Conservative Policy Iteration

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i.e., pick an action that has the largest advantage against π_{old} at every state s,

Maximize advantage is great, as it gives monotonic improvement:
$Q^{\pi_{new}}(s, a) \geq Q^{\pi_{old}}(s, a), \forall s, a$

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The advantage against π_{old} averaged over π_{new}'s own distribution

Today:
Conservative Policy Iteration

Q: How to enforce incremental policy update and ensure monotonic improvement

Outline

1. Greedy Policy Selection (via reduction to regression) and recap of API

2. Conservative Policy Iteration

3. Monotonic Improvement of CPI

Setting and Notation

Discounted infinite horizon MDP:

$$\mathcal{M} = \{S, A, \gamma, r, P, \mu\}$$

→ initial $s_0 \sim \mu$

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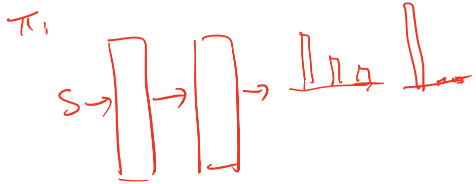
$$\Pi = \{\pi : S \mapsto A\}$$



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(We can think about each policy as a classifier)

Note that the optimal policy $\pi^\star$ may not be in Π

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As we will consider large scale unknown MDP here, we start with a (restricted) function class Π:

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From now on, think about deterministic policy as a special stochastic policy

(We can think about each policy as a classifier)

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Greedy Policy Selector

How to implement such greedy policy selector?
We talked about a regression process..

Implementing Approximate Greedy Policy Selector via Regression

We can do a **reduction to Regression** via Advantage function approximation

$$\mathscr{F} = \{f : S \times A \mapsto \mathbb{R}\} \quad (\approx A^{\pi^t})$$

$A^{\pi^t}(s,a) = Q^{\pi^t}(s,a) - V^{\pi^t}(s)$

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$$\Pi = \{\pi(s) = \arg\max_a f(s,a) : f \in \mathcal{F}\}$$ ← implicit

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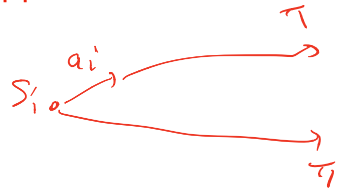

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$$\{s_i, a_i, y_i\}, s_i \sim d_\mu^{\pi^t}, a_i \sim U(A), \mathbb{E}\left[y_i\right] = A^{\pi^t}(s_i, a_i)$$

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$$\widehat{A}^t = \arg\min_{f \in \mathcal{F}} \sum_i \left(f(s_i, a_i) - y_i\right)^2$$

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$\arg\max_a A^{\pi}(s,a)$

$= \arg\max_a Q^{\pi}(s,a)$

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Assume this regression is successful, i.e.,

$$\mathbb{E}_{s \sim d_\mu^{\pi^t}, a \sim U(A)} \left(\widehat{A}^t(s, a) - A^{\pi^t}(s, a) \right)^2 \leq \delta \sim \sqrt{\frac{1}{N}} \text{ or } \frac{1}{N}$$

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Then, $\widehat{\pi}(s) = \arg\max_a \widehat{A}^t(s,a)$ is an approximate greedy policy:

$$\mathbb{E}_{s\sim d_\mu^{\pi^t}} A^{\pi^t}(s, \widehat{\pi}(s)) \geq \max_{\pi \in \Pi} \mathbb{E}_{s \sim d_\mu^{\pi^t}} A^{\pi^t}(s, \pi(s)) - O\left(\sqrt{\delta}\right)$$

$\tilde{\pi}$ $\quad \Delta \quad \to 0 \quad \delta \approx \frac{1}{N}$

$$\mathbb{E}_{s\sim d_\mu^{\pi^t}} A^{\pi^t}(s, \hat\pi(s)) - \mathbb{E}_{s\sim d_\mu^{\pi^t}} A^{\pi^t}(s, \tilde\pi(s))$$

$$= \mathbb{E}_{s\sim d_\mu^{\pi^t}} \left[A^{\pi^t}(s,\hat\pi(s)) - \hat A^t(s,\hat\pi(s)) + \hat A^t(s,\hat\pi(s)) - A^{\pi^t}(s,\tilde\pi(s)) \right]$$

$$\geq \hat A^t(s,\tilde\pi(s)) - A^{\pi^t}(s,\tilde\pi(s))$$

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By reduction to Supervised Learning (i.e., via Regression to approximate A^{π^t} under $d_\mu^{\pi^t}$), we can expect to find an approximate greedy optimizer $\widehat{\pi}$, s.t.,

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PI: $A^{\pi^t}(s,a), \forall s,a$

$\arg\max_a A^{\pi^t}(s,a)$

API:

For $t = 0, \ldots T$

$\pi^{t+1} = \arg\max_{\pi \in \Pi} \mathbb{E}_{s \sim d_\mu^{\pi^t}} \left[A^{\pi^t}(s, \pi(s)) \right]$

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from iteration t

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Throughout this lecture,

we will simply assume we can achieve $\arg\max_{\pi \in \Pi} \mathbb{E}_{s \sim d_\mu^{\pi^t}} \left[A^{\pi^t}(s, \pi(s)) \right]$

(think about $N \to \infty$

Greedy policy selector

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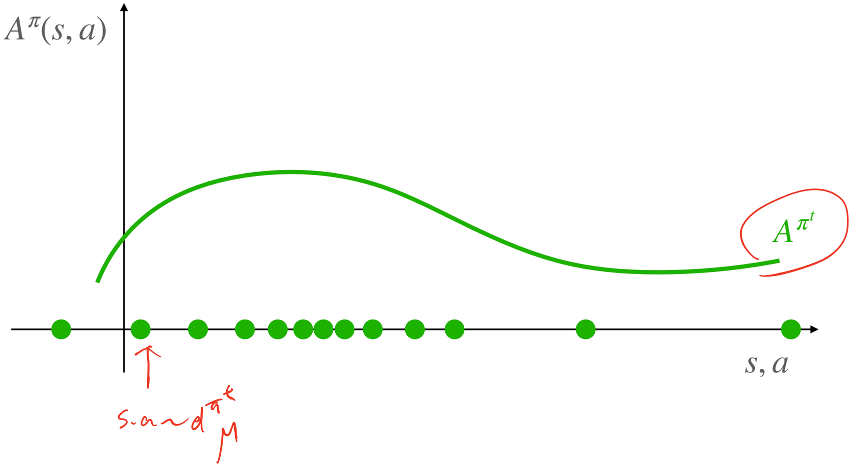
The Failure case of API: Abrupt distribution change

API cannot guarantee to succeed (let's think about advantage function approximation setting)



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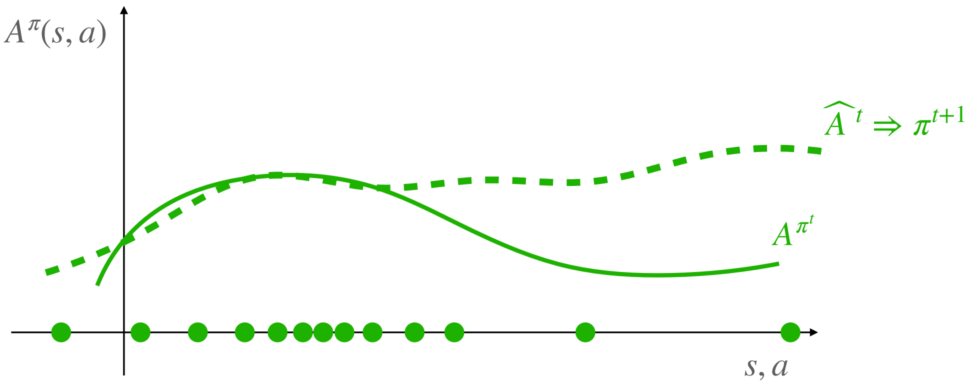

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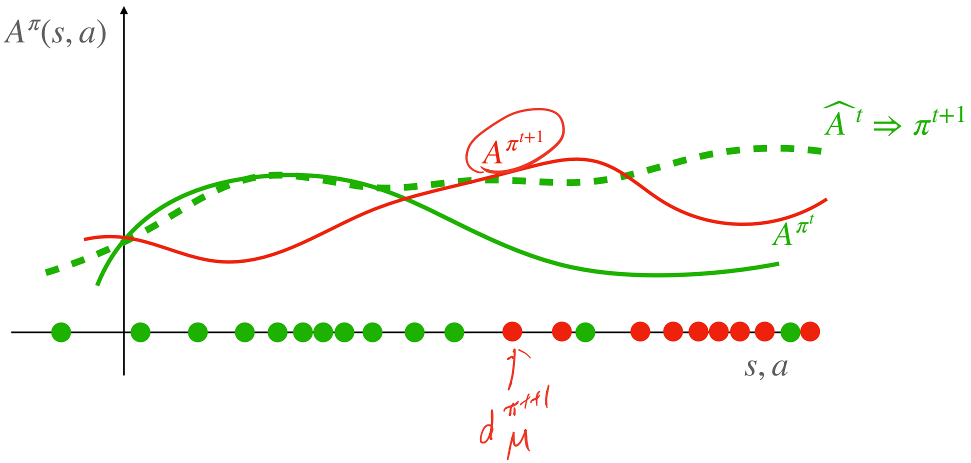

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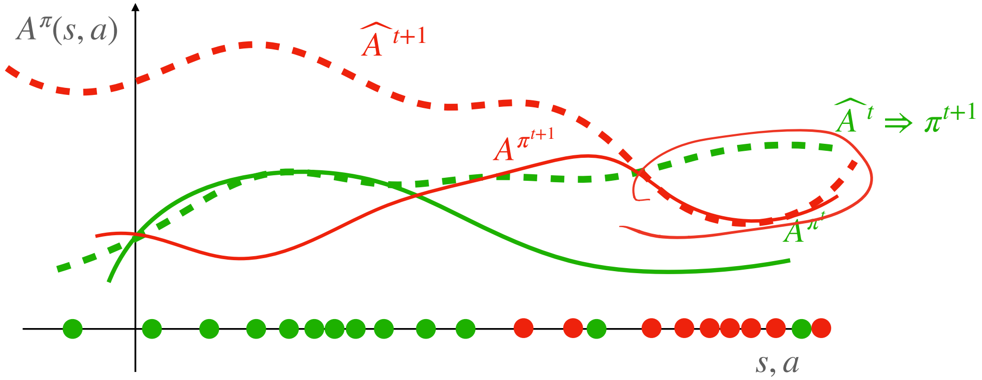

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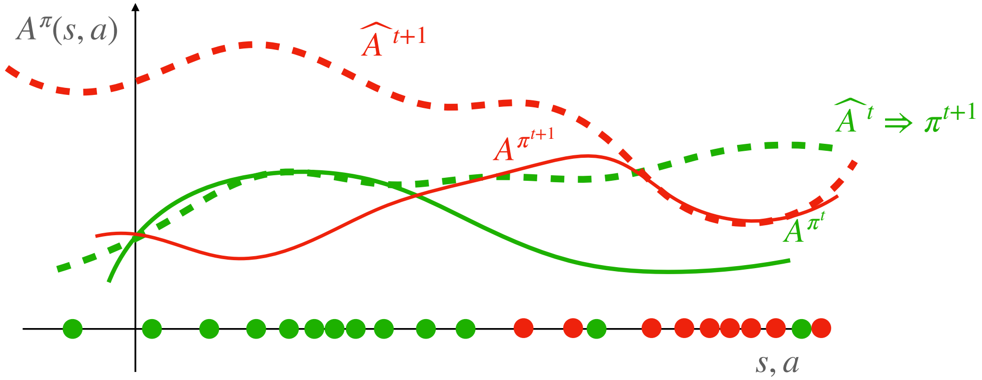

Oscillation between two updates:
No monotonic improvement

Key Idea of CPI: Incremental Update—No Abrupt Distribution Change

Let's design policy update rule such that $d_\mu^{\pi^{t+1}}$ and $d_\mu^{\pi^t}$ are not that different!

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somehow: $d_\mu^{\pi^t} \approx d_\mu^{\pi^{t+1}}$

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$$d^{\pi^t} \approx d^{\pi^{t+1}}$$

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This we know how to optimize: the Greedy Policy Selector

(Recall Regression)

The CPI Algorithm

Initialize π^0

For t = 0 …

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1. Greedy Policy Selector:

$$\pi' \in \arg\max_{\pi\in\Pi} \mathbb{E}_{s\sim d_\mu^{\pi^t}} \left[A^{\pi^t}(s, \pi(s)) \right]$$

local - Adv against π^t

API: $\pi^{t+1} = \pi'$

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   Return π^t

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3. Incremental Update:

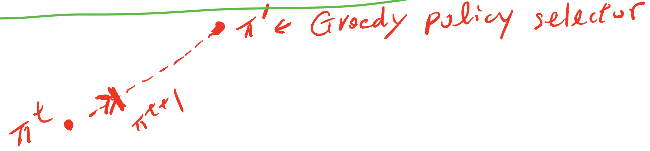
$$\pi^{t+1}(\cdot \mid s) = (1 - \alpha)\pi^t(\cdot \mid s) + \alpha \pi'(\cdot \mid s), \forall s$$

With prob $1-\alpha$, $a \sim \pi^t(\cdot|s)$

w/ prob α, $a \sim \pi'(\cdot|s)$

$\alpha = 1 \rightarrow$ API

$\alpha = 0 \rightarrow \pi^{t+1} = \pi^t$



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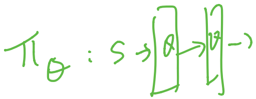
Q: Why this is incremental? In what sense?

Q: Can we get monotonic policy improvement?



Today: Policy Optimization

1. Greedy Policy Selection

2. Conservative Policy Iteration

3. Monotonic Improvement of CPI

Q1: Why this is incremental? In what sense?

Greed policy selector

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$$\pi^{t+1}(\cdot \mid s) = (1-\alpha)\pi^t(\cdot \mid s) + \alpha\pi'(\cdot \mid s), \forall s$$

$$\sum_a |\pi^{t+1}(a|s) - \pi^t(a|s)|$$

Key observation 1:

For any state s, we have $\|\pi^{t+1}(\cdot \mid s) - \pi^t(\cdot \mid s)\|_1 \leq 2\alpha$

$$\|\pi^{t+1}(\cdot|s) - \pi^t(\cdot|s)\|_1$$

$$= \|-\alpha\pi^t(\cdot|s) + \alpha\pi'(\cdot|s)\|_1$$

$$\leq \alpha\|\pi^t(\cdot|s)\|_1 + \alpha\|\pi'(\cdot|s)\|_1$$

$$= \alpha + \alpha = 2\alpha$$

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Key observation 2 (Lemma 12.1 in AJKS)

For any two policies π and π', if $\|\pi(\cdot \mid s) - \pi'(\cdot \mid s)\|_1 \leq \delta, \forall s$, then $\|d^{\pi}_{\mu}(\cdot) - d^{\pi'}_{\mu}(\cdot)\|_1 \leq \frac{\gamma\delta}{1-\gamma}$

State-Distribution

Q1: Why this is incremental? In what sense?

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Key observation 2 (Lemma 12.1 in AJKS)

For any two policies π and π', if $\|\pi(\cdot \mid s) - \pi'(\cdot \mid s)\|_1 \leq \delta, \forall s$, then $\|d_\mu^{\pi}(\cdot) - d_\mu^{\pi'}(\cdot)\|_1 \leq \dfrac{\gamma\delta}{1-\gamma}$

CPI ensures incremental update, i.e., $\|d_\mu^{\pi^{t+1}}(\cdot) - d_\mu^{\pi^t}(\cdot)\|_1 \leq \dfrac{2\gamma\alpha}{1-\gamma}$

Monotonic Improvement before Termination:

Before terminate, we have non-trivial avg **local advantage**:

$$\mathbb{A} := \mathbb{E}_{d_\mu^{\pi^t}}\left[A^{\pi^t}(s, \pi'(s))\right] \geq \varepsilon$$

$\mathbb{A}$ Max local Adv against π^t

Recall CPI:

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← Def of π^{t+1}

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plug in def of π^{t+1}

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Why?

$$\sum_a (1-\alpha)\pi^t(a|s) A^{\pi^t}(s,a) + \sum_a \alpha \pi'(a|s) A^{\pi^t}(s,a)$$

$= 0$

$$A^{\pi^t}(s,a) = Q^{\pi}(s,a) - V^{\pi}(s)$$

$$\mathbb{E}_{a\sim\pi^t(\cdot|a)} A^{\pi^t}(s,a) = V^{\pi}(s) - V^{\pi}(s) = 0$$

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$\to d_\mu^{\pi^t}$

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$\mathbb{A}$ $\nwarrow \alpha$ (Local-Avg) $\mathbb{A}$

$\leq \alpha \max_{s,a} \left|A^{\pi^t}(s,a)\right| \left\|d_\mu^{\pi^{t+1}} - d_\mu^{\pi^t}\right\|_1$ $\leq \frac{1}{1-\gamma}$

$\left|\mathbb{E}_{x\sim P} f(x) - \mathbb{E}_{x\sim Q} f(x)\right|$

$\leq \max_x |f(x)| \, \|P-Q\|_1$

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$\alpha \cdot [\text{local\_Adv}]$

$$\| d_\mu^{\pi^{t+1}} - d_\mu^{\pi^t} \|_1 \leq \frac{\gamma \cdot 2\alpha}{1-\gamma}$$

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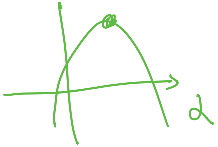
$= \mathbb{A}$

$$\geq \alpha \mathbb{A} - \frac{2\gamma\alpha^2}{(1-\gamma)^2}$$

max-local-Adv



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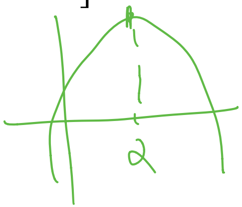
(Set $\alpha = \dfrac{(1-\gamma)^2\mathbb{A}}{4\gamma}$)



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✓ PDL

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← Incremental-update

$$\geq \alpha\mathbb{A} - \frac{2\gamma\alpha^2}{(1-\gamma)^2} \quad = \frac{\mathbb{A}^2(1-\gamma)}{8\gamma} > 0 \quad \left(\text{Set } \alpha = \frac{(1-\gamma)^2\mathbb{A}}{4\gamma}\right)$$

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$\geq \frac{\mathbb{A}^2}{\gamma}$ $\quad \mathbb{A} \geq \varepsilon$

$\geq \frac{\varepsilon^2}{\gamma}$

Summary of CPI so far:

1. Incremental update (Lemma 12.1 in AJKS)

$$\|d_\mu^{\pi^{t+1}} - d_\mu^{\pi^t}\|_1 \leq \frac{2\gamma\alpha}{1-\gamma}$$

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max-local-Adv

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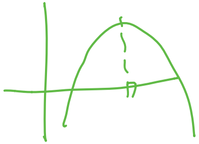


Summary for today:

1. Algorithm: Conservative Policy Iteration:
Find the local greedy policy, and move towards it a little bit

2. Small change in policies results small change in state distributions

2. Unlike API, incremental policy update ensures monotonic improvement

Finite Horizon H steps

$$\|\mathbb{P}_h^{\pi^t} - \mathbb{P}_h^{\pi^{t+1}}\|_1 \le \delta \cdot h$$

$$\underbrace{(0 + \delta + 2\delta + 3\delta \cdots (H-1)\delta)}_{\delta \cdot H^2/2} / H \approx H \cdot \delta$$

$$\|\mathbb{P}_h^{\pi^t}(s) - \mathbb{P}_h^{\pi^{t+1}}(s)\|_1 \approx h \cdot \delta$$

$$(1-\gamma)\left[0 + \gamma\delta + \gamma^2 2\delta + \gamma^3 3\delta \ldots\right]$$

$$V^{\pi^{new}}(s_0) - V^{\pi^{old}}(s_0) = \frac{1}{1-\gamma} \mathbb{E}_{s\sim d^{\pi^{new}}_{s_0}} \left[A^{\pi^{old}}(s, \pi^{new}(s)) \right]$$

$$\mathbb{E}_{s_0\sim\mu_0} \left[V^{\pi^{new}}(s_0) - V^{\pi^{old}}(s_0) \right] = \frac{1}{1-\gamma} \mathbb{E}_{\substack{s_0\sim\mu \\ s\sim d^{\pi^{new}}_{s_0}}} \left[\cdots \right]$$

$$\downarrow$$

$$d^{\pi^{new}}_{\mu}$$

$$s' = \underset{\Delta}{f}(s,a) + \varepsilon \rightarrow P(s' \mid s,a) \leftarrow \text{CPI (PG)}$$

$$\rightarrow Q^{\pi}(s,a), \ \forall s,a \ ??$$

$$\min_{\pi} \sum_{h=0}^{H-1} c(s_h, a_h)$$

$$\text{s.t. } s_{h+1} = \underset{\Delta}{f}(s_h, a_h), \ a_h = \pi(s_h)$$

$$\pi^{t+1} = (1-\alpha)\pi^t + \alpha\pi' = \pi^t - \alpha\pi^t + \alpha\pi'$$

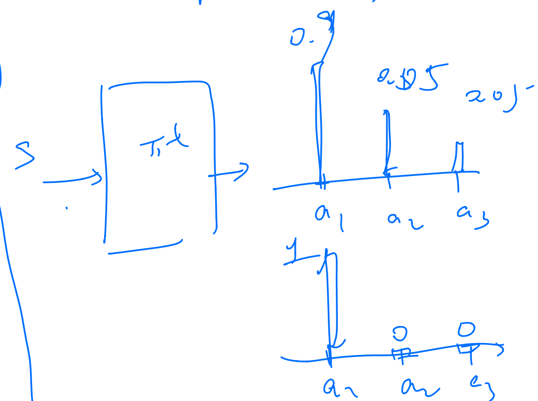
$$\|\pi^{t+1} - \pi^t\|_1 = \|-\alpha\pi^t + \alpha\pi'\|_1$$

$$\le \alpha\|\pi^t\|_1 + \alpha\|\pi'\|_1$$

$$\le \alpha + \alpha$$

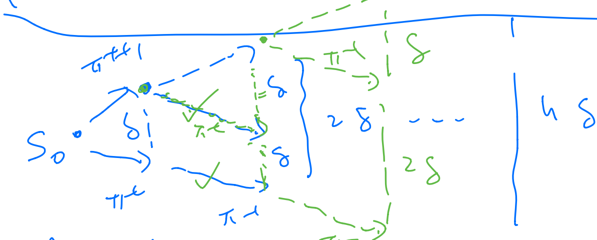
$\|\pi^t(\cdot|s)\|_1 = 1$

$\pi^t(\cdot|s) \in \Delta(A)$



Finite Horizon H

✓ $\|\mathbb{P}_h^{\pi^{t+1}}(\cdot) - \mathbb{P}_h^{\pi^t}(\cdot)\| \le h\cdot\delta$



$$(0 + \delta + 2\delta + \dots (H-1)\delta)/H$$

$$\approx \frac{\delta H(H-1)}{2}/H \approx \delta H$$