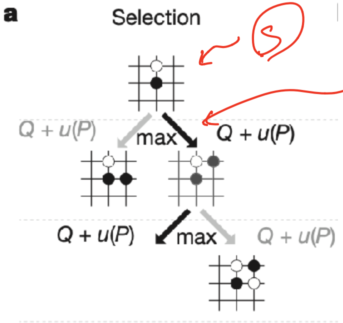

s

$$Q(s,a) + \sqrt{\frac{1}{N(s,a)}}$$

← # of times we tried (s,a)

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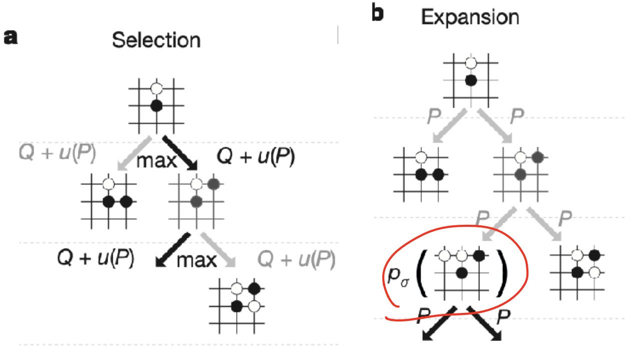

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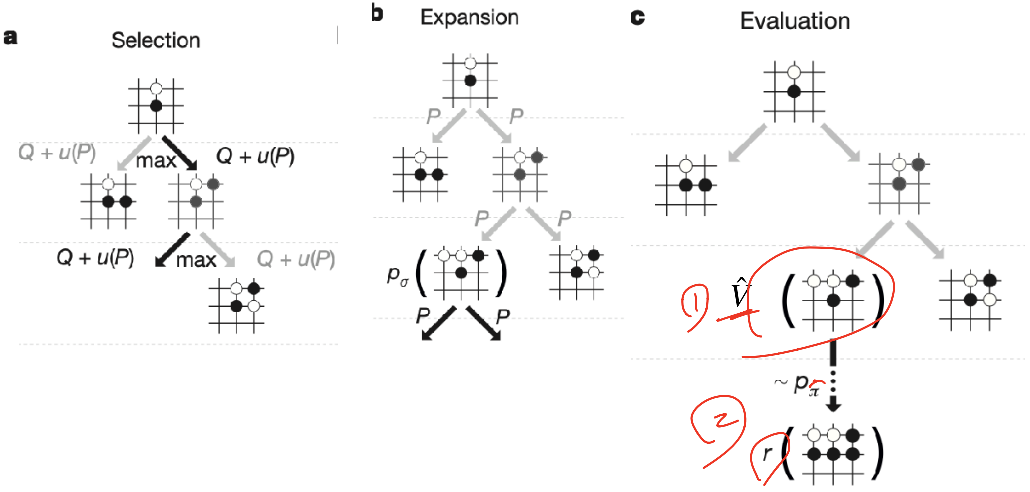

$$\lambda \hat{V}(s') + (1-\lambda) r$$

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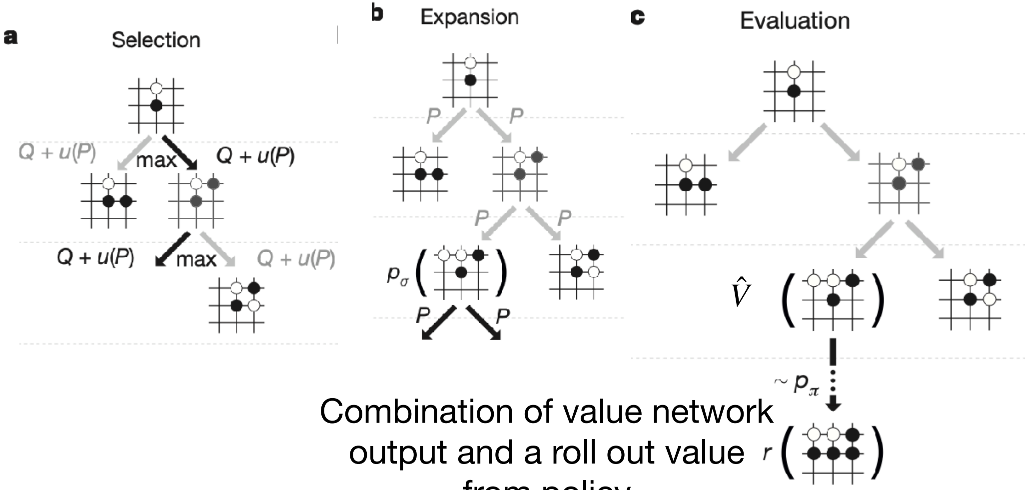

Combination of value network output and a roll out value from policy

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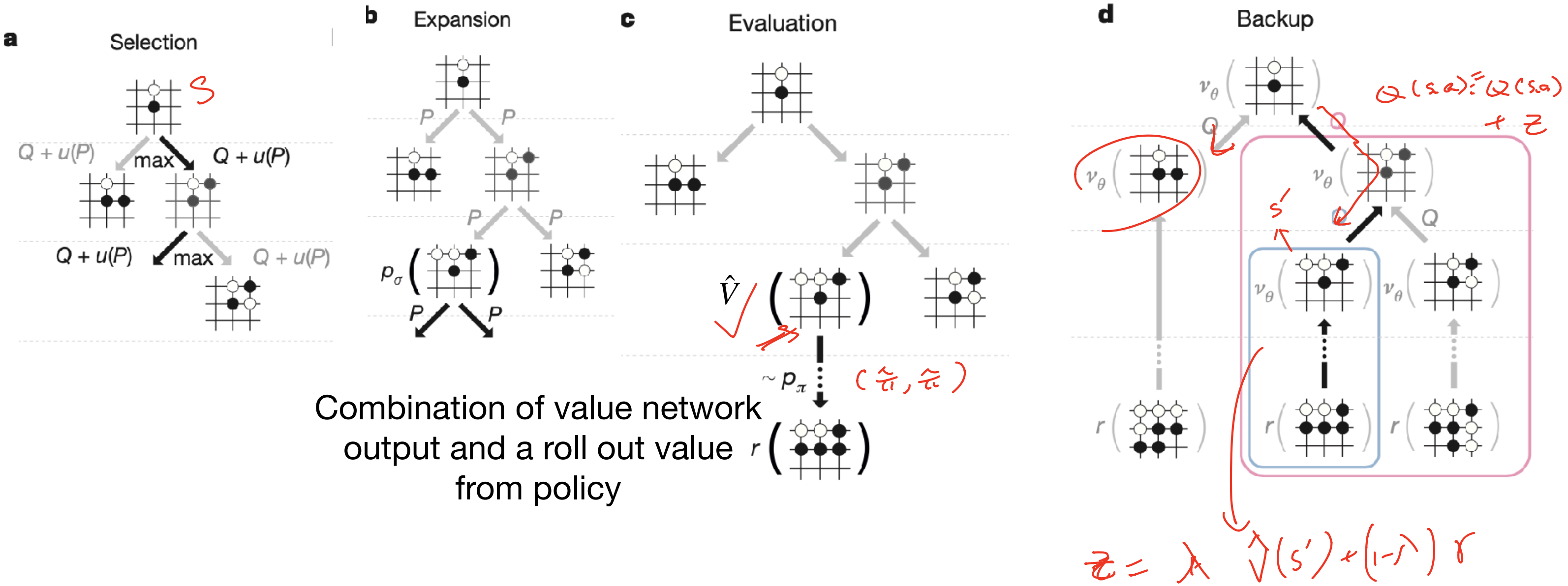

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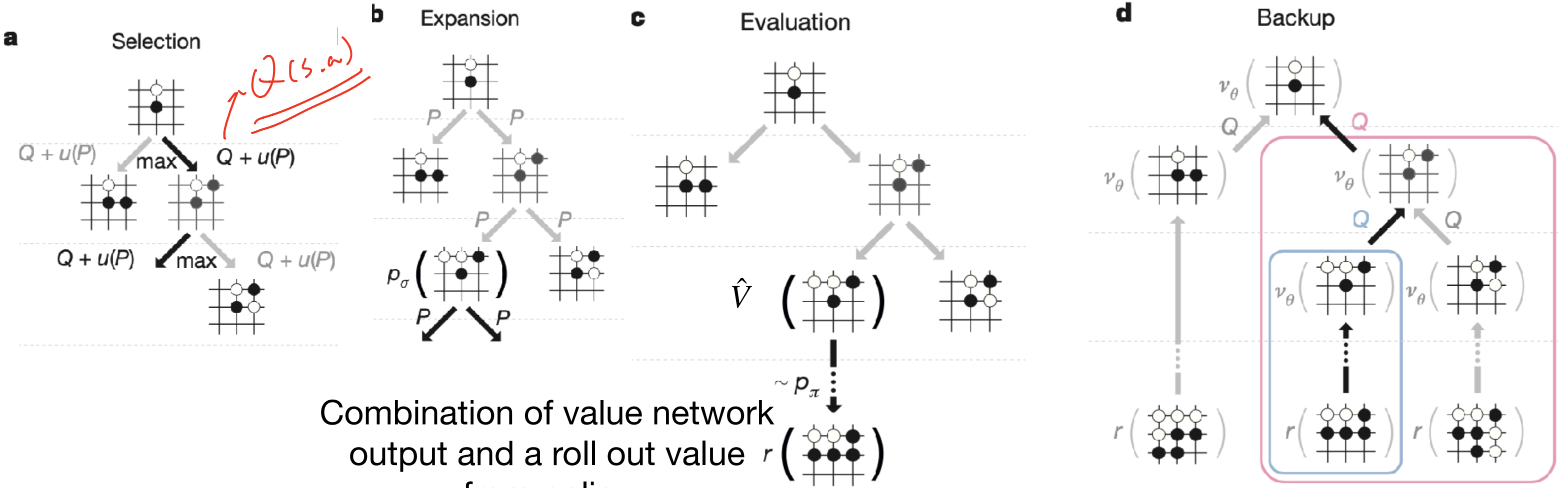

Combination of value network output and a roll out value from policy

i.e., we enumerate and plan for several steps into the future, and bottom up by a predicted outcome

Summary of the AlphaGo Program

1. Behavior cloning on 30m expert data samples

2. Classic Policy gradient on self-play games

3. Train a value network $\hat{V}$ to predict PG policy's outcome (on 30m self-played games)

4. Build search tree and use $\hat{V}$ to significantly reduce the search tree depth