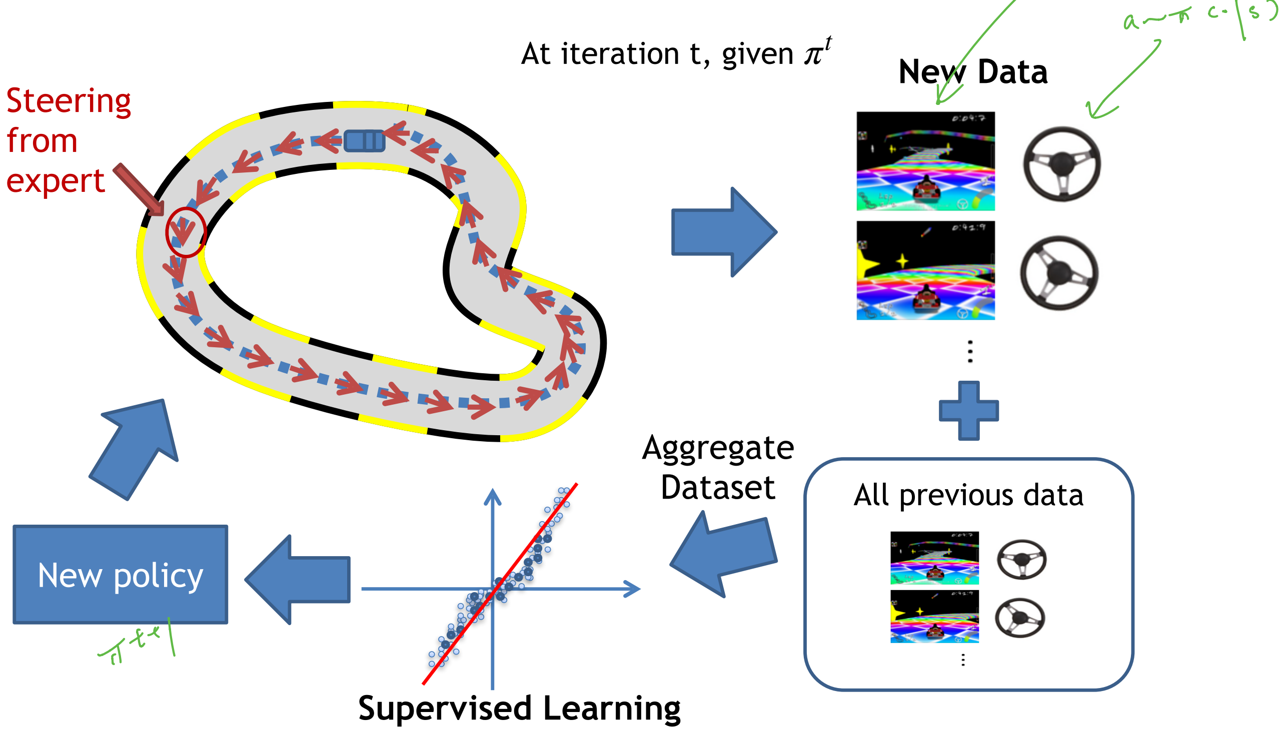
# Maximum Entropy IRL

# DAgger Recap




**Data Aggregation = Follow-the-Regularized-Leader Online Learner**

# DAgger Performance Recap:

DAgger finds a policy $\widehat{\pi}$ such that it **matches to $\pi^\star$ under its own $d_\mu^{\widehat{\pi}}$**

$$\mathbb{E}_{s \sim d_\mu^{\widehat{\pi}}} \left[ \mathbf{1}\{ \widehat{\pi}(s) \neq \pi^\star(s) \} \right] \leq \epsilon_{reg} = O(1/\sqrt{T})$$

$\Delta$

T: # of iterations

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If expert herself can **quickly recover from a deviation**, i.e., $|Q^{\pi^\star}(s,a) - V^{\pi^\star}(s)|$ is small for all $s$,

$$V^{\pi^\star} - V^{\hat{\pi}} \leq O\left(\underbrace{\frac{1}{1-\gamma}}_{\text{linear}} \cdot \epsilon_{reg}\right)$$

BC: $\frac{1}{(1-\gamma)^2}\epsilon$

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If expert herself can **quickly recover from a deviation**, i.e., $|Q^{\pi^\star}(s,a) - V^{\pi^\star}(s)|$ is small for all $s$,

$$V^{\pi^\star} - V^{\pi^t} \leq O\left( \frac{1}{1-\gamma} \cdot \epsilon_{reg} \right)$$

**This is a significant improvement over BC in both theory and practice**

# Plan for Today:

1. The principle of Maximum Entropy

2. Constrained Optimization

2. The Algorithm: Maximum Entropy Inverse RL

# Setting

cost $c(s,a) \to [0, 1]$

Finite horizon MDP $\mathscr{M} = \{S, A, H, c, P, \mu, \pi^\star\}$

$\Delta$

expert policy

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Finite horizon MDP $\mathscr{M} = \{S, A, H, c, P, \mu, \pi^\star\}$

(1) Ground truth cost $c(s, a)$ is unknown;

(2) assume expert is the optimal policy $\pi^\star$ of the cost $c$

(3) **transition P is known**

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**Key Assumption on cost:**
$c(s, a) = \langle \theta^\star, \underline{\phi(s, a)} \rangle$, **linear w.r.t feature** $\phi(s, a)$

learn $\theta^\star$ ??

$\phi : S \times A \mapsto \mathbb{R}^d$

# Running Example: Define feature map

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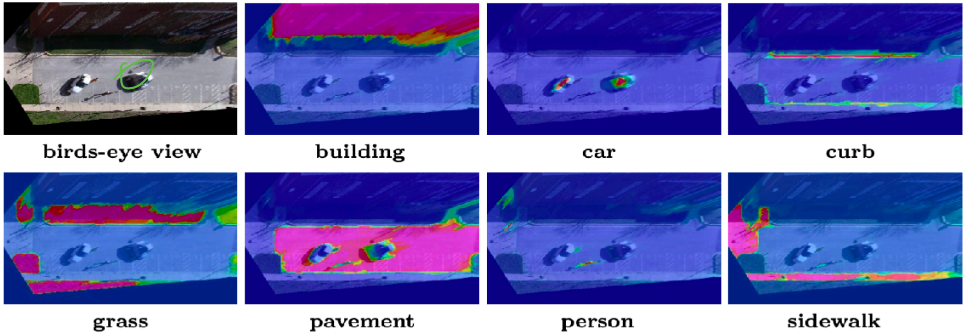


**Fig. 4.** Classifier feature response maps. Top left is the original image.

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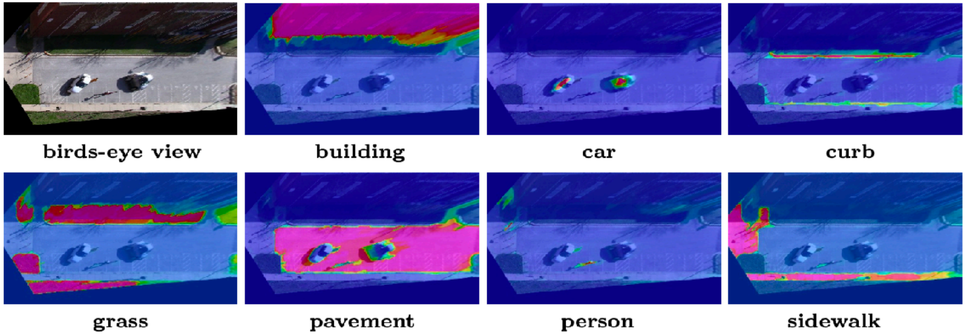
State $s$: pixel or a group of neighboring pixels in image)

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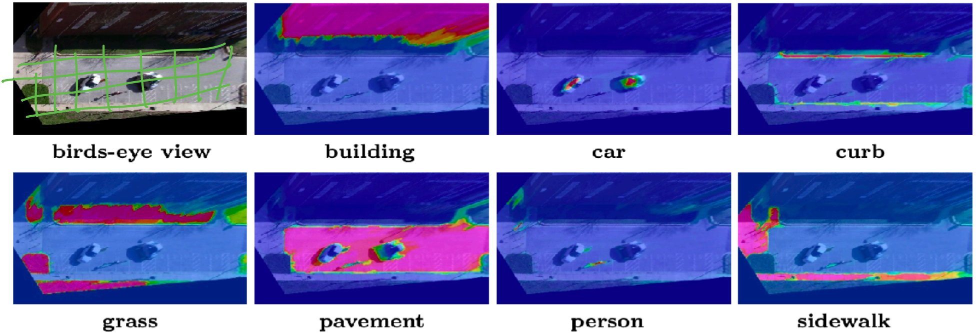


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State $s$: pixel or a group of neighboring pixels in image)

pixel

$$\phi(s,a) = \begin{bmatrix} \mathbb{P}(\text{pixels being building}) \\ \mathbb{P}(\text{pixels being grass}) \\ \mathbb{P}(\text{pixels being sidewalk}) \checkmark \\ \mathbb{P}(\text{pixels being car}) \checkmark \\ \cdots \end{bmatrix}$$

$$\mathbb{E}_{s,a \sim d_\mu^\pi} \phi(s,a) \approx \mathbb{E}_{s,a \sim d_\mu^{\pi^*}} \left[ \phi(s,a) \right] \checkmark$$

$$\theta^{*T} \left( \mathbb{E}_{s,a \sim d_\mu^\pi} \phi(s,a) \right) \approx \theta^{*T} \left( \mathbb{E}_{s,a \sim d_\mu^{\pi^*}} \phi(s,a) \right)$$

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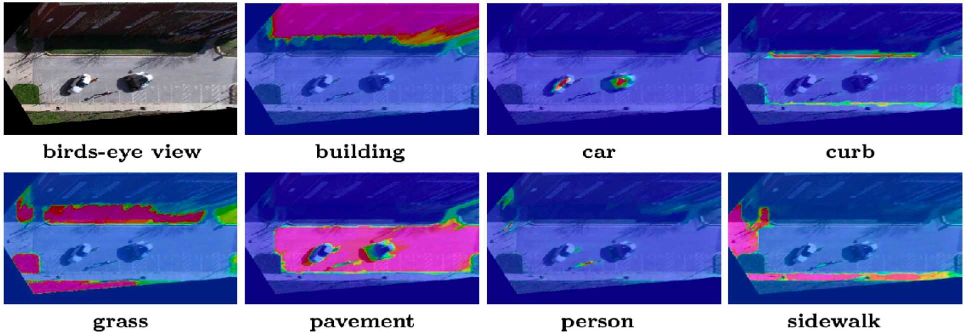


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Maybe colliding with cars or buildings has **high** cost, but walking on sideway or grass has **low** cost

$\theta^{\star T} \phi(s,a)$

# Notation on Distributions

$\mathbb{P}_h^\pi(s, a; \mu)$: probability of visiting $(s, a)$ at time step $h$ following $\pi$

$d_\mu^\pi(s, a) = \sum_{h=0}^{H-1} \mathbb{P}_h^\pi(s, a; \mu)/H$: average state-action distribution

$\rho^\pi(\tau) := \mu_0(s_0)\pi(a_0 \mid s_0)P(s_1 \mid s_0, a_0)\pi(a_1 \mid s_1)\ldots\pi(a_{H-1} \mid s_{H-1})P(s_H \mid s_{H-1}, a_{H-1})$:

Likelihood of the trajectory $\tau$ under $\pi$, i.e., the prob of $\pi$ generating $\tau$

# Detour: Principle of Maximum Entropy

Definition of the Entropy of a distribution:

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Discrete set

Definition of the Entropy of a distribution:

Given a distribution $P \in \Delta(X)$, the entropy is defined as:

$$\text{Entropy}(P) = -\sum_{x} P(x) \cdot \underline{\ln P(x)} = \mathop{\mathbb{E}}_{x \sim P} -\ln P(x)$$

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$U(X)$ is uniform Dist over $X$

Uniform distribution has the highest entropy,

i.e., $\text{Entropy}(U(X)) = -\sum_x \underline{(1/|X|)}\ln(1/|X|) = \ln(|X|)$ ✓

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Deterministic distribution has zero entropy:

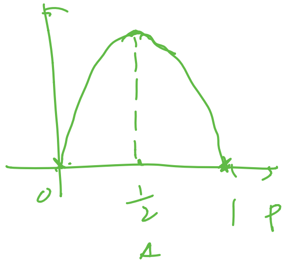
i.e., $\text{Entropy}(\delta(x_0)) = -\underbrace{1 \cdot \ln 1}_{x_0} - \sum_{x \neq x_0} 0 \ln 0 = 0$

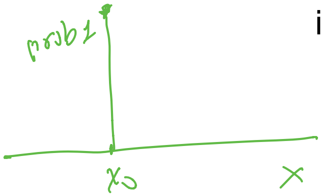
with

$P \in [0,1]$, $+1$

with

$1-P$, $-1$



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$\mathbb{E}_{x\sim p}\left[(x-\mu)(x-\mu)^T\right]$

$$\max_{P \in \Delta(X)} \text{entropy}(P), \quad \text{s.t.,} \quad \mathbb{E}_{x\sim P}[x] = \mu, \quad \mathbb{E}_{x\sim P}[xx^\top] = \underset{\Delta}{\Sigma} + \mu\mu^\top$$

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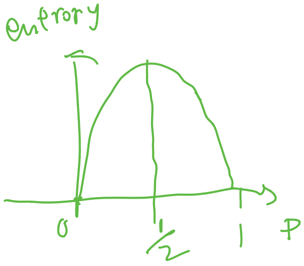
Solution: $P^\star = \mathcal{N}(\mu, \Sigma)$
(proof: out of scope)

# Detour: Principle of Maximum Entropy

In summary:

Maximum Entropy Principle says that:

Among the distributions that satisfy pre-defined constraints (mean & variance),
let's pick the one that is the most uncertain
(uncertainty measured in entropy)



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1. The principle of Maximum Entropy

2. Constrained Optimization

3. The Algorithm: Maximum Entropy Inverse RL

# Constrained Optimization:

Consider the following constrained optimization problem:

$$\min_{x} f(x)$$

$$s.t., g_1(x) = 0, \quad g_2(x) = 0$$

$$\min_{x,y} \; x+y$$

$$s.t. \; x^2+y^2-1=0$$

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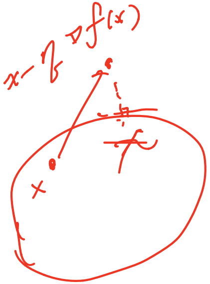
Denote $x^\star$ as the optimal solution here.



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How to solve such constrained optimization problem?

# Constrained Optimization:

$g_1$ $g_2$

Define two Lagrange multiplier $w_1, w_2 \in \mathbb{R}$, we consider the following Lagrange formulation:

$$\min_{x} \left[ \max_{w_1, w_2} f(x) + w_1 g_1(x) + w_2 g_2(x) \right]$$

$\min f(x)$

s.t. $g_1(x) = 0$

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$$\min_{x} \left[ \max_{w_1, w_2} f(x) + w_1 g_1(x) + w_2 g_2(x) \right]$$

For any $x$ that does not satisfy constraints, i.e., $g_1(x) \neq 0$ or $g_2(x) \neq 0$,

we must have: $\underline{\max_{w_1, w_2}} f(x) + w_1 g_1(x) + w_2 g_2(x) = +\infty$

$g_1(x) = 5$

$w_1 = +\infty$

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infeasible

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feasible

For any $x$ that satisfies constraints, i.e., $g_1(x) = 0$ and $g_2(x) = 0$,

we must have: $\max_{w_1,w_2} f(x) + w_1 \underbrace{g_1(x)}_{g_1(x)=0} + w_2 \underbrace{g_2(x)}_{g_2(x)=0} = f(x)$

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$$\min_{x} \left[ \max_{w_1, w_2} f(x) + w_1 g_1(x) + w_2 g_2(x) \right]$$

In other words,

$$\underline{\max_{w_1, w_2} f(x) + w_1 g_1(x) + w_2 g_2(x)} = \begin{cases} +\infty & g_1(x) \neq 0 \text{ or } g_2(x) \neq 0 \text{ i.e, infeasible} \quad (1) \\ f(x) & g_1(x) = g_2(x) = 0 \text{ i.e., feasible} \quad (2) \end{cases}$$

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$-\infty$ infeasible

$f(x)$ feasible

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Thus, solving the Lagrange formulation is equivalent to the original formulation:

$$\arg\min_x \left[ \max_{w_1,w_2} f(x) + w_1 g_1(x) + w_2 g_2(x) \right] = x^\star$$

# Constrained Optimization:

In summary, we have that

$$\arg\min_{x} \left[ \max_{w_1, w_2} f(x) + w_1 g_1(x) + w_2 g_2(x) \right] = x^\star$$

Where $x^\star$ is the optimal solution of the original constrained program:

$$\min_{x} f(x)$$

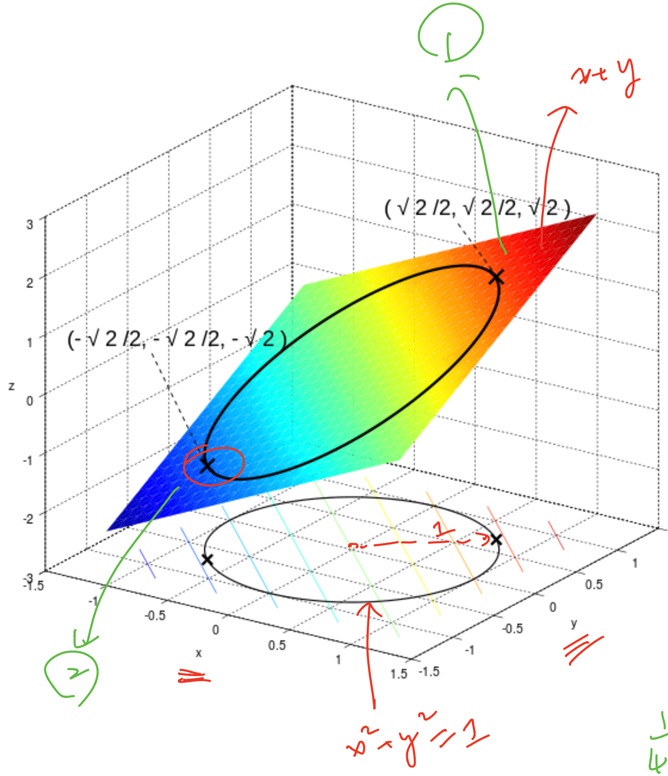
$$s.t., g_1(x) = 0, \quad g_2(x) = 0$$

## Example:

$x, y \in \mathbb{R}$

Introduce $w \in \mathbb{R}$

$$\min_{x,y} x + y, \text{ s.t.}, x^2 + y^2 = 1$$




$$\min_{x,y} \max_{w} \left( x+y + w(x^2+y^2-1) \right)$$

$$\Leftrightarrow \min_{x,y} \max_{w} \; l(x, y, w)$$

$$\frac{dl}{dx}(x,y,w) = 1 + 2wx = 0 \Rightarrow x = -\frac{1}{2w}$$

$$\frac{dl}{dy}(x,y,w) = 1 + 2wy = 0 \Rightarrow y = -\frac{1}{2w}$$

$$\frac{dl}{dw}(x,y,w) = x^2 + y^2 - 1 = 0 \Rightarrow x^2 + y^2 = 1$$

$$\frac{1}{4w^2} + \frac{1}{4w^2} = 1 \Rightarrow w = \pm\sqrt{\frac{1}{2}} \Rightarrow (x,y) = \begin{cases} \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) & (1) \\ \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) & (2) \end{cases}$$

# Constrained Optimization:

We will often be interested in solving the dual version, i.e.,

$$\max_{w_1,w_2} \min_x \underbrace{f(x) + w_1 g_1(x) + w_2 g_2(x)}_{:=\ell(x,w)}$$

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s.t. $g_1(x) \le 0$

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For $t = 0 \to T - 1$

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$$w^t = \begin{bmatrix} w_1^t \\ w_2^t \end{bmatrix}$$

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For $t = 0 \to T - 1$

$$x^t = \arg\min_x f(x) + w_1^t g_1(x) + w_2^t g_2(x) \quad \text{(\# best response: } \arg\min_x \ell(x, w^t))$$

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$$w_1^{t+1} = w_1^t + \eta g_1(x^t)$$

$$w_2^{t+1} = w_2^t + \eta g_2(x^t)$$

(#incremental update: $w^{t+1} = w^t + \eta \underbrace{\nabla_w \ell(x^t, w)}_{A}\Big|_{w=w^t}$)

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**Return**: $\bar{x} = \sum_{t=0}^{T-1} x_t / T$

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$$w_1^{t+1} = w_1^t + \eta g_1(x^t)$$

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$$w_2^{t+1} = w_2^t + \eta g_2(x^t)$$

**Return**: $\bar{x} = \sum_{t=0}^{T-1} x_t / T$

Informal theorem: when $f, g$ are convex, $\bar{x} \to x^\star$, as $T \to \infty$

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1. The principle of Maximum Entropy

2. Constrained Optimization

$\min f(x)$

s.t.

$g(x) \leq 0$

$h(x) = 0$

3. The Algorithm: Maximum Entropy Inverse RL

# Maximum Entropy Inverse RL:

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$$(s_i^\star, a_i^\star) \sim d_\mu^{\pi^\star}$$

$$\approx \frac{1}{N}\sum_{i=1}^{N} \phi(s_i^\star, a_i^\star)$$

Q: we want to find a policy $\pi$ such that $\mathbb{E}_{s,a\sim d_\mu^\pi}\phi(s,a) = \mathbb{E}_{s,a\sim d_\mu^{\pi^\star}}\phi(s,a)$

(Note **linear cost assumption** implies $\pi$ is as good as $\pi^\star$)

But there are potentially many such policies…

$$\mathbb{E}_{s,a\sim d_\mu^{\pi}}\left(\theta^{\star\top}\phi(s,a)\right) - \mathbb{E}_{s,a\sim d_\mu^{\pi^\star}}\left(\theta^{\star\top}\phi(s,a)\right)$$

$$= \theta^{\star\top}\underbrace{\left[\mathbb{E}_{s,a\sim d_\mu^{\pi}}\phi(s,a) - \mathbb{E}_{s,a\sim d_\mu^{\pi^\star}}\phi(s,a)\right]}_{=0}$$

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**The principle of Maximum Entropy:**

Find a policy $\pi$ that maximizes some entropy while subject to the constraint:

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Q: we want to find a policy $\pi$ such that $\mathbb{E}_{s,a\sim d_\mu^\pi}\phi(s,a) = \mathbb{E}_{s,a\sim d_\mu^{\pi^\star}}\phi(s,a)$

(Note **linear cost assumption** implies $\pi$ is as good as $\pi^\star$)

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encourage diverse

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This can be estimated using expert data:

$$\sum_{i=1}^N \phi(s_i^\star, a_i^\star)/N$$

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$$\arg\max_{\pi} \mathbb{E}_{s\sim d_\mu^\pi}\left[\text{entropy}(\pi(\cdot\,|\,s))\right] = \arg\min_{\pi} \mathbb{E}_{s,a\sim d_\mu^\pi} \ln \pi(a\,|\,s)$$

# Maximum Entropy Inverse RL:

We arrive at the following constraint optimization problem:

$$\arg\min_{\pi} \mathbb{E}_{s,a\sim d_\mu^\pi} \ln \pi(a \mid s)$$

$$s.t, \mathbb{E}_{s,a\sim d_\mu^\pi} \phi(s,a) = \mathbb{E}_{s,a\sim d_\mu^{\pi^\star}} \phi(s,a)$$

Introduce the Lagrange multiplier $w \in \mathbb{R}^d$ (we have d many constraints), consider the max-min dual version:

$$\max_{w\in\mathbb{R}^d} \min_{\pi} \mathbb{E}_{s,a\sim d_\mu^\pi} \ln \pi(a \mid s) + w^\top \left( \mathbb{E}_{s,a\sim d_\mu^\pi} \phi(s,a) - \mathbb{E}_{s,a\sim d_\mu^{\pi^\star}} \phi(s,a) \right)$$

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Next lecture,
we will design algorithm (in high level, it is the iterative algorithm framework)
for this $\max - \min$ problem