Policy Gradient
(continue)

Recap: Policy Parameterization

Recall that we consider parameterized policy $\pi_\theta(\cdot \mid s) \in \Delta(A), \forall s$

1. Softmax linear Policy (We will try this in HW2)

Feature vector $\phi(s, a) \in \mathbb{R}^d$, and parameter $\theta \in \mathbb{R}^d$

$a \in \{+1, -1\}$

$$\pi_\theta(a \mid s) = \frac{\exp(\theta^\top \phi(s, a))}{\sum_{a'} \exp(\theta^\top \phi(s, a'))}$$

$$\frac{\exp(\theta^\top \phi(s,a))}{1 + \exp(\theta^\top \phi(s,a))}$$

2. Neural Policy:

Neural network $f_\theta : S \times A \mapsto \mathbb{R}$

$f_\theta \in [-\infty, \infty]$

$$\pi_\theta(a \mid s) = \boxed{\frac{\exp(f_\theta(s, a))}{\sum_{a'} \exp(f_\theta(s, a'))}}$$

In high level, think about π_θ as a classifier which has its parameters to be optimized

Recap: the REINFORCE Algorithm

$$\tau = \{s_0, a_0, s_1, a_1, \ldots, s_{H-1}, a_{H-1}\}$$

$$\rho_\theta(\tau) = \mu(s_0)\pi_\theta(a_0 \mid s_0)P(s_1 \mid s_0, a_0)\pi_\theta(a_1 \mid s_1)\ldots$$

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$$J(\pi_\theta) = \mathbb{E}_{\tau\sim\rho_\theta(\tau)} \underbrace{\left[\sum_{h=0}^{H-1} r(s_h, a_h)\right]}_{R(\tau)}$$

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$$\rho_\theta(\tau) = \mu(s_0)\pi_\theta(a_0 \mid s_0)P(s_1 \mid s_0, a_0)\pi_\theta(a_1 \mid s_1)\ldots$$

$$\nabla \ln \rho_\theta(\tau) = \nabla \left[\ln \mu(s_0) + \ln \pi_\theta(a_0 \mid s_0) + \ln P(s_1 \mid s_0, a_0) \ \ldots \right]$$

$$\nabla_\theta J(\pi_\theta)\big|_{\theta=\theta_0} := \mathbb{E}_{\tau \sim \rho_{\theta_0}(\tau)} \left[\left(\sum_{h=0}^{H-1} \nabla_\theta \ln \pi_{\theta_0}(a_h \mid s_h) \right) R(\tau) \right]$$

$$\nabla_\theta \ln \pi_\theta(a_h \mid s_h)\big|_{\theta=\theta_0}$$

$$\nabla_\theta J(\pi_\theta)\big|_{\theta=\theta_0}$$

$$= \nabla_\theta \mathop{E}_{\tau \sim \rho_\theta} [R(\tau)] \Big|_{\theta=\theta_0}$$

$$= \nabla_\theta \mathop{E}_{\tau \sim \rho_{\theta_0}} \frac{\rho_\theta(\tau)}{\rho_{\theta_0}(\tau)} R(\tau) \Big|_{\theta=\theta_0} = \mathop{E}_{\tau \sim \rho_{\theta_0}} \left[\frac{\nabla_\theta \rho_\theta(\tau)\big|_{\theta=\theta_0}}{\rho_{\theta_0}(\tau)} \right] R(\tau) = \mathop{E}_{\tau \sim \rho_{\theta_0}} \nabla_\theta \ln \rho_\theta(\tau) \cdot R(\tau)$$

$$\parallel \quad \nabla_\theta \ln \rho_\theta(\tau)\big|_{\theta=\theta_0}$$

Recap: the REINFORCE Algorithm

$$\nabla_\theta J(\pi_\theta)\,|_{\theta=\theta_0} := \mathbb{E}_{\tau\sim\rho_{\theta_0}(\tau)}\left[\left(\sum_{h=0}^{H-1}\nabla_\theta \ln \pi_{\theta_0}(a_h\,|\,s_h)\right)R(\tau)\right]$$

How to get an unbiased estimate of the PG?

Recap: the REINFORCE Algorithm

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How to get an unbiased estimate of the PG?

$$\tau \sim \rho_{\theta_0}$$

~~ex~~ execute π_{θ_0} from $s_0 \sim \mu$

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$$g := \sum_{h=0}^{H-1}\left[\nabla \ln \pi_{\theta_0}(a_h\,|\,s_h)R(\tau)\right]$$

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How to get an unbiased estimate of the PG?

$$\tau \sim \rho_{\theta_0}$$

$\checkmark\ \pi_\theta(a|s) \propto \exp\left(\theta^\top \phi(s,a)\right)$

$\exp\left(\underline{f_\theta(s,a)}\right)$

$$g := \sum_{h=0}^{H-1}\left[\nabla \ln \pi_{\theta_0}(a_h|s_h)R(\tau)\right]$$

We have: $\mathbb{E}[g] = \nabla_\theta J(\pi_{\theta_0})$

Recap: the REINFORCE Algorithm

$$\nabla_\theta J(\pi_\theta)\,|_{\theta=\theta_0} := \mathbb{E}_{\tau\sim\rho_{\theta_0}(\tau)}\left[\left(\sum_{h=0}^{H-1}\nabla_\theta \ln \pi_{\theta_0}(a_h\,|\,s_h)\right)R(\tau)\right]$$

$Var(g)$
$= E[g^2] - (E(g))^2$

How to get an unbiased estimate of the PG?

$E[g^2]$

$\approx \sum_{h=0}^{H-1} \underbrace{E(\nabla \ln \pi(a_h|s_h) \cdot R(\tau))^2}_{\approx H^2}$

$\approx H^3$

$\tau \sim \rho_{\theta_0}$

$$g := \sum_{h=0}^{H-1}\left[\nabla \ln \pi_{\theta_0}(a_h\,|\,s_h) \underbrace{R(\tau)}_{\approx H}\right]$$

Δ

$E[g^2] \approx \sum_{h=0}^{H-1} H^2 = H^3$

$R(\tau)^2 \approx H^2$

This formulation has large variance, i.e., $\mathbb{E}\left[\|g - \nabla_\theta J(\pi_{\theta_0})\|_2^2\right]$ could be as large as H^3

(In practice, no one uses it)

We have: $\mathbb{E}[g] = \nabla_\theta J(\pi_{\theta_0})$

Today's Question:

How to reduce Variance in Policy Gradient?

Outline:

1. A $Q^{\pi}(s, a)$ based Policy Gradient

2. Variance Reduction via A Baseline
(i.e., an $A^{\pi}(s, a)$ based PG)

$A^{\pi}(s,a) = Q^{\pi}(s,a) - V^{\pi}(s)$

3. Algorithm: Put everything together

Notations

$$\mathscr{M} = \{P, r, \gamma, \mu, S, A\} \quad \text{where } s_0 \sim \mu$$

$$V^\pi(s) = \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r(s_h, a_h) \,|\, s_0 = s, a_h \sim \pi\right]$$

$$\text{Objective: } J(\pi) := \mathbb{E}_{s_0 \sim \mu}\left[V^\pi(s_0)\right]$$

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$$d_\mu^\pi(s, a) = (1 - \gamma)\sum_{h=0}^{\infty} \gamma^h \mathbb{P}_h^\pi(s, a; \mu)$$

$$A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s, a)$$

Derivation of Policy Gradient w/ Q^π

Recall definition of value function $V^{\pi_\theta}(s)$

$\nabla \ln \pi_\theta(a_h|s_h) \cdot \boxed{R(\tau)}$

maybe

$\sum_{\tau=h}^{\infty} \gamma^{\tau} r(s_\tau, a_\tau)$

$\nabla \ln \pi_\theta(a_h|s_h) \cdot Q^{\pi_\theta}(s_h, a_h)$

Derivation of Policy Gradient w/ Q^π

Recall definition of value function $V^{\pi_\theta}(s)$

$$\nabla_\theta J(\pi_\theta) = \nabla_\theta \mathbb{E}_{s_0 \sim \mu}\left[V^{\pi_\theta}(s_0)\right]$$

Derivation of Policy Gradient w/ Q^π

$a_0 \sim \pi_\theta(s_0)$

$:= a_0 \sim \pi_\theta(\cdot \mid s_0)$

Recall definition of value function $V^{\pi_\theta}(s)$

$(fg)' = f'g + f \cdot g'$

$$\nabla_\theta J(\pi_\theta) = \nabla_\theta \mathbb{E}_{s_0 \sim \mu}\left[V^{\pi_\theta}(s_0)\right]$$

$$= \mathbb{E}_{s_0 \sim \mu}\left[\nabla_\theta \mathbb{E}_{a_0 \sim \pi_\theta(s_0)} Q^{\pi_\theta}(s_0, a_0)\right]$$

Derivation of Policy Gradient w/ Q^π

Recall definition of value function $V^{\pi_\theta}(s)$

$$\nabla_\theta J(\pi_\theta) = \nabla_\theta \mathbb{E}_{s_0 \sim \mu}\left[V^{\pi_\theta}(s_0)\right]$$

$$= \mathbb{E}_{s_0 \sim \mu}\left[\nabla_\theta \mathbb{E}_{a_0 \sim \pi_\theta(s_0)} Q^{\pi_\theta}(s_0, a_0)\right] = \mathbb{E}_{s_0 \sim \mu}\left[\sum_{a_0} \nabla_\theta \pi_\theta(a_0 \mid s_0) \cdot Q^{\pi_\theta}(s_0, a_0) + \sum_{a_0} \pi_\theta(a_0 \mid s_0) \cdot \nabla_\theta Q^{\pi_\theta}(s_0, a_0)\right]$$

$$= \nabla_\theta \left[\sum_{a_0} \pi_\theta(a_0 \mid s_0)\, Q^{\pi_\theta}(s_0, a_0)\right]$$

product rule

$$= \sum_a \nabla_\theta \pi_\theta(a_0 \mid s_0) \cdot Q^{\pi_\theta}(s_0, a_0)$$

$$+ \sum_a \pi_\theta(a_0 \mid s_0) \cdot \nabla_\theta Q^{\pi_\theta}(s_0, a_0)$$

$(fg)'$

$= f' \cdot g + f \cdot g'$

Derivation of Policy Gradient w/ Q^π

Recall definition of value function $V^{\pi_\theta}(s)$

$$\nabla_\theta Q^{\pi_\theta}(s_0, a_0)$$
$$= \nabla_\theta \left[r(s_0, a_0) + \gamma \mathbb{E}_{s_1 \sim P(s_0, a_0)} V^{\pi_\theta}(s_1) \right]$$
$$= \gamma \nabla_\theta \mathbb{E}_{s_1 \sim P(s_0, a_0)} V^{\pi_\theta}(s_1)$$
$$= \gamma \mathbb{E}_{s_1 \sim P(s_0, a_0)} \nabla_\theta V^{\pi_\theta}(s_1)$$

product Rule:

$$\nabla_\theta J(\pi_\theta) = \nabla_\theta \mathbb{E}_{s_0 \sim \mu} \left[V^{\pi_\theta}(s_0) \right]$$

$$= \mathbb{E}_{s_0 \sim \mu} \left[\nabla_\theta \mathbb{E}_{a_0 \sim \pi_\theta(s_0)} Q^{\pi_\theta}(s_0, a_0) \right] = \mathbb{E}_{s_0 \sim \mu} \left[\sum_{a_0} \nabla_\theta \pi_\theta(a_0 \mid s_0) \cdot Q^{\pi_\theta}(s_0, a_0) + \sum_{a_0} \pi_\theta(a_0 \mid s_0) \cdot \underline{\nabla_\theta Q^{\pi_\theta}(s_0, a_0)} \right]$$

$$= \mathbb{E}_{s_0 \sim \mu} \left[\underbrace{\sum_{a_0 \in A} \pi_\theta(a_0 \mid s_0) \left[\frac{\nabla_\theta \pi_\theta(a_0 \mid s_0)}{\pi_\theta(a_0 \mid s_0)} \right]} \cdot Q^{\pi_\theta}(s_0, a_0) + \gamma \sum_{a_0} \pi_\theta(a_0 \mid s_0) \mathbb{E}_{s_1 \sim P_{s_0, a_0}} \nabla_\theta V^{\pi_\theta}(s_1) \right]$$

✓

$$= \mathbb{E}_{a_0 \sim \pi_\theta(a_0 \mid s_0)} \nabla_\theta \ln \pi_\theta(a_0 \mid s_0)$$

Derivation of Policy Gradient w/ Q^π

Recall definition of value function $V^{\pi_\theta}(s)$

$$\nabla_\theta J(\pi_\theta) = \nabla_\theta \mathbb{E}_{s_0 \sim \mu} \left[V^{\pi_\theta}(s_0) \right]$$

$$= \mathbb{E}_{s_0 \sim \mu} \left[\nabla_\theta \mathbb{E}_{a_0 \sim \pi_\theta(s_0)} Q^{\pi_\theta}(s_0, a_0) \right] = \mathbb{E}_{s_0 \sim \mu} \left[\sum_{a_0} \nabla_\theta \pi_\theta(a_0 \,|\, s_0) \cdot Q^{\pi_\theta}(s_0, a_0) + \sum_{a_0} \pi_\theta(a_0 \,|\, s_0) \cdot \nabla_\theta Q^{\pi_\theta}(s_0, a_0) \right]$$

$$= \mathbb{E}_{s_0 \sim \mu} \left[\sum_{a_0 \in A} \pi_\theta(a_0 \,|\, s_0) \left[\frac{\nabla_\theta \pi_\theta(a_0 \,|\, s_0)}{\pi_\theta(a_0 \,|\, s_0)} \right] \cdot Q^{\pi_\theta}(s_0, a_0) + \gamma \sum_{a_0} \pi_\theta(a_0 \,|\, s_0) \mathbb{E}_{s_1 \sim P_{s_0, a_0}} \nabla_\theta V^{\pi_\theta}(s_1) \right]$$

chain rule for ln

$$= \mathbb{E}_{s_0 \sim \mu} \left[\mathbb{E}_{a_0 \sim \pi_\theta(a_0|s_0)} \nabla_\theta \ln \pi_\theta(a_0 \,|\, s_0) \cdot Q^{\pi_\theta}(s_0, a_0) \right] + \gamma \mathbb{E}_{s_1 \sim \mathbb{P}_1^{\pi_\theta}} \nabla_\theta V^{\pi_\theta}(s_1)$$

← Repeat ; Recursion

Derivation of Policy Gradient w/ Q^π

Recall definition of value function $V^{\pi_\theta}(s)$

$$\nabla_\theta J(\pi_\theta) = \nabla_\theta \mathbb{E}_{s_0 \sim \mu}\left[V^{\pi_\theta}(s_0)\right]$$

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$$= \mathbb{E}_{s_0 \sim \mu}\left[\sum_{a_0 \in A} \pi_\theta(a_0 \mid s_0)\left[\frac{\nabla_\theta \pi_\theta(a_0 \mid s_0)}{\pi_\theta(a_0 \mid s_0)}\right] \cdot Q^{\pi_\theta}(s_0, a_0) + \gamma \sum_{a_0} \pi_\theta(a_0 \mid s_0) \mathbb{E}_{s_1 \sim P_{s_0, a_0}} \nabla_\theta V^{\pi_\theta}(s_1)\right]$$

$$= \mathbb{E}_{s_0 \sim \mu}\left[\mathbb{E}_{a_0 \sim \pi_\theta(a_0 \mid s_0)} \nabla_\theta \ln \pi_\theta(a_0 \mid s_0) \cdot Q^{\pi_\theta}(s_0, a_0)\right] + \boxed{\gamma \mathbb{E}_{s_1 \sim \mathbb{P}_1^{\pi_\theta}} \nabla_\theta V^{\pi_\theta}(s_1)}$$

Repeat on this term

$$= \mathbb{E}_{s_0 \sim \mu}\left[\mathbb{E}_{a_0 \sim \pi_\theta(a_0 \mid s_0)} \nabla_\theta \ln \pi_\theta(a_0 \mid s_0) Q^{\pi_\theta}(s_0, a_0)\right] + \gamma \mathbb{E}_{s_1 \sim \mathbb{P}_1^{\pi_\theta}}\left[\mathbb{E}_{a_1 \sim \pi_\theta(a_1 \mid s_1)} \nabla_\theta \ln \pi_\theta(a_1 \mid s_1) Q^{\pi_\theta}(s_1, a_1)\right] + \boxed{\gamma^2 \mathbb{E}_{s_2 \sim \mathbb{P}_2^{\pi_\theta}} \nabla_\theta V^{\pi_\theta}(s_2)}$$

Repeat

Derivation of Policy Gradient w/ Q^π

Recall definition of value function $V^{\pi_\theta}(s)$

$$\nabla_\theta J(\pi_\theta) = \nabla_\theta \mathbb{E}_{s_0\sim\mu}\left[V^{\pi_\theta}(s_0)\right]$$

$$= \mathbb{E}_{s_0\sim\mu}\left[\nabla_\theta \mathbb{E}_{a_0\sim\pi_\theta(s_0)} Q^{\pi_\theta}(s_0, a_0)\right] = \mathbb{E}_{s_0\sim\mu}\left[\sum_{a_0} \nabla_\theta \pi_\theta(a_0 \mid s_0)\cdot Q^{\pi_\theta}(s_0, a_0) + \sum_{a_0} \pi_\theta(a_0 \mid s_0)\cdot \nabla_\theta Q^{\pi_\theta}(s_0, a_0)\right]$$

$$= \mathbb{E}_{s_0\sim\mu}\left[\sum_{a_0\in A} \pi_\theta(a_0 \mid s_0)\left[\frac{\nabla_\theta \pi_\theta(a_0 \mid s_0)}{\pi_\theta(a_0 \mid s_0)}\right]\cdot Q^{\pi_\theta}(s_0, a_0) + \gamma \sum_{a_0} \pi_\theta(a_0 \mid s_0)\mathbb{E}_{s_1\sim P_{s_0,a_0}} \nabla_\theta V^{\pi_\theta}(s_1)\right]$$

$$= \mathbb{E}_{s_0\sim\mu}\left[\mathbb{E}_{a_0\sim\pi_\theta(a_0|s_0)} \nabla_\theta \ln \pi_\theta(a_0 \mid s_0)\cdot Q^{\pi_\theta}(s_0, a_0)\right] + \gamma \mathbb{E}_{s_1\sim\mathbb{P}_1^{\pi_\theta}} \nabla_\theta V^{\pi_\theta}(s_1)$$

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$$= \sum_{h=0}^{\infty} \gamma^h \mathbb{E}_{s_h,a_h\sim\mathbb{P}_h^{\pi_\theta}} \nabla_\theta \ln \pi_\theta(a_h \mid s_h)\cdot Q^{\pi_\theta}(s_h, a_h)$$

Derivation of Policy Gradient w/ Q^{π}

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$$= \sum_{h=0}^{\infty}\gamma^h \mathbb{E}_{s_h,a_h\sim\mathbb{P}_h^{\pi_\theta}}\nabla_\theta \ln \pi_\theta(a_h\mid s_h)\cdot Q^{\pi_\theta}(s_h,a_h) \quad = \frac{1}{1-\gamma}\mathbb{E}_{s,a\sim d_{\mu}^{\pi_\theta}}\nabla_\theta \ln\pi_\theta(a\mid s)\cdot Q^{\pi_\theta}(s,a)$$

Summary so far:

Product rule + Important weighting + Recursion:

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$$\nabla_\theta J(\pi_\theta) = \sum_{h=0}^{\infty} \gamma^h \mathbb{E}_{s,a\sim\mathbb{P}_h^{\pi_\theta}} \nabla_\theta \ln \pi_\theta(a \mid s) \cdot Q^{\pi_\theta}(s,a)$$

Def wrt $d_\mu^{\pi_\theta}$

$$\triangleq \frac{1}{1-\gamma} \mathbb{E}_{s,a\sim d_\mu^{\pi_\theta}} \left[\nabla_\theta \ln \pi_\theta(a \mid s) \cdot Q^{\pi_\theta}(s,a) \right]$$

Summary so far:

Product rule + Important weighting + Recursion:

$$\nabla_\theta J(\pi_\theta) = \sum_{h=0}^{\infty} \gamma^h \mathbb{E}_{s,a \sim \mathbb{P}_h^{\pi_\theta}} \nabla_\theta \ln \pi_\theta(a \mid s) \cdot Q^{\pi_\theta}(s,a)$$

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π^* is determinise

$$\pi^*(a|s) = \begin{cases} 1, & a = \pi^*(s) \\ 0, & \text{else} \end{cases}$$

$s.a \sim d^{\pi^*}$

$$\underline{\underbrace{\nabla_\theta \ln \pi^*(a|s)}_{=0} \cdot \underbrace{Q^*(s.a)}_{=0}}$$

For finite horizon setting, we have:

$$\nabla_\theta J(\pi_\theta) = \sum_{h=0}^{H-1} \mathbb{E}_{s_h, a_h \sim \mathbb{P}_h^{\pi_\theta}} \left[\nabla \ln \pi_\theta(a_h \mid s_h) \cdot Q_h^{\pi_\theta}(s_h, a_h) \right]$$

Outline:

1. A $Q(s, a)$ based Policy Gradient

2. Variance Reduction via A Baseline
(i.e., an $A(s, a)$ based PG)

3. Algorithm: Put everything together

Intuition behind Q-based PG:

$$\nabla_\theta J(\pi_\theta) = \sum_{h=0}^{H-1} \mathbb{E}_{s_h, a_h \sim \mathbb{P}_h^{\pi_\theta}} \left[\nabla_\theta \ln \pi_\theta(a_h \mid s_h) \cdot Q_h^{\pi_\theta}(s_h, a_h) \right]$$

We want to slowly adjust policy,
such that $\pi_\theta(a \mid s)$ is large at action a with large $Q^{\pi_\theta}(s, a)$

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After all, recall PI, we know that $\arg\max_a A^{\pi_\theta}(s, a)$ can work

(subject to knowing A^{π_θ} everywhere)

PI:

$\pi' = \arg\max_a A^{\pi_\theta}(s, a)$

$\pi' \geq \pi_\theta$

The Advantage-based PG:

$$\nabla_\theta J(\pi_\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s,a\sim d_\mu^{\pi_\theta}}\left[\nabla_\theta \ln \pi_\theta(a \,|\, s) \cdot A^{\pi_\theta}(s,a)\right]$$

The Advantage-based PG:

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$b: S \mapsto \mathbb{R}$

We will prove a more general version, denote $b(s)$ as a state-dependent ***baseline, we have:***

(Action-independent)

$$\nabla_\theta J(\pi_\theta) = \frac{1}{1-\gamma} \mathbb{E}_{s,a \sim d_\mu^{\pi_\theta}} \left[\nabla_\theta \ln \pi_\theta(a|s) \cdot Q^{\pi_\theta}(s,a) \right]$$

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$=0$

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$\forall s$;

$\mathbb{E}_{a\sim\pi_\theta(\cdot|s)} \nabla_\theta \ln \pi_\theta(a\,|\,s) b(s) = 0$

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$$\mathbb{E}_{a\sim\pi_\theta(\cdot|s)}\nabla_\theta \ln \pi_\theta(a\,|\,s) b(s)$$

chain Rule:

$$\frac{\nabla_\theta \pi_\theta(a|s)}{\pi_\theta(a|s)}$$

$$= \sum_a \pi_\theta\cancel{(a\,|\,s)}\frac{\nabla \pi_\theta(a\,|\,s)}{\pi_\theta\cancel{(a\,|\,s)}}b(s)$$

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$$\mathbb{E}_{a\sim\pi_\theta(\cdot|s)}\nabla_\theta \ln \pi_\theta(a\,|\,s)b(s)$$

$$= \sum_a \cancel{\pi_\theta(a\,|\,s)}\frac{\nabla \pi_\theta(a\,|\,s)}{\cancel{\pi_\theta(a\,|\,s)}}b(s) = b(s)\sum_a \nabla_A \pi_\theta(a\,|\,s) = b(s)\nabla\left[\overbrace{\sum_a \pi_\theta(a\,|\,s)}^{=1}\right]$$

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$$\mathbb{E}_{a \sim \pi_\theta(\cdot \mid s)} \nabla_\theta \ln \pi_\theta(a \mid s) b(s) = 0$$

$$= \sum_a \pi_\theta(a \mid s) \frac{\nabla \pi_\theta(a \mid s)}{\pi_\theta(a \mid s)} b(s) \quad = b(s) \sum_a \nabla \pi_\theta(a \mid s) = b(s) \nabla \left[\sum_a \pi_\theta(a \mid s) \right] \quad = b(s) \nabla 1 = 0$$

Summary so far:

$b(s) \Leftarrow$ action-independent

By a Baseline (proof undoes the importance weighting trick), we have:

$$\nabla_\theta J(\pi_\theta) = \frac{1}{1-\gamma} \mathbb{E}_{s,a \sim d_\mu^{\pi_\theta}} \left[\nabla_\theta \ln \pi_\theta(a \mid s) \cdot \left(Q^{\pi_\theta}(s,a) - \underline{b(s)} \right) \right]$$

$$\because \mathbb{E}_{a \sim \pi_\theta(\cdot|s)} \nabla_\theta \ln \pi_\theta(a|s)\, b(s) = 0$$

Summary so far:

By a Baseline (proof undoes the importance weighting trick), we have:

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This holds for any baseline as long as it is action-independent
(thus we can set $b(s) = V^{\pi_\theta}(s)$—the most common thing)

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Summary so far:

$Q^{\pi_\theta}(s,a)$ is L-Lip function wrt a

$|Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)|$

$\leq L \cdot |a - \pi_\theta(s)|$

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$Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)$

This holds for any baseline as long as it is action-independent
(thus we can set $b(s) = V^{\pi_\theta}(s)$—the most common thing)

Baseline helps variance reduction (formal proof out of scope)

Outline:

1. A $Q(s, a)$ based Policy Gradient

2. Variance Reduction via A Baseline (i.e., an $A(s, a)$ based PG)

3. Algorithm: Put everything together

Algorithm that relies on Stochastic Gradient Ascent

Recall the PG: $$\nabla_\theta J(\pi_\theta) = \frac{1}{1-\gamma}\underbrace{\mathbb{E}_{s,a\sim d_\mu^{\pi_\theta}}}_{\text{by samples}}\left[\nabla_\theta \ln \pi_\theta(a \mid s) \cdot \left(Q^{\pi_\theta}(s,a) - b(s)\right)\right]$$

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To get unbiased estimate of gradient, recall we can roll-in $(s,a) \sim d_\mu^{\pi_\theta}$, and roll out to get y w/ $\mathbb{E}[y] = Q^{\pi_\theta}(s,a)$

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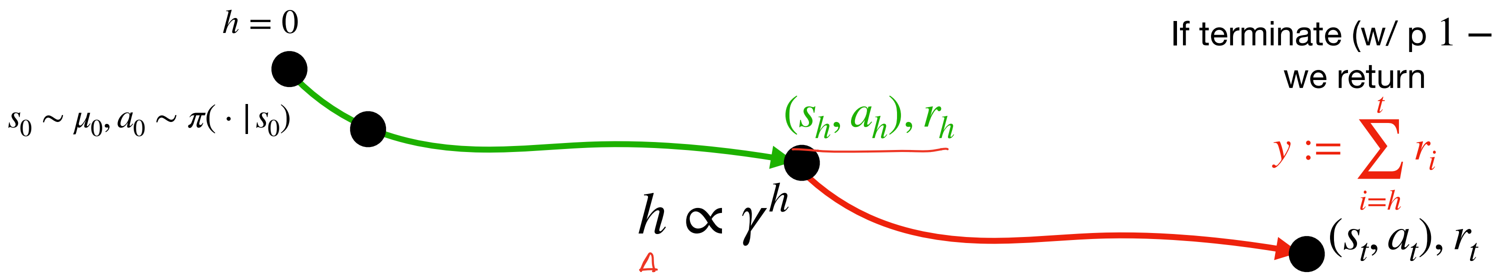

If terminate (w/ p $1 - \gamma$), we return

$$y := \sum_{i=h}^{t} r_i$$

(s, a, y) s.t. $s, a \sim d_\mu^{\pi_\theta}$

$E[y] = Q^{\pi_\theta}(s, a)$

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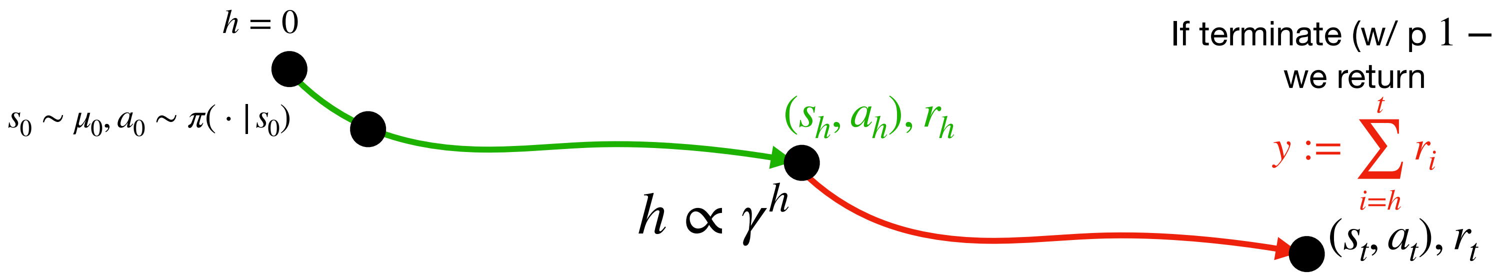

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Repeat roll-in & roll-out N times, with the mini-batch $\{s^i, a^i, y^i\}_{i=1}^N$,

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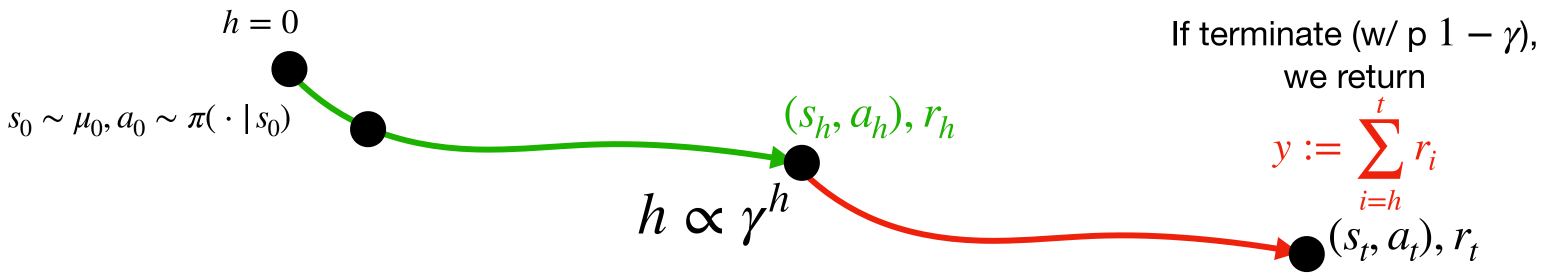

$y^i - b(s^i)$

Repeat roll-in & roll-out N times, with the mini-batch $\{s^i, a^i, y^i\}_{i=1}^N$,

$$g = \sum_{i=1}^{N}\frac{1}{N}\left[\nabla_\theta \ln \pi_\theta(a^i\,|\,s^i)\cdot y^i\right] \cdot \frac{1}{1-\gamma}$$

$\mathbb{E}[y^i] = Q^{\pi_\theta}(s^i, a^i)$

Algorithm that relies on Stochastic Gradient Ascent

Initialization θ_0

For t = 0, …

Sample $\underbrace{\{s^i, a^i, y^i\}_{i=1}^N}$, w/ $s^i, a^i \sim d_\mu^{\pi_{\theta_t}}, \mathbb{E}[y^i] = Q^{\pi_{\theta_t}}(s^i, a^i)$

Form gradient estimate: $g_t = \frac{1}{1-\gamma} \sum_{i=1}^{N} \nabla_\theta \ln \pi_{\theta_t}(a^i \mid s^i) \cdot y^i / N$ ✓ unbiased estimate

Stochastic GA: $\theta_{t+1} = \theta_t + \eta g_t$

↳ $\mathbb{E}[g_t] = \nabla_\theta J(\pi_\theta)\big|_{\theta=\theta_t}$

In practice, we often use supervised learning to estimate Q^{π_θ}:

$$\nabla_\theta J(\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s,a\sim d^{\pi_\theta}}\left[\nabla_\theta \ln \pi_\theta(a\,|\,s)\left(Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)\right)\right]$$

$$\theta_{t+1} = \theta_t + \underbrace{\left[\eta \cdot g_t\right]}_{A} \rightsquigarrow \boxed{g_t = \nabla_\theta J(\theta_t)}$$

$\eta = 0.5$

$E[0.5\, g_t] = 0.5\, E(g_1)$

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$$\hat{f} = \arg\min_f \mathbb{E}_{\underline{s\sim d_\mu^{\pi_\theta}, a\sim U(A)}}\left(f(s,a) - \underbrace{Q^{\pi_\theta}(s,a)}_{y^{T}_{\text{Roll-out}}}\right)^2$$ (e.g., regression oracle!)

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We can form an approximated Gradient **(could be unbiased) using $\hat{f}$:**

$$\nabla_\theta \ln \pi_\theta(a_h \mid s_h)\left(\underbrace{\hat{f}(s_h,a_h)}_{\approx Q^{\pi_\theta}(s_h,a_h)} - \underbrace{\mathbb{E}_{a'\sim\pi_\theta(a'|s_h)}\hat{f}(s_h,a')}_{\approx V^{\pi_\theta}(s_h)}\right)$$

Actor-critic

In practice, we often use supervised learning to estimate Q^{π_θ}:

π_θ $\hat{f}$

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We can form an approximated Gradient **(could be unbiased) using $\hat{f}$:**

$$\nabla_\theta \ln \pi_\theta(a_h\,|\,s_h)\left(\hat{f}(s_h,a_h) - \mathbb{E}_{a'\sim\pi_\theta(a'|s_h)}\hat{f}(s_h,a')\right)$$

$\theta \le 90$

gradient

Bisa-variance tradeoff

(our $\hat{f}$ is a function now, we no-longer rely on a roll-out)

Δ

Summary for PG:

Three common PG formulations:

REINFORCE ← Important weighting

$$\nabla J(\theta_t) = \mathbb{E}_{\tau \sim \rho_{\theta_t}(\tau)} \left[\left(\sum_{h=0}^{\infty} \nabla_\theta \ln \pi_{\theta_t}(a_h \mid s_h) \right) R(\tau) \right]$$

Summary for PG:

Three common PG formulations:

REINFORCE

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PG w/ Q function

$$\nabla_\theta J(\theta_t) = \frac{1}{1-\gamma} \mathbb{E}_{s,a \sim d^{\pi_{\theta_t}}} \left[\nabla_\theta \ln \pi_{\theta_t}(a | s) \left(Q^{\pi_{\theta_t}}(s, a) \right) \right]$$

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PG w/ A function (use $V^\pi(s)$ as a baseline)

$$\nabla_\theta J(\theta_t) = \frac{1}{1-\gamma} \mathbb{E}_{s,a \sim d^{\pi_{\theta_t}}} \left[\nabla_\theta \ln \pi_{\theta_t}(a | s) \left(A^{\pi_{\theta_t}}(s, a) \right) \right]$$

Next lecture:

Trust-region policy optimization (Natural Policy Gradient)

Natural Gradient ←