Review

What we covered this semester:

1. Basics of Markov Decision Process

2. Planning in MDP: VI and PI

3. Learning: Model-based RL, Policy Optimization, Bandit

4. Imitation

Basics of MDP

Understanding those widely used notations and terminologies:

$$\pi, V^{\pi}, Q^{\pi}, \pi^{\star}, V^{\star}, Q^{\star}$$

$$\mathbb{P}_h^{\pi}(s; \mu), d_{\mu}^{\pi}(s)$$

Basics of MDP

Bellman Equation and Bellman Optimality:

$$\forall s, a: \quad Q^\pi(s,a) = r(s,a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} V^\pi(s')$$

$$Q^\star(s,a) = r(s,a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} \max_{a'} Q^\star(s',a'), \forall s, a$$

Planning in MDP

Q: When P and r are known, we can compute $\pi^\star$ via VI or PI

Algorithm 1: Value Iteration

$$\forall s, a : Q^{t+1}(s, a) = r(s, a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} \max_{a'} Q^t(s', a')$$

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$$\|Q^{t+1} - Q^\star\|_\infty = \|\mathcal{T} Q^t - \mathcal{T} Q^\star\|_\infty \leq \gamma \|Q^t - Q^\star\|_\infty$$

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1. Policy Evaluation: $Q^{\pi^t}(s, a), \forall s, a$

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2. Policy Improvement $\pi^{t+1}(s) := \arg\max_a Q^{\pi^t}(s,a), \forall s;$

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Key Properties:

Monotonic improvement + hit $\pi^\star$ in at most A^S many iterations (hw1)

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2. Reset from fixed initial state distribution μ;

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1. Model fitting:

$\forall s, a$: collect N next states, $s'_i \sim P(\cdot \mid s, a), i \in [N]$; set

$$\widehat{P}(s' \mid s, a) = \frac{\sum_{i=1}^{N} \mathbf{1}\{s'_i = s'\}}{N};$$

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2. Planning w/ the learned model:

$$\widehat{\pi}^\star = \textbf{PI}\left(\widehat{P}, r\right)$$

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An Important Lemma that is widely used in model-based approach

Simulation Lemma:

$$\widehat{V}^{\pi}(s_0) - V^{\pi}(s_0) = \frac{\gamma}{1-\gamma}\mathbb{E}_{s,a\sim d^{\pi}_{s_0}}\left[\mathbb{E}_{s'\sim \widehat{P}(\cdot|s,a)}\widehat{V}^{\pi}(s') - \mathbb{E}_{s'\sim P(\cdot|s,a)}\widehat{V}^{\pi}(s')\right]$$

$$\leq \frac{\gamma}{(1-\gamma)^2}\mathbb{E}_{s,a\sim d^{\pi}_{s_0}}\left\| \widehat{P}(\cdot\,|\,s,a) - P(\cdot\,|\,s,a)\right\|_1$$

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REINFORCE:

$$\nabla_\theta J(\pi_\theta) = \mathbb{E}_{\tau \sim \rho^{\pi_\theta}} \left[R(\tau) \sum_{h=0}^{H-1} \nabla_\theta \ln \pi_\theta(a_h \mid s_h) \right]$$

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The Q-version:

$$\nabla_\theta J(\pi_\theta) = \frac{1}{1-\gamma} \mathbb{E}_{s,a \sim d_\mu^{\pi_\theta}} \left[\nabla_\theta \ln \pi_\theta(a \mid s) \left(Q^{\pi_\theta}(s, a) - b(s) \right) \right]$$

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Given a differentiable parameterized policy $\pi_\theta(a \mid s)$, w/ $\theta \in \mathbb{R}^d$:

The Natural Policy Gradient:

$$\theta_{t+1} = \theta_t + \eta F_{\theta_t}^{-1} \nabla_\theta J(\pi_{\theta_t})$$

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Instead of using Euclidean distance metric, we use local geometry metric

$$d(\theta, \theta_t) := (\theta - \theta_t)^\top F_{\theta_t}(\theta - \theta_t)$$

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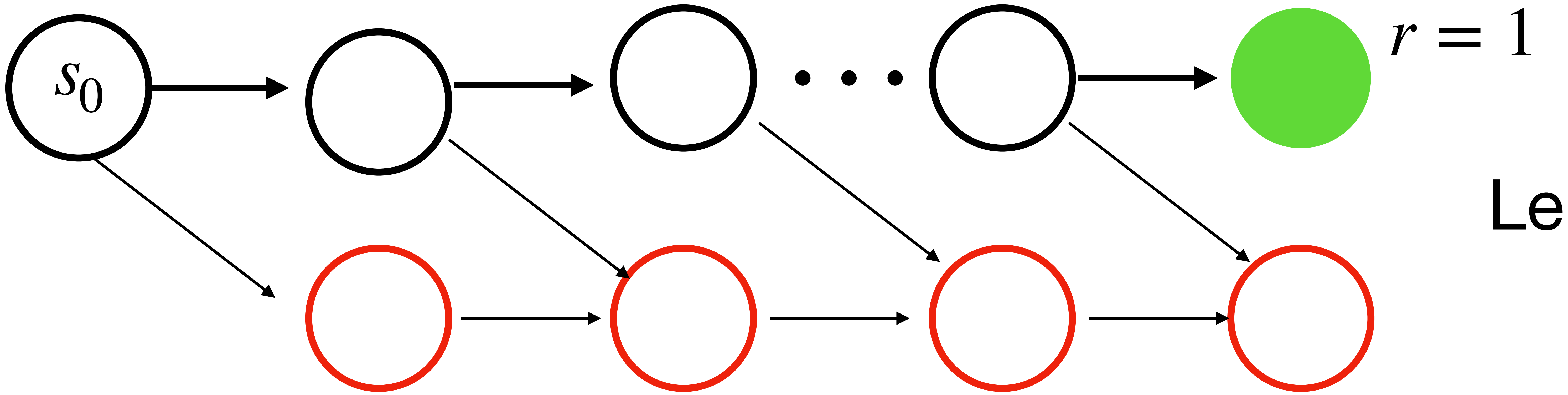
However, PG fails on problems that require exploration..



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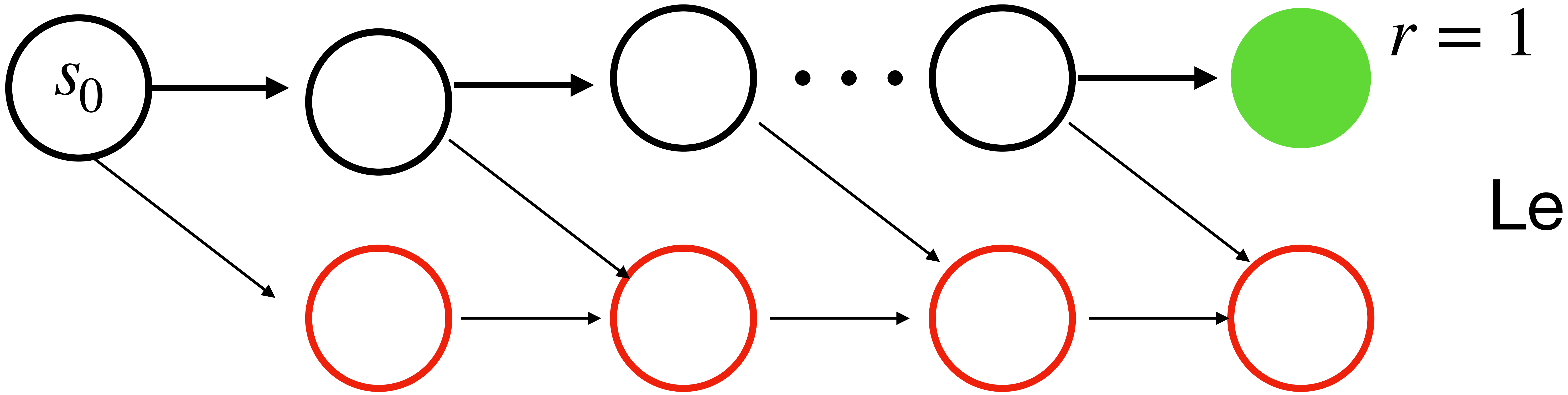


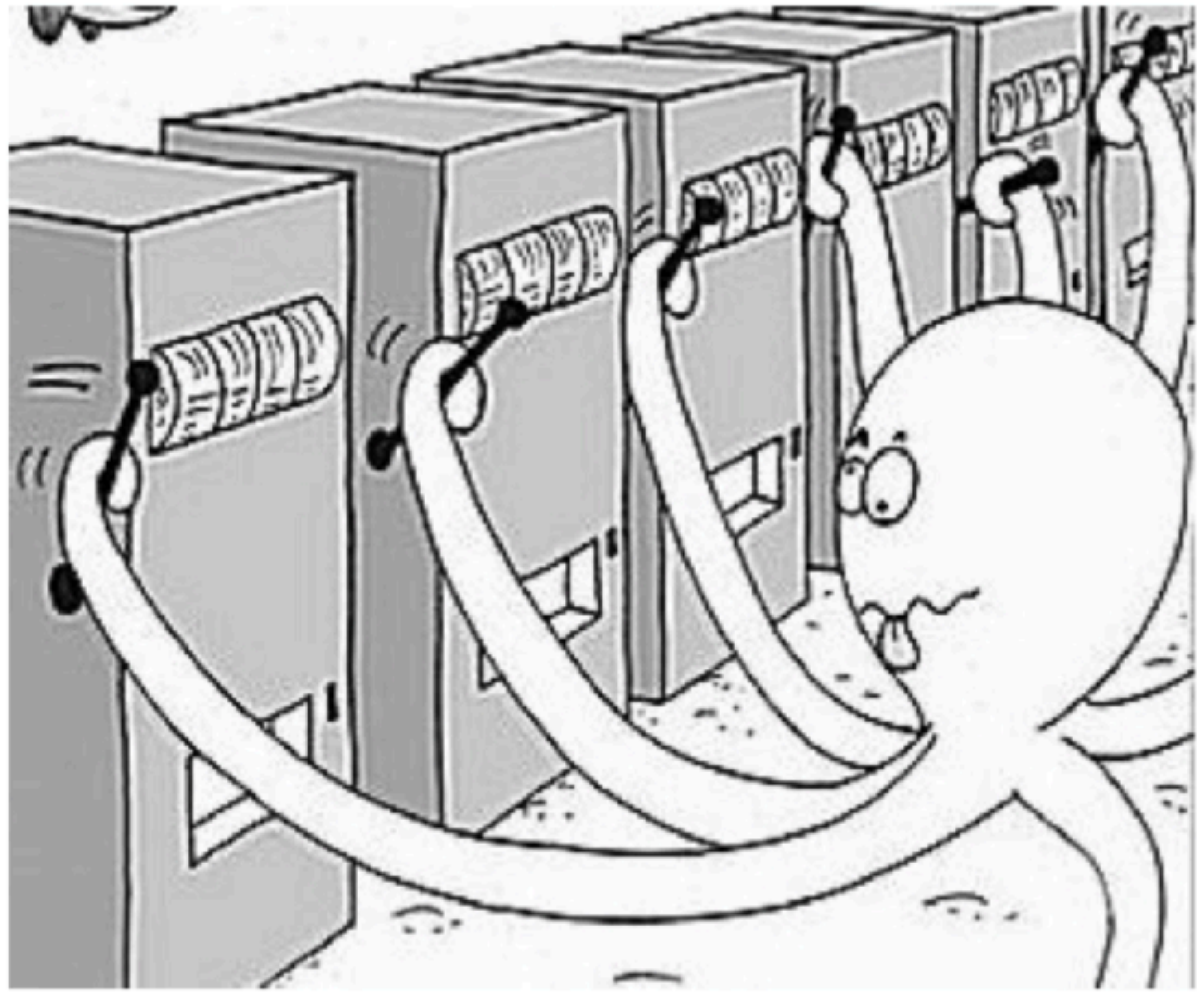
Length: H

What is the probability of a random policy generating a trajectory that hits the goal?

Learning:

Q: How to learn efficient (i.e., balance explore and exploit) in Multi-armed Bandit setting:

We have K many arms (or actions): $a_1, \ldots, a_K$



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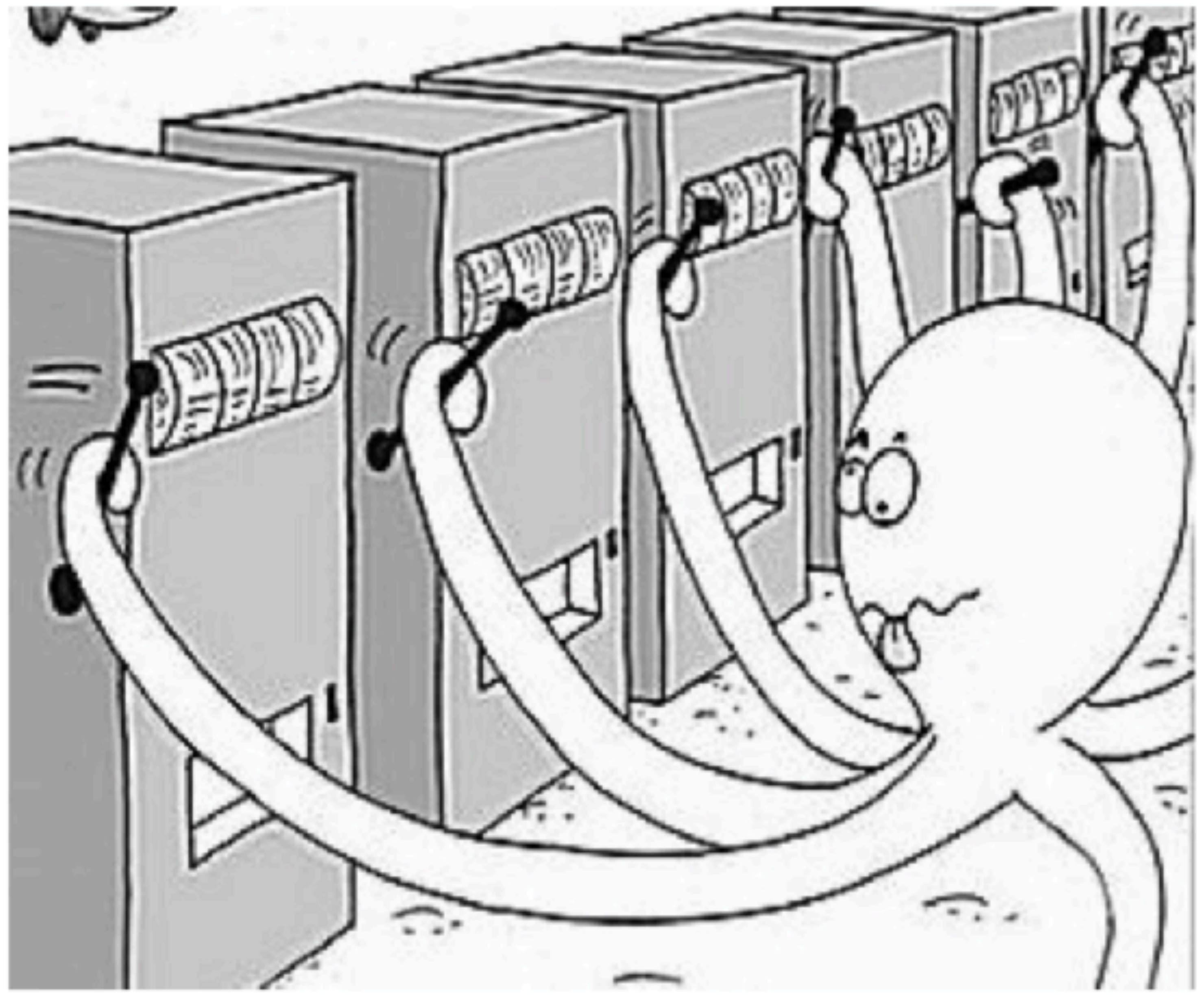
We have K many arms (or actions): $a_1, \ldots, a_K$

Each arm has a unknown reward distribution,
i.e., $\nu_i \in \Delta([0,1])$,
w/ mean $\mu_i = \mathbb{E}_{r \sim \nu_i}[r]$



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2. For all future T-KN rounds, play the best empirical arm $\hat{I} = \arg\max_{i \in [K]} \hat{\mu}_i$

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2. The Upper Confidence Bound Algorithm

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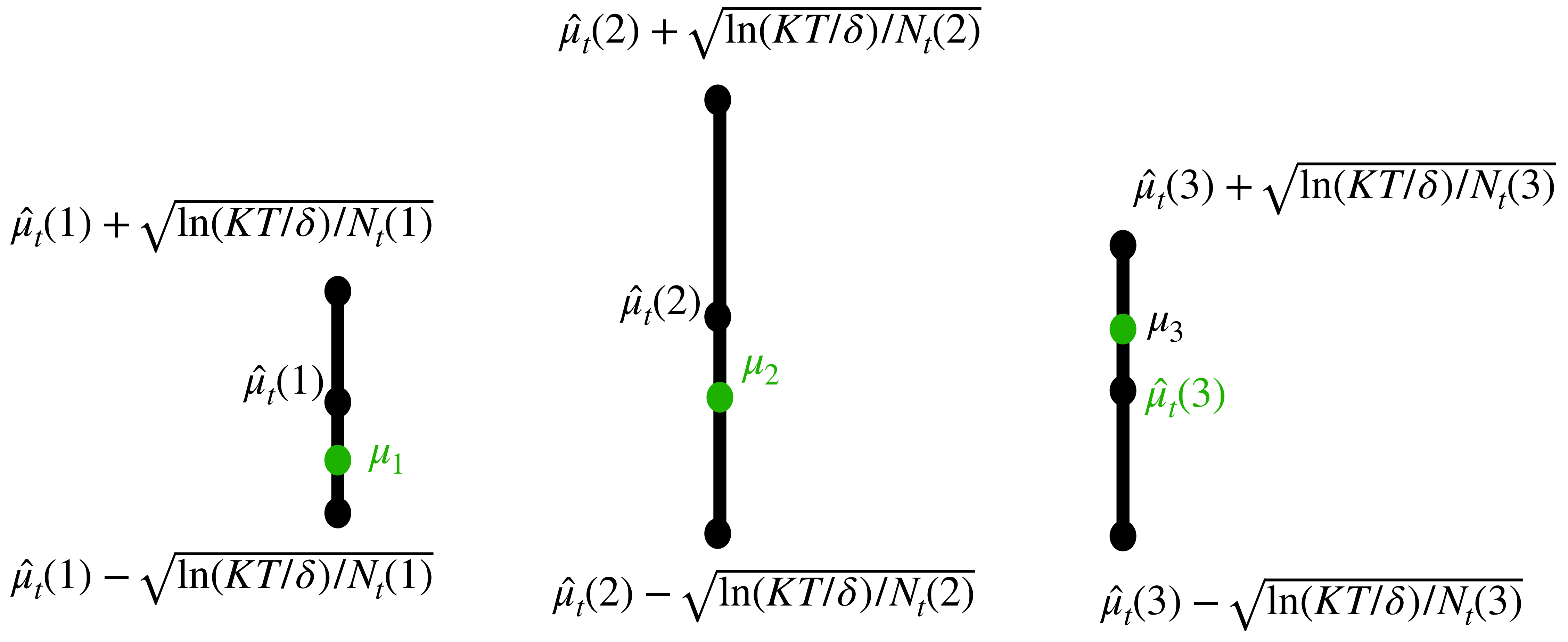
For $t = 0 \rightarrow T-1$:

$$I_t = \arg\max_{i \in [K]} \left(\hat{\mu}_t(i) + \sqrt{\frac{\ln(KT/\delta)}{N_t(i)}} \right)$$

(# Upper-conf-bound of arm i)



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1. Explore and Commit (or ϵ-greedy)

Importance weighting + Reward-sensitive Classification

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Importance weighting + Reward-sensitive Classification

1. For the first N rounds, randomly try actions to construct a classification dataset:

$$\{x_i, \hat{\mathbf{r}}_i\}_{i=0}^{N-1}$$

$$\hat{\mathbf{r}} := \begin{bmatrix} 0 \\ 0 \\ \cdots \\ r_t/p(a_t) \\ 0, \\ \cdots \\ 0 \end{bmatrix}$$

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2. **Call RSC oracle:** $\hat{\pi} = \arg\max_{\pi \in \Pi} \sum_{i=0}^{N-1} \hat{\mathbf{r}}_i[\pi(x_i)]$

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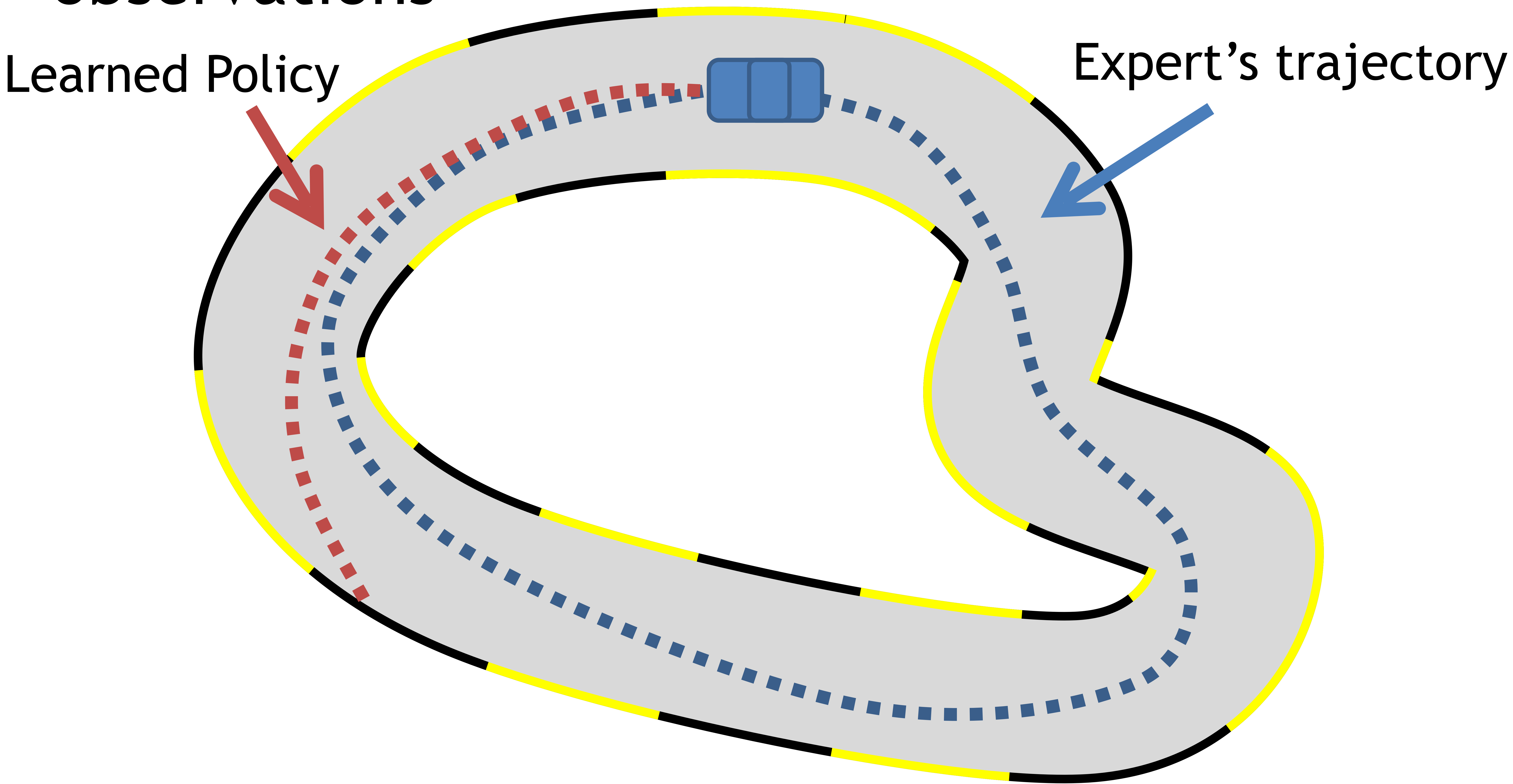
e.g., Negative log-likelihood (NLL): $\ell(\pi, s, a^\star) = -\ln \pi(a^\star \mid s^\star)$ (used in AlphaGo)

Distribution shift!

[Pomerleau89,Daume09]

- Predictions affect future inputs/ observations



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(Recap the connection to online learning and how it avoids distribution shift..)

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Assume the ground truth reward $r(s,a) = (\theta^\star)^\top \phi(s,a)$

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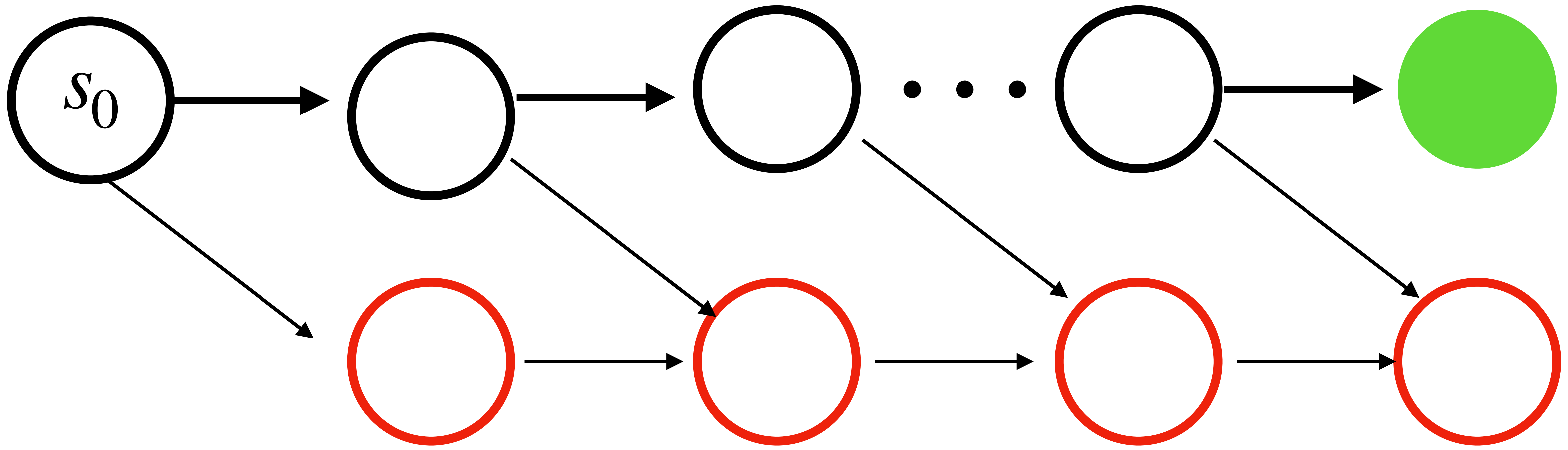
Iterate: $w_{t+1} = w_t + \eta\nabla_w \ell(w_t,\pi_t)$, $\pi^{t+1} = \arg\min_\pi \ell(w_{t+1},\pi)$ via Soft-VI

What we did not cover:

1. How to do strategic exploration in RL? Can we do it in poly time?



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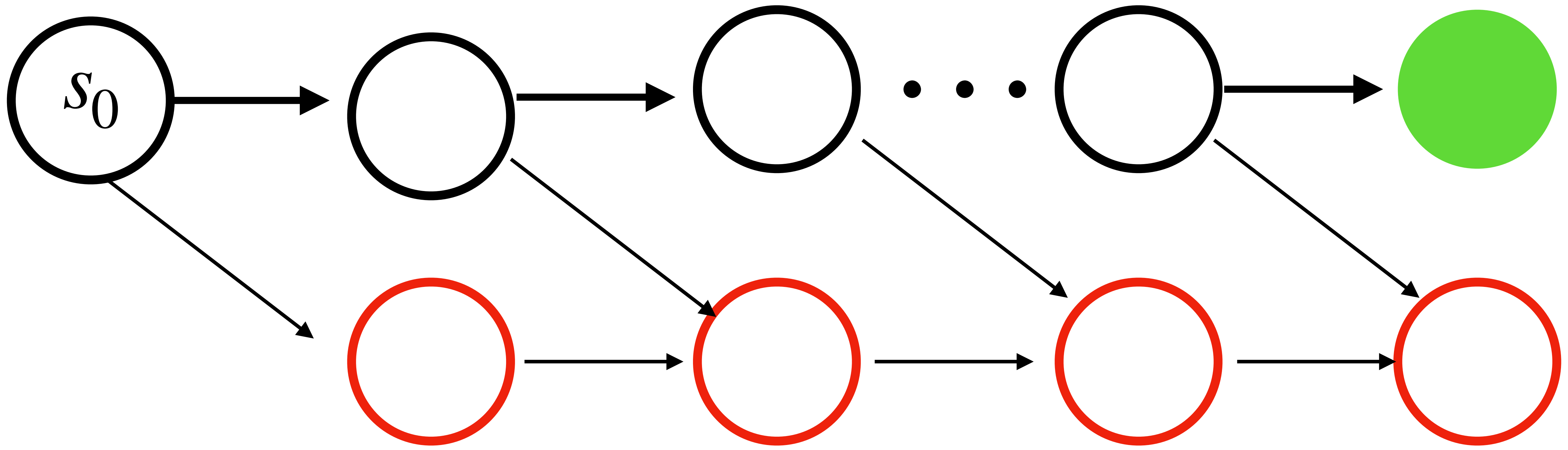

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Though the RL objective function is non-convex wrt policy, under some cases, PG provably converges to global optimal policies!

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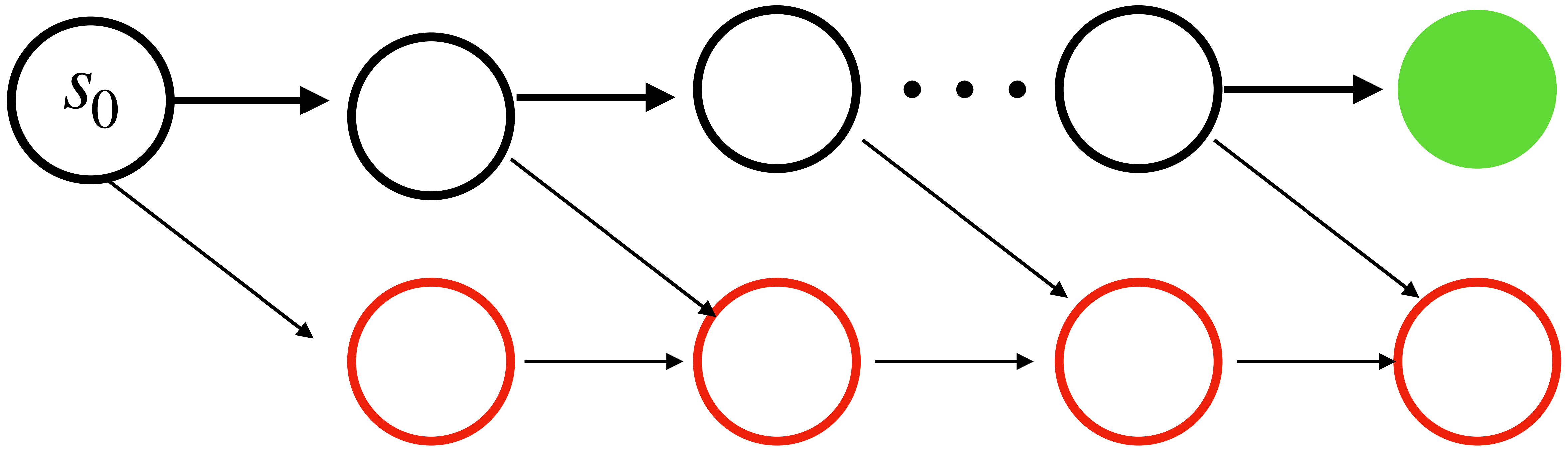

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3. Deep Reinforcement Learning

Most of the time, it is Deep nets (e.g., policies) + RL