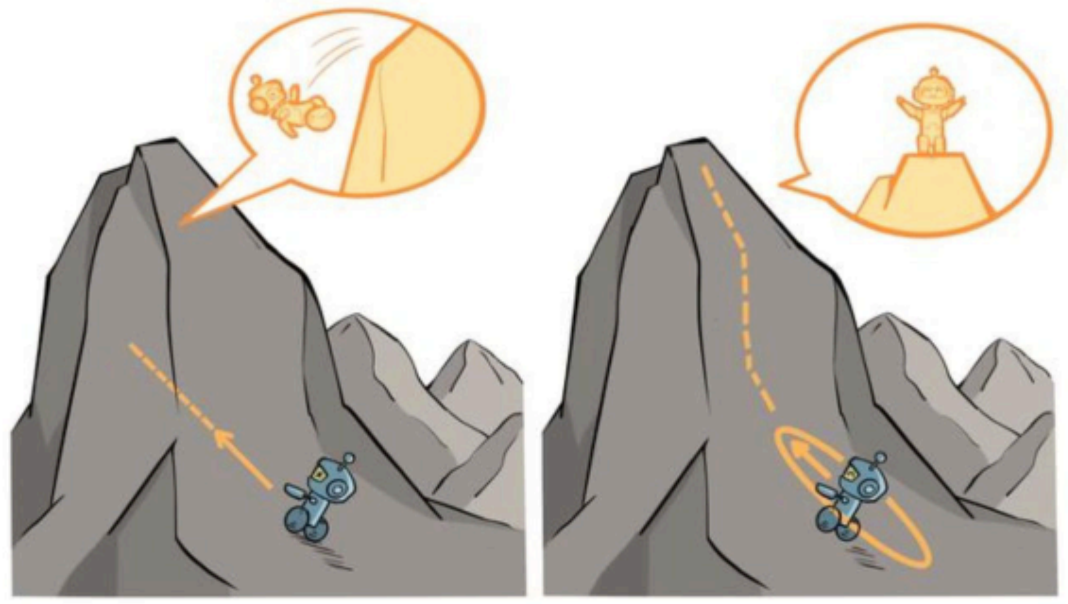
Trust Region Policy Optimization



Nicolas Espinosa Dice

Slides adapted from Wen Sun
(with inspiration from Benjamin Eysenbach)

Improving Policy Gradient

Lecture 10: **Policy gradient**

Lecture 11: **Variance Reduction** via advantage estimation

Lecture 12 (today!): **Leverage the geometry** via Natural Policy Gradient (NPG)

Recap Policy Gradient

$$J(\pi_\theta) = \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r(s_h, a_h) \,|\, s_0 \sim \mu, a \sim \pi_\theta\right]$$

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The most commonly used formulation:

$$\nabla_\theta J(\pi_{\theta_t}) = \mathbb{E}_{s,a \sim d_\mu^{\pi_{\theta_t}}}\left[\nabla_\theta \ln \pi_{\theta_t}(a \,|\, s) A^{\pi_{\theta_t}}(s, a)\right]$$

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Algorithm: Stochastic Gradient Ascent

Policy Parameterization

Recall that we consider parameterized policy $\pi_\theta(\cdot \mid s) \in \Delta(A), \forall s$

1. Softmax linear Policy

Feature vector $\phi(s, a) \in \mathbb{R}^d$, and parameter $\theta \in \mathbb{R}^d$

$$\pi_\theta(a \mid s) = \frac{\exp(\theta^\top \phi(s, a))}{\sum_{a'} \exp(\theta^\top \phi(s, a'))}$$

2. Neural Policy:

Neural network $f_\theta : S \times A \mapsto \mathbb{R}$

$$\pi_\theta(a \mid s) = \frac{\exp(f_\theta(s, a))}{\sum_{a'} \exp(f_\theta(s, a'))}$$

Outline

1. Motivation behind trust-region policy optimization

2. Quick intro on KL-divergence

3. A Trust-Region Formulation for Policy Optimization

4. Algorithm: Natural Policy Gradient

Two Observations

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Observation 1: Policy gradient estimates have high variance

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Today's Question

Two Observations

Observation 1: Policy gradient estimates have high variance
Observation 2: Small changes in policy's parameters can lead to *large changes in policy*

Today's Question

Can we optimize the policy's parameters without drastically changing the policy?

Intuition Behind Observation #2

Observation 2: Small changes in policy's parameters can lead to *large changes in policy*

Example

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Train a robot to “run” forward as fast as possible

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State: joint angles, center of mass, velocity, etc

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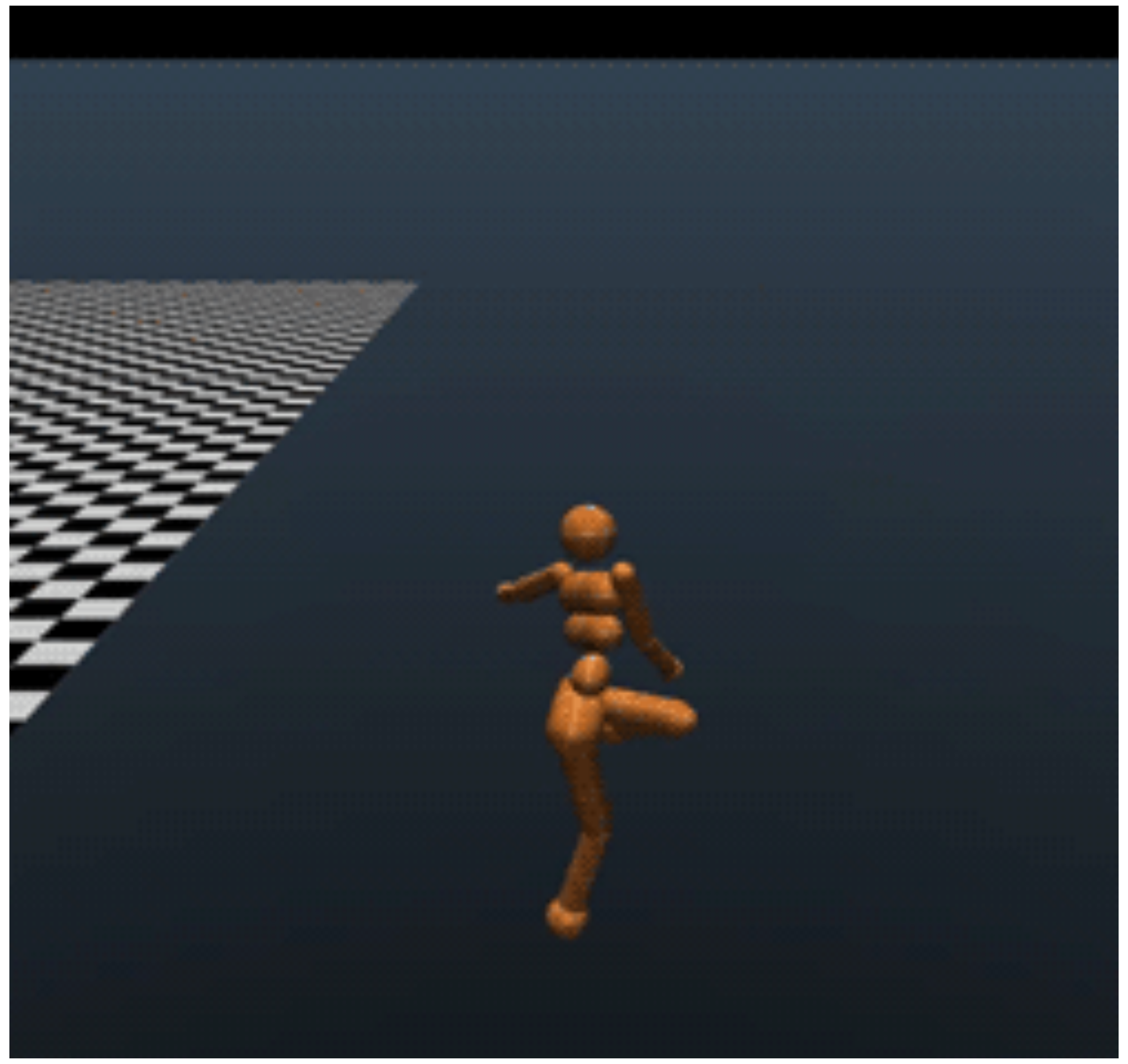

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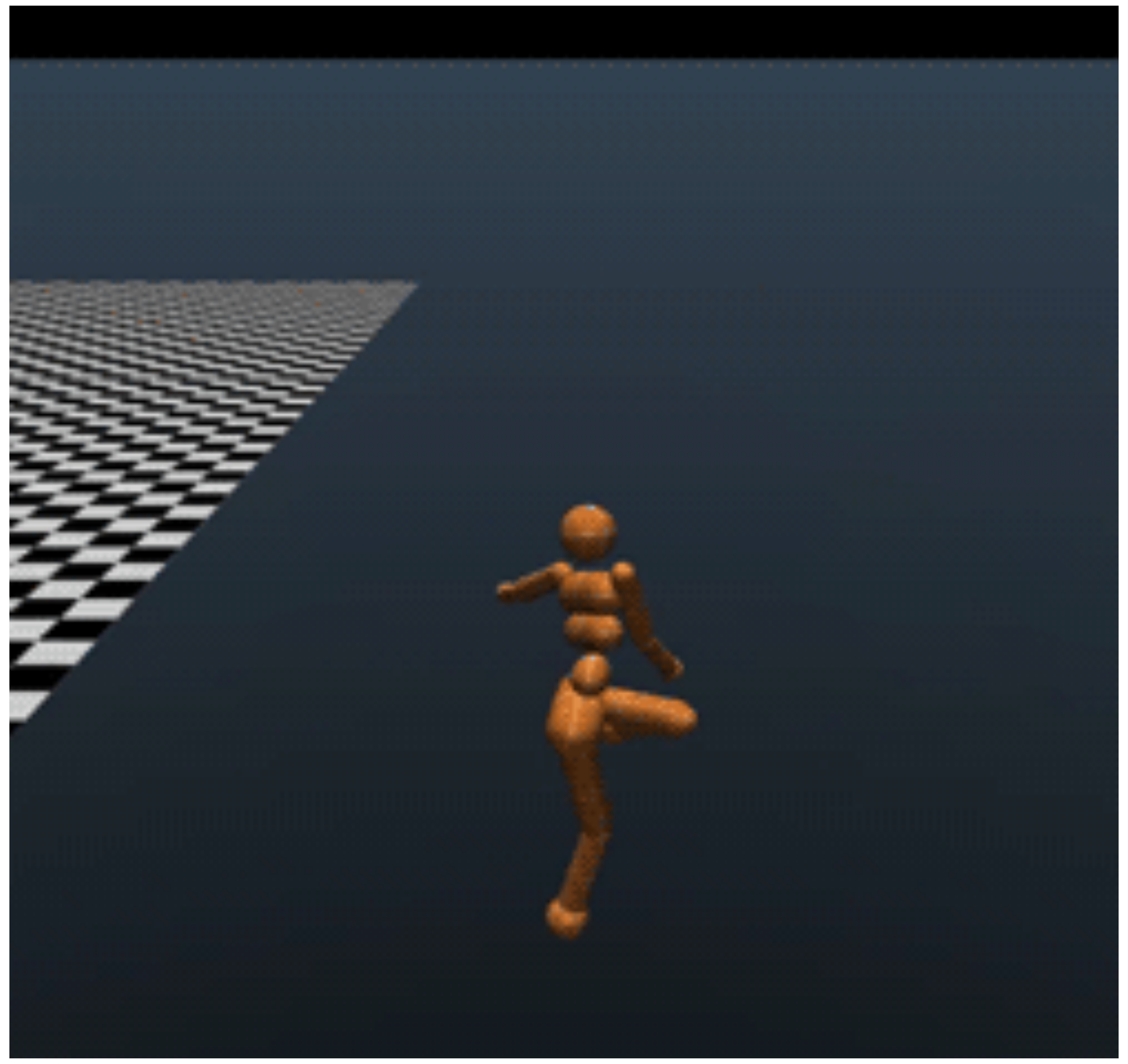

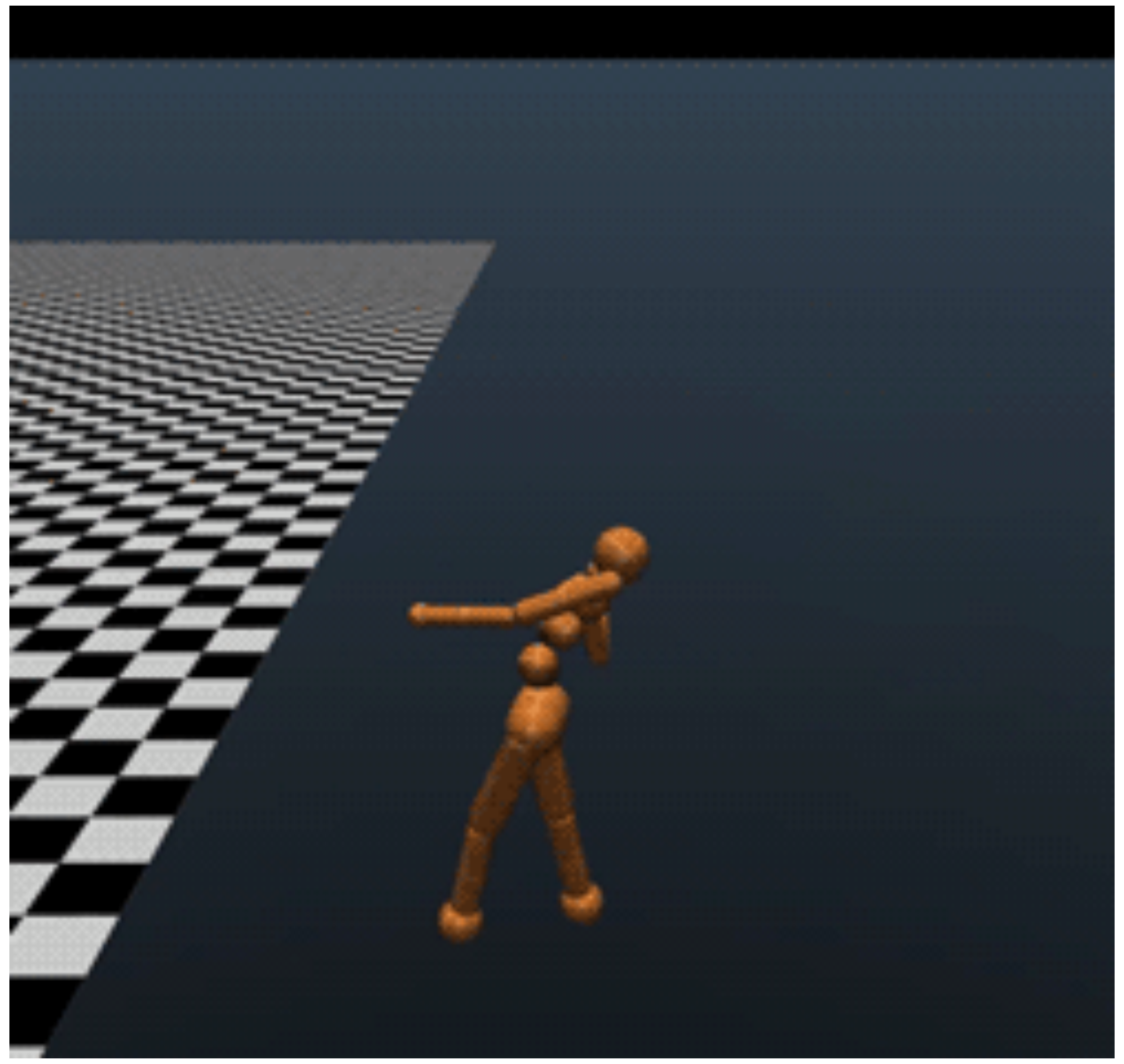

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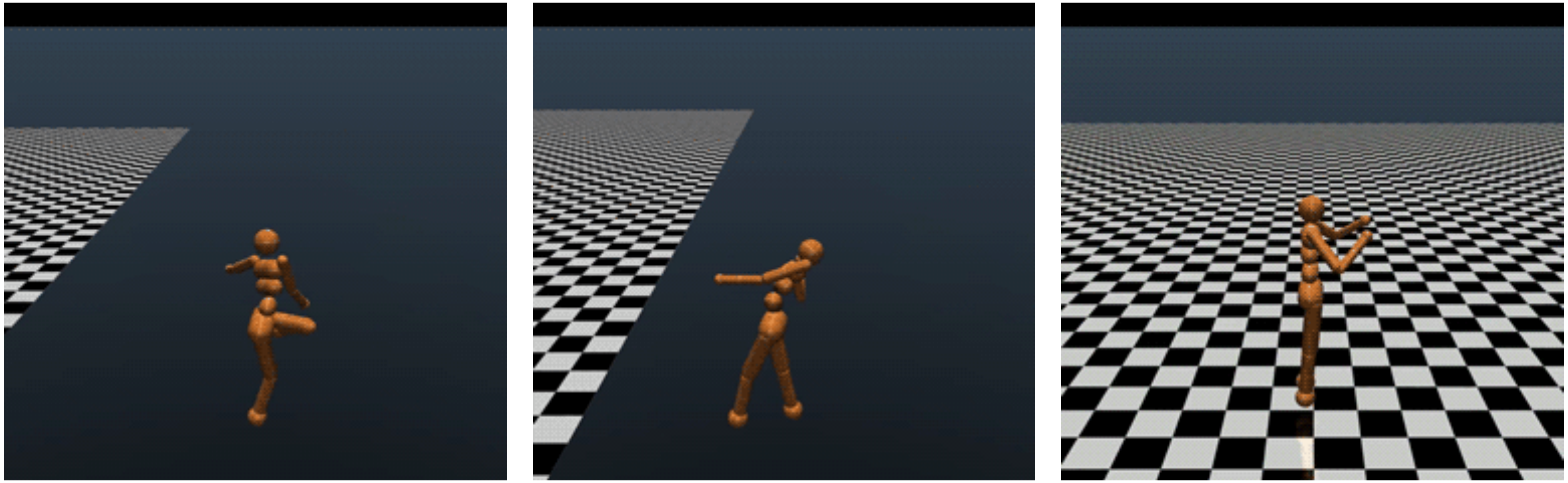

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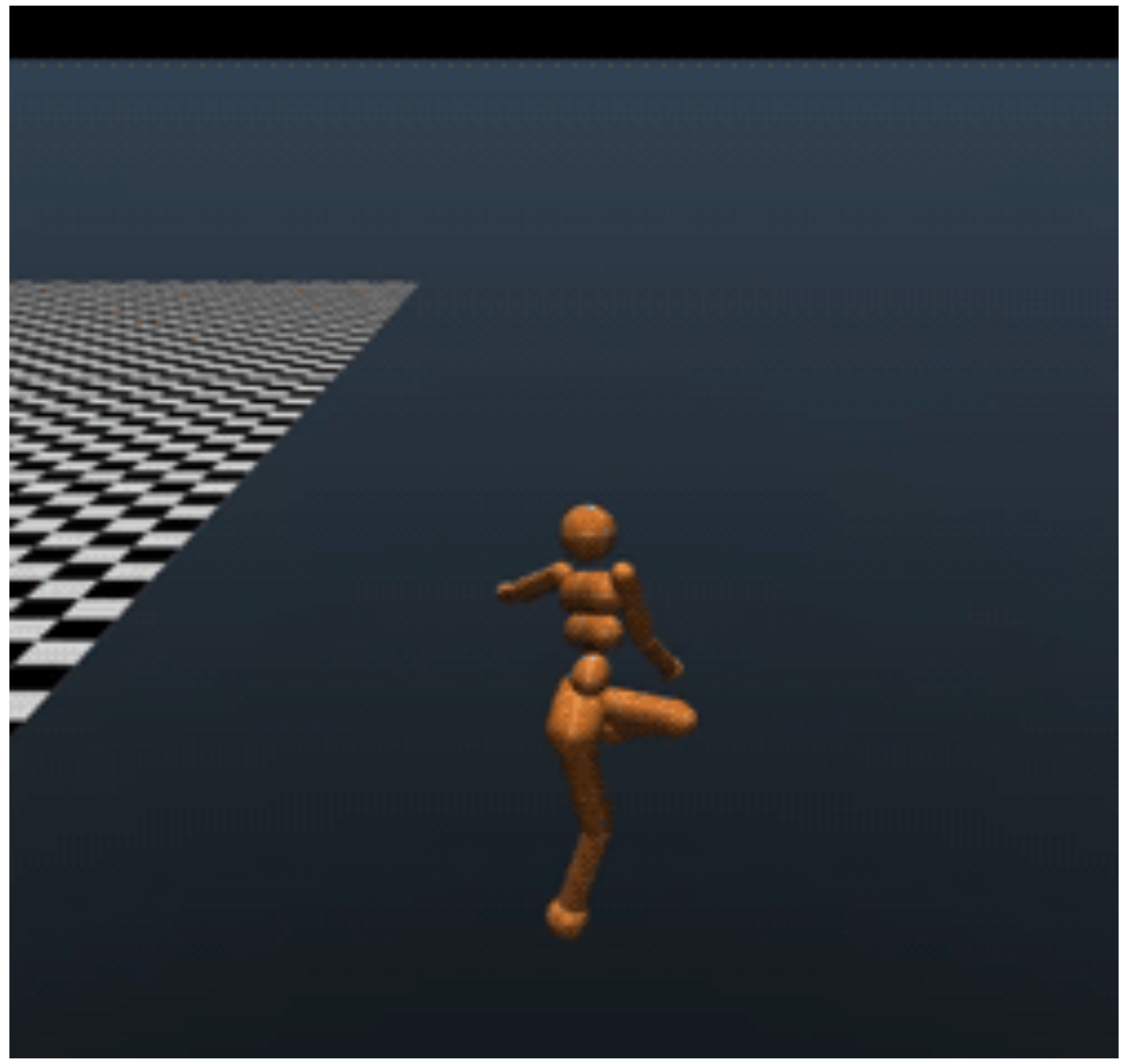
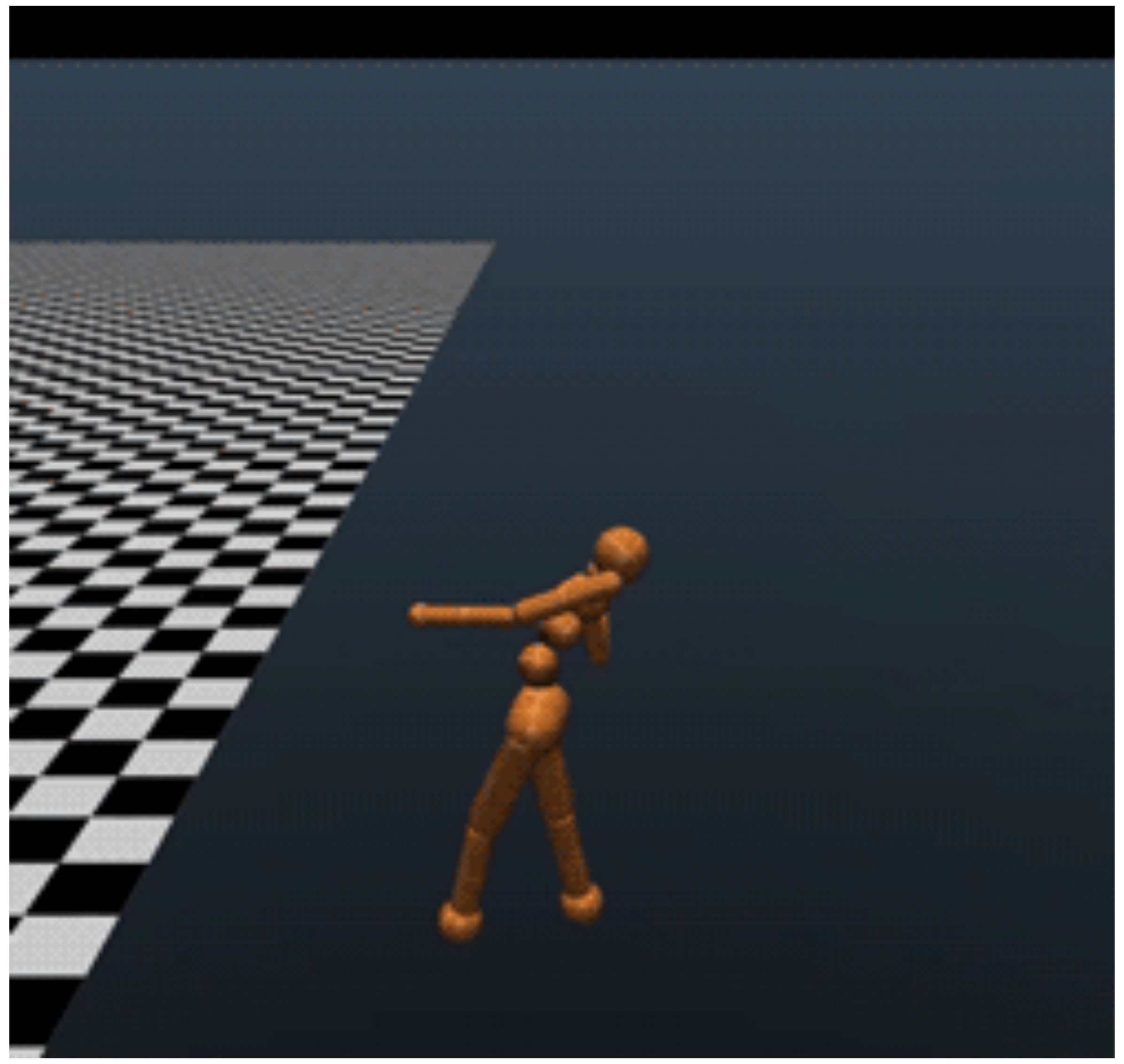
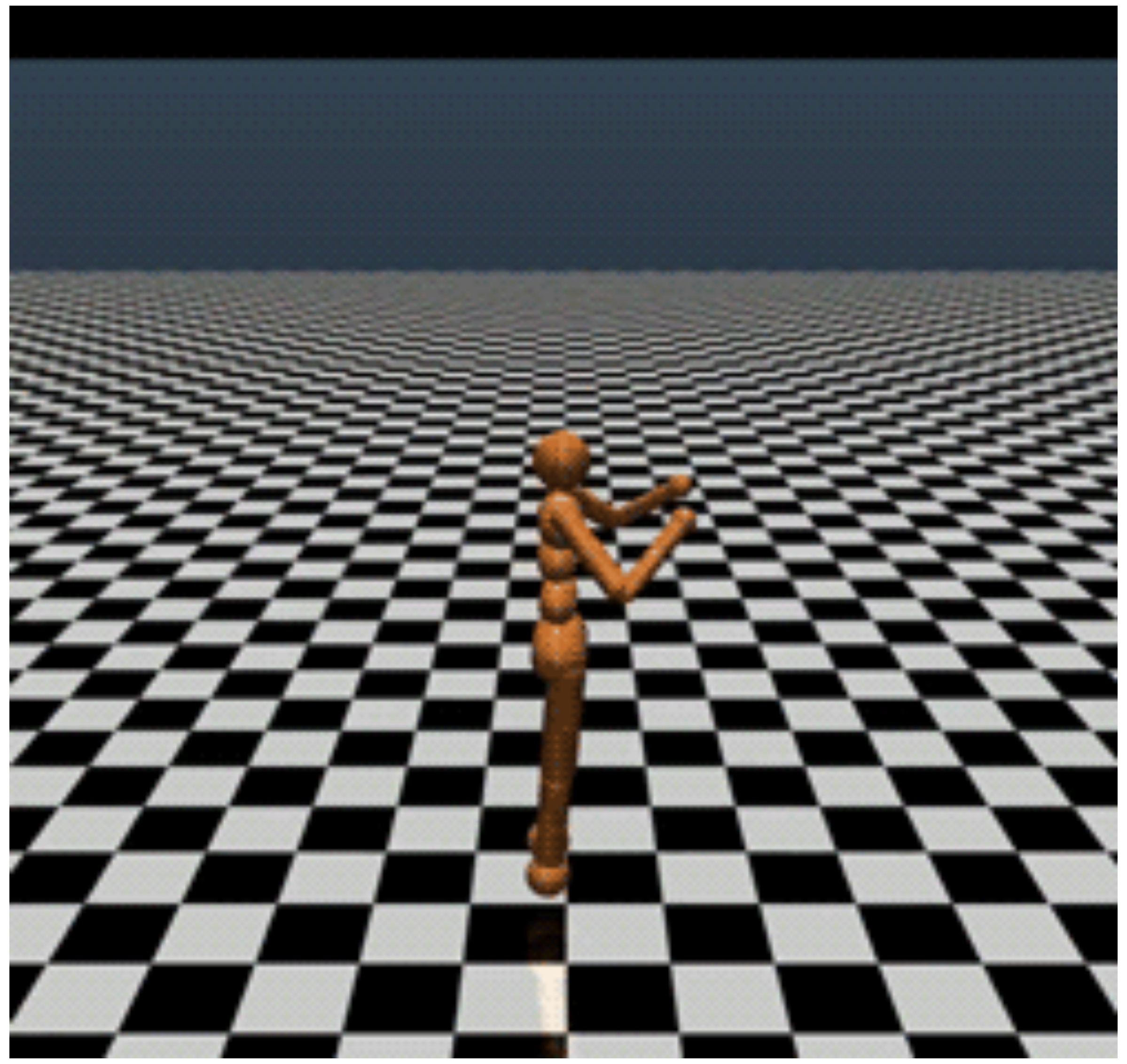
Note: All three robots achieve high reward!







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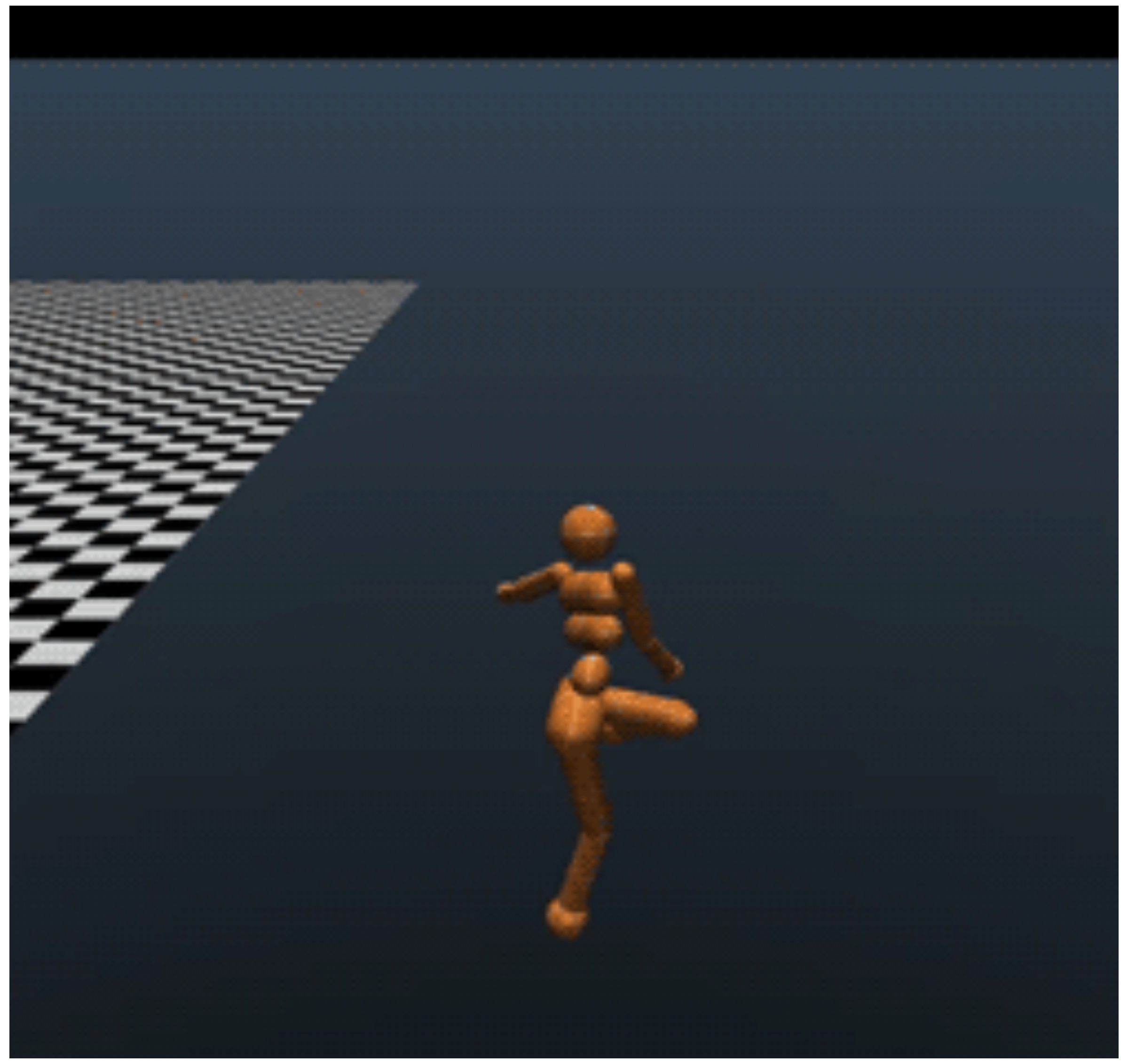

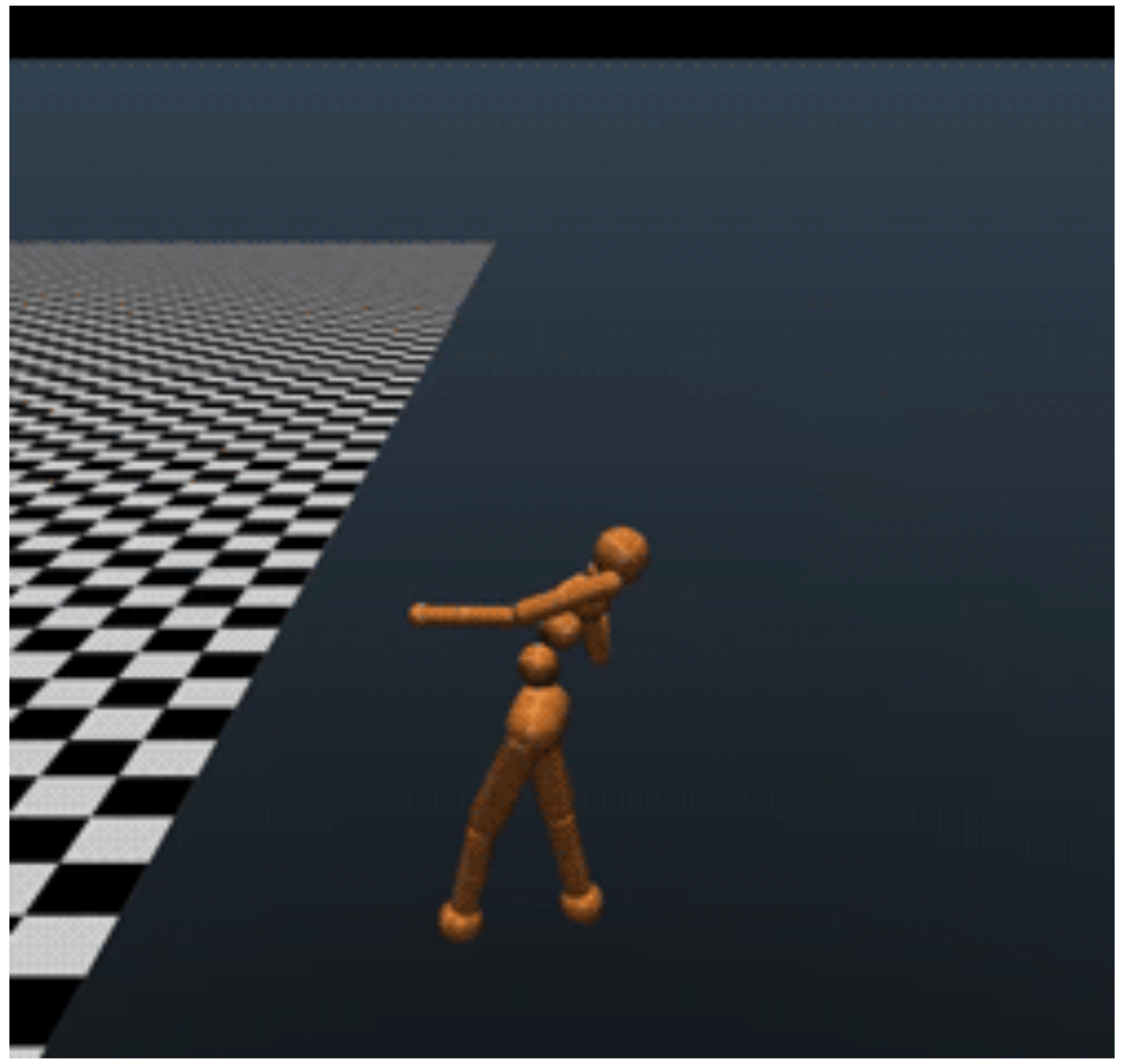

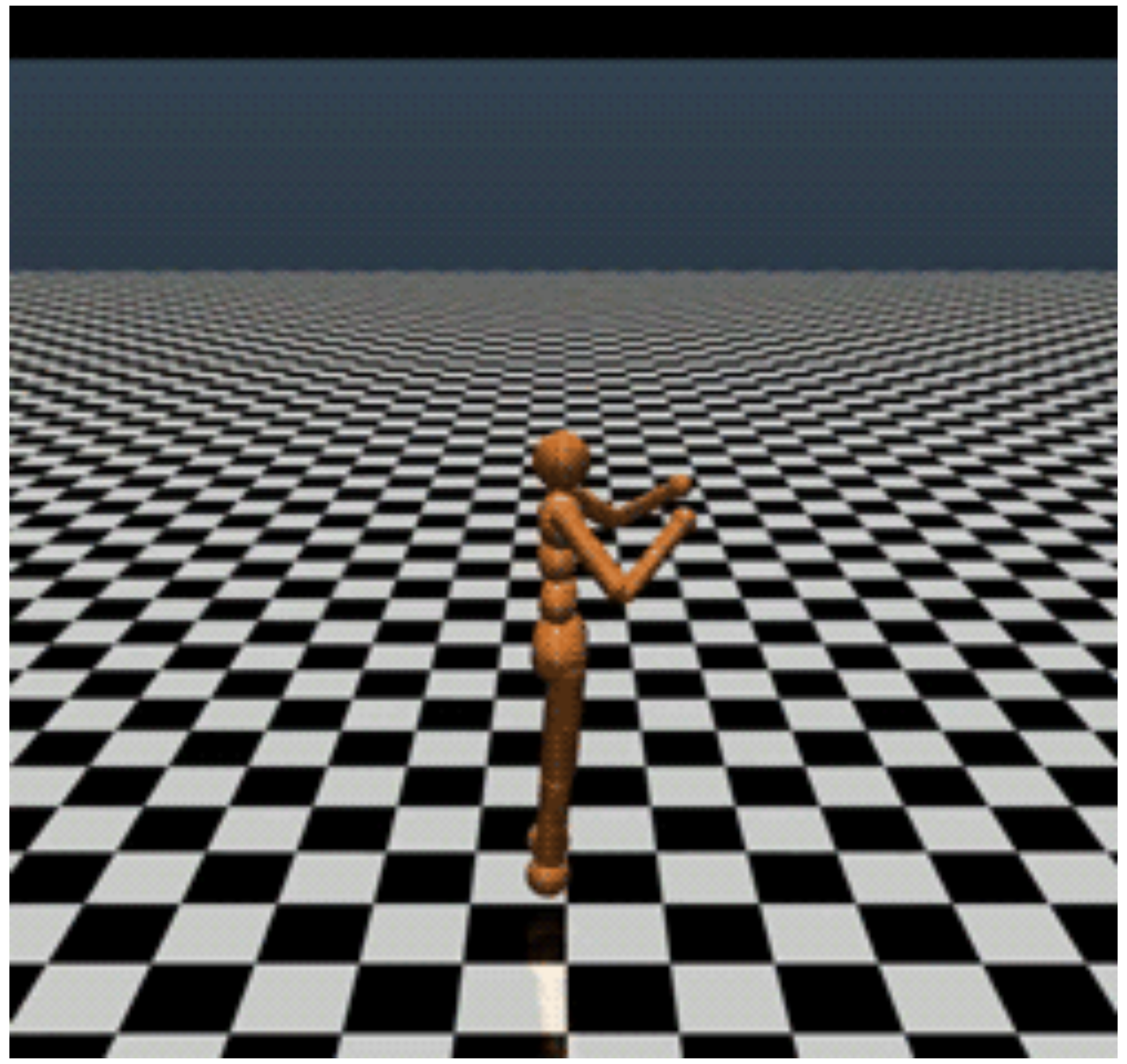

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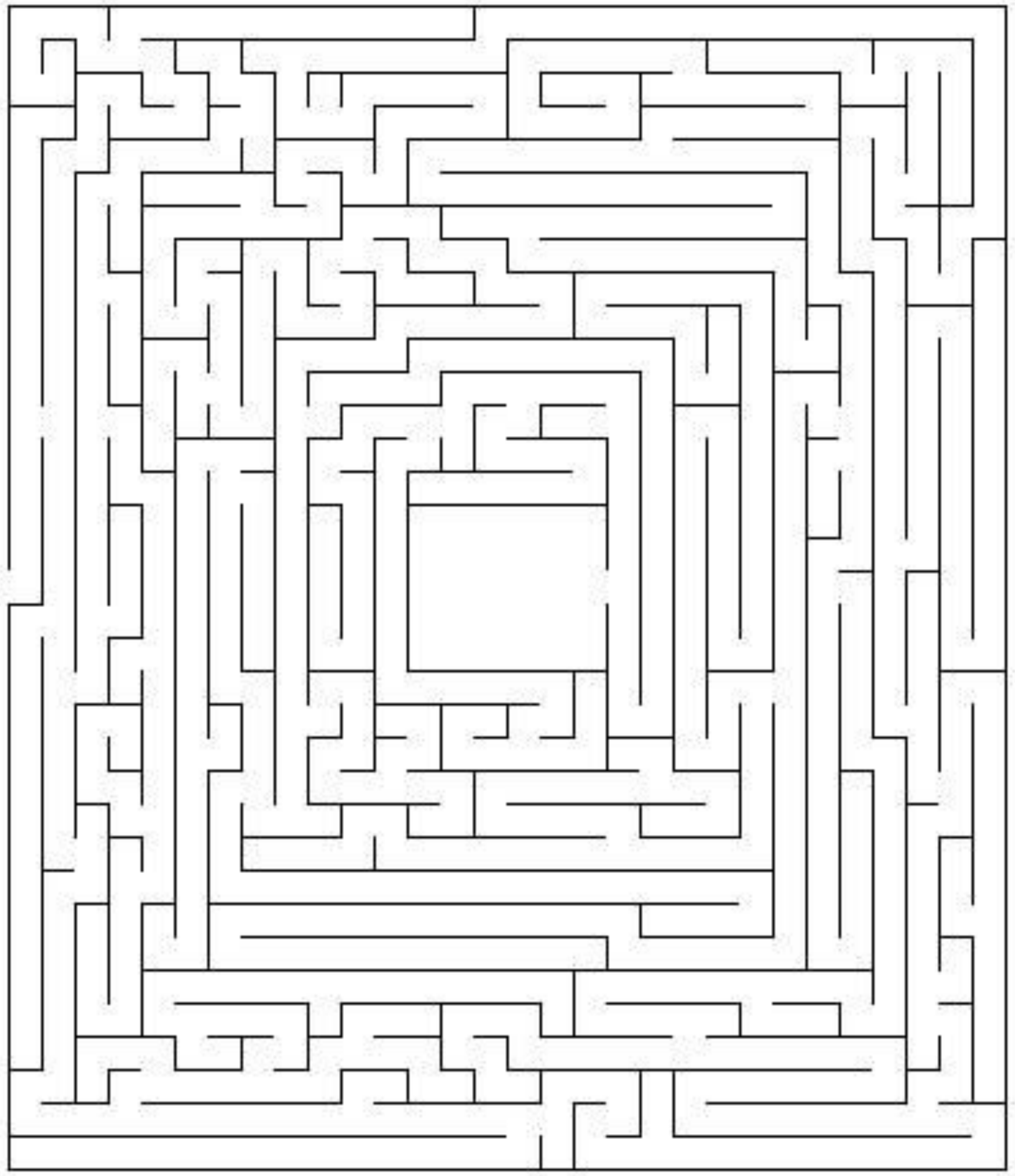
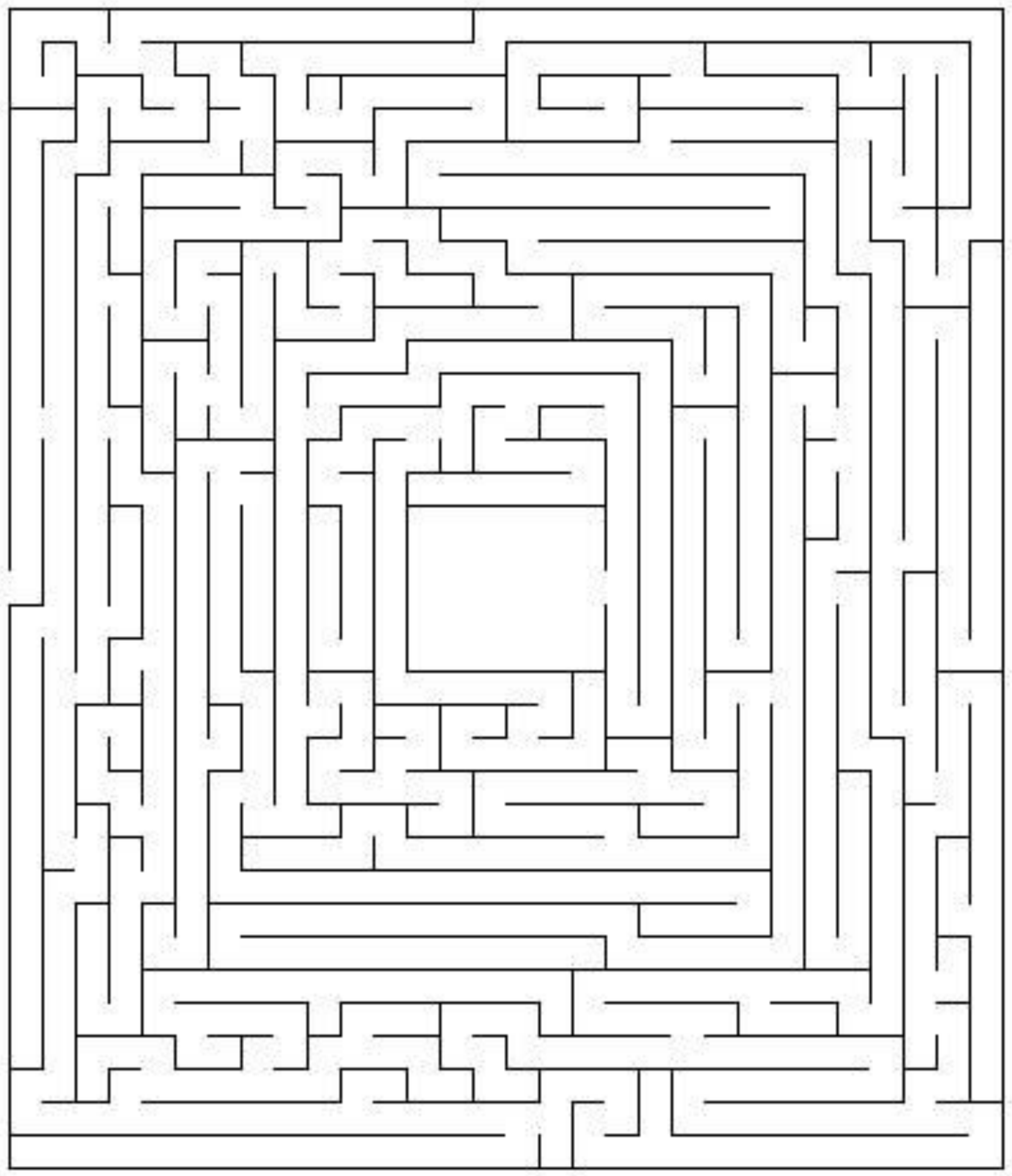
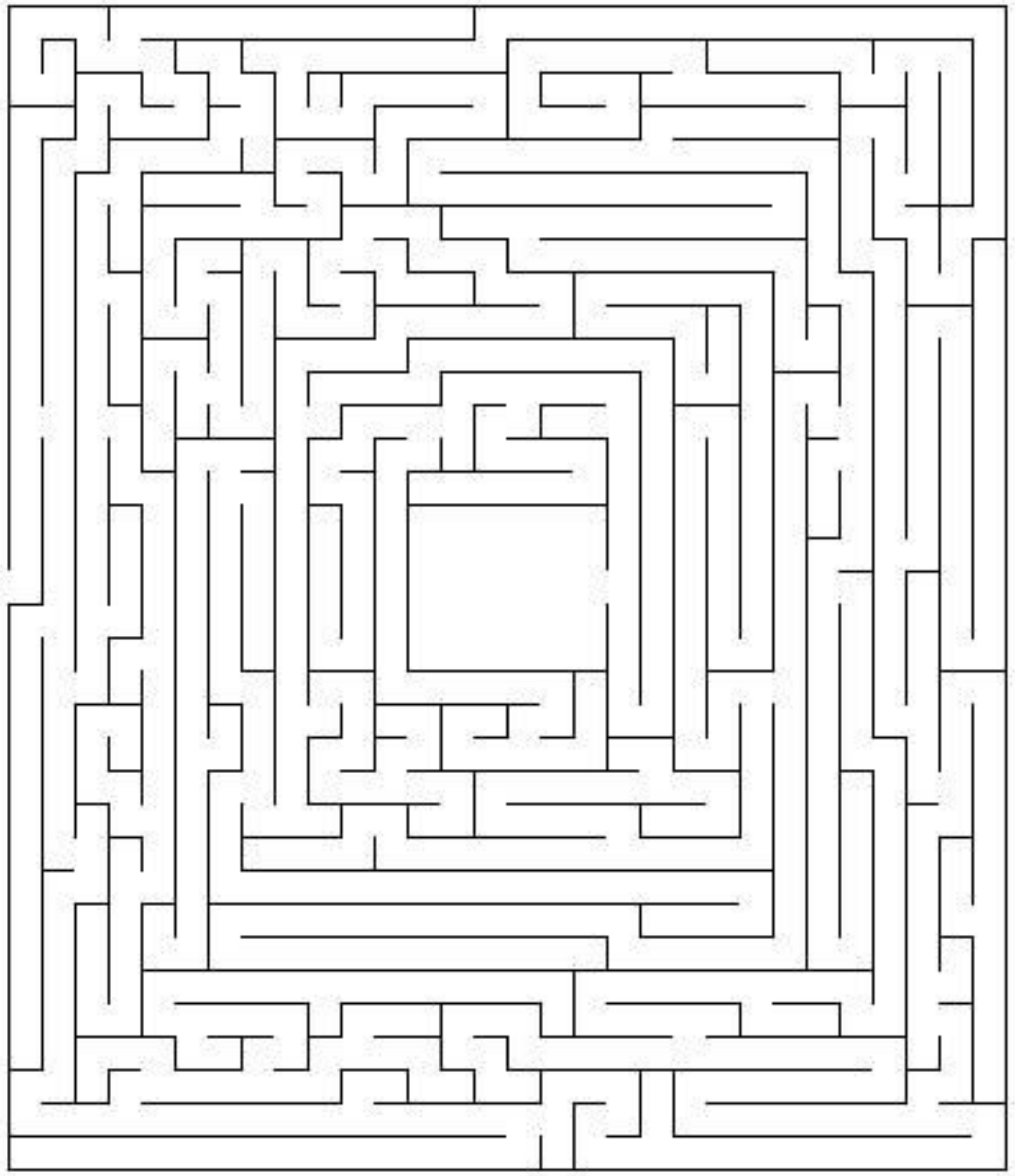




Policy: direction to move in at S_1



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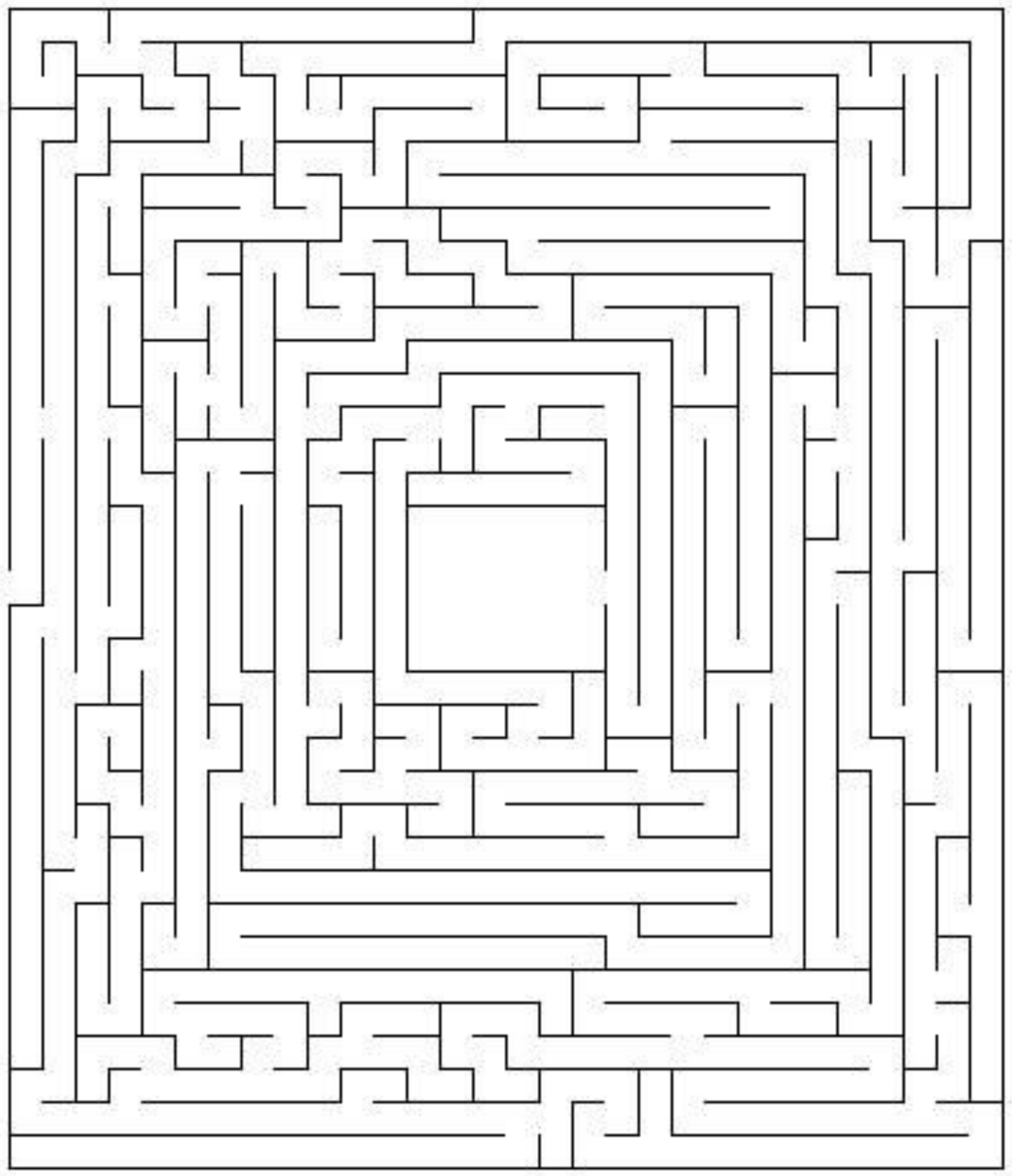

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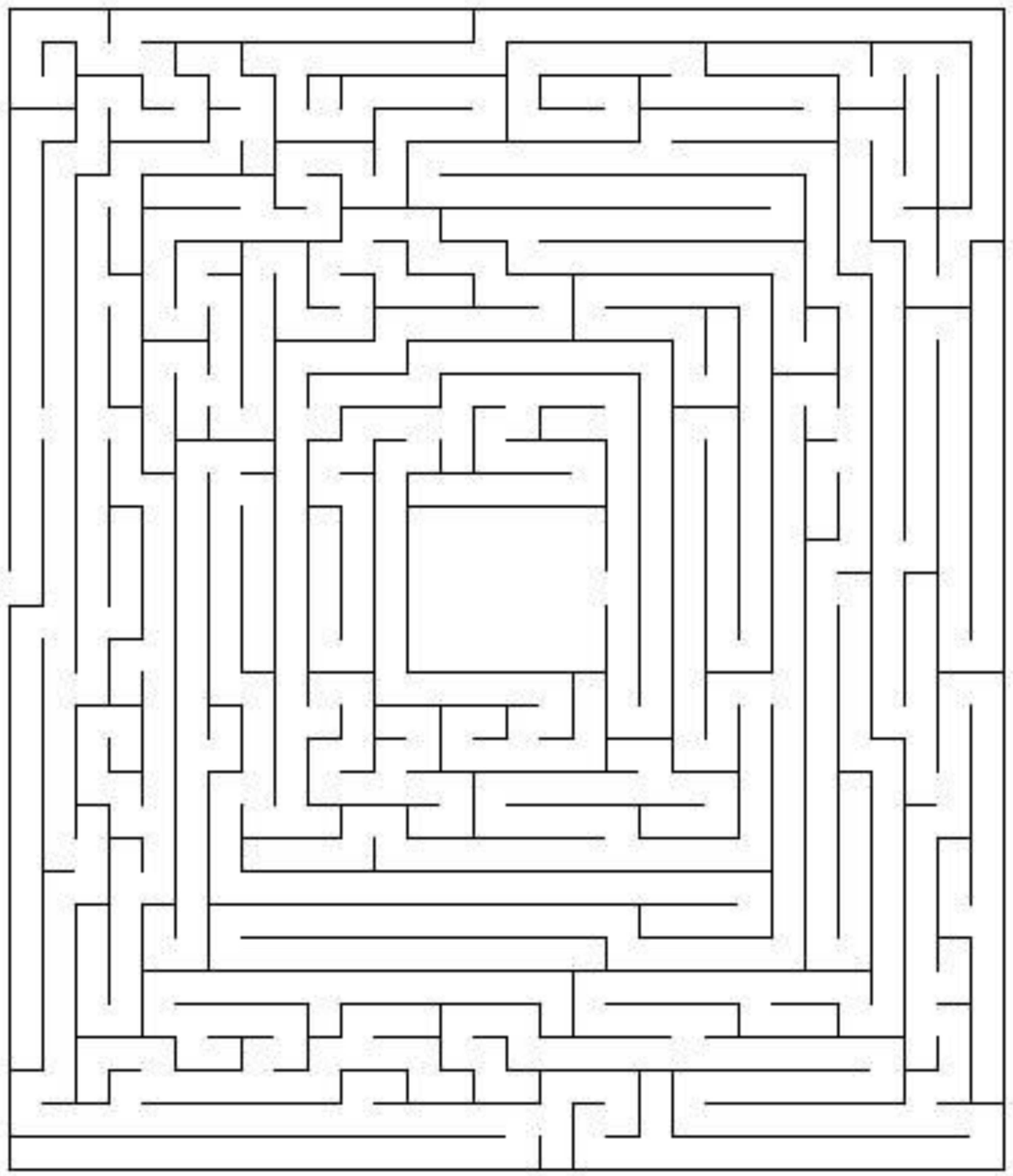

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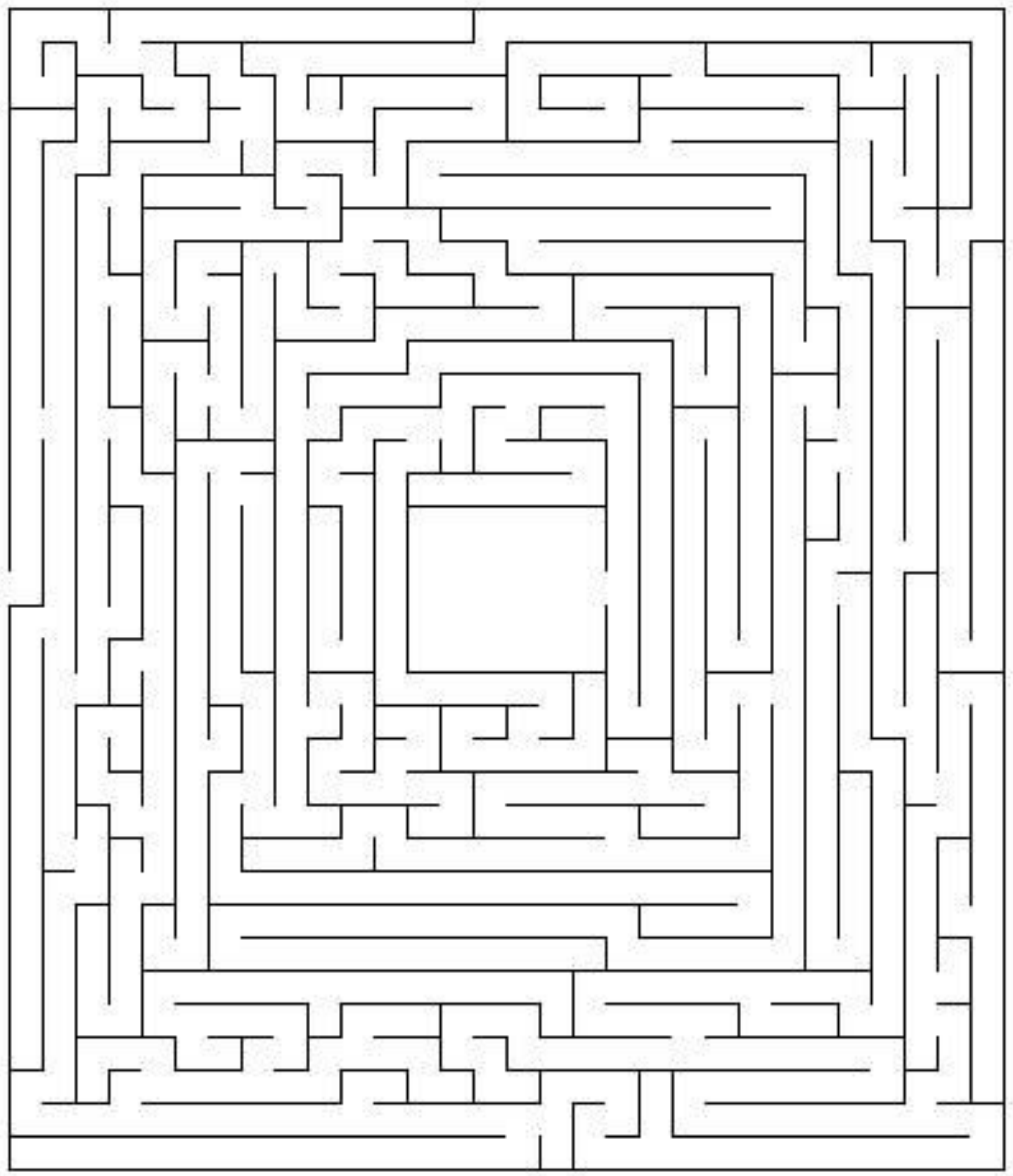

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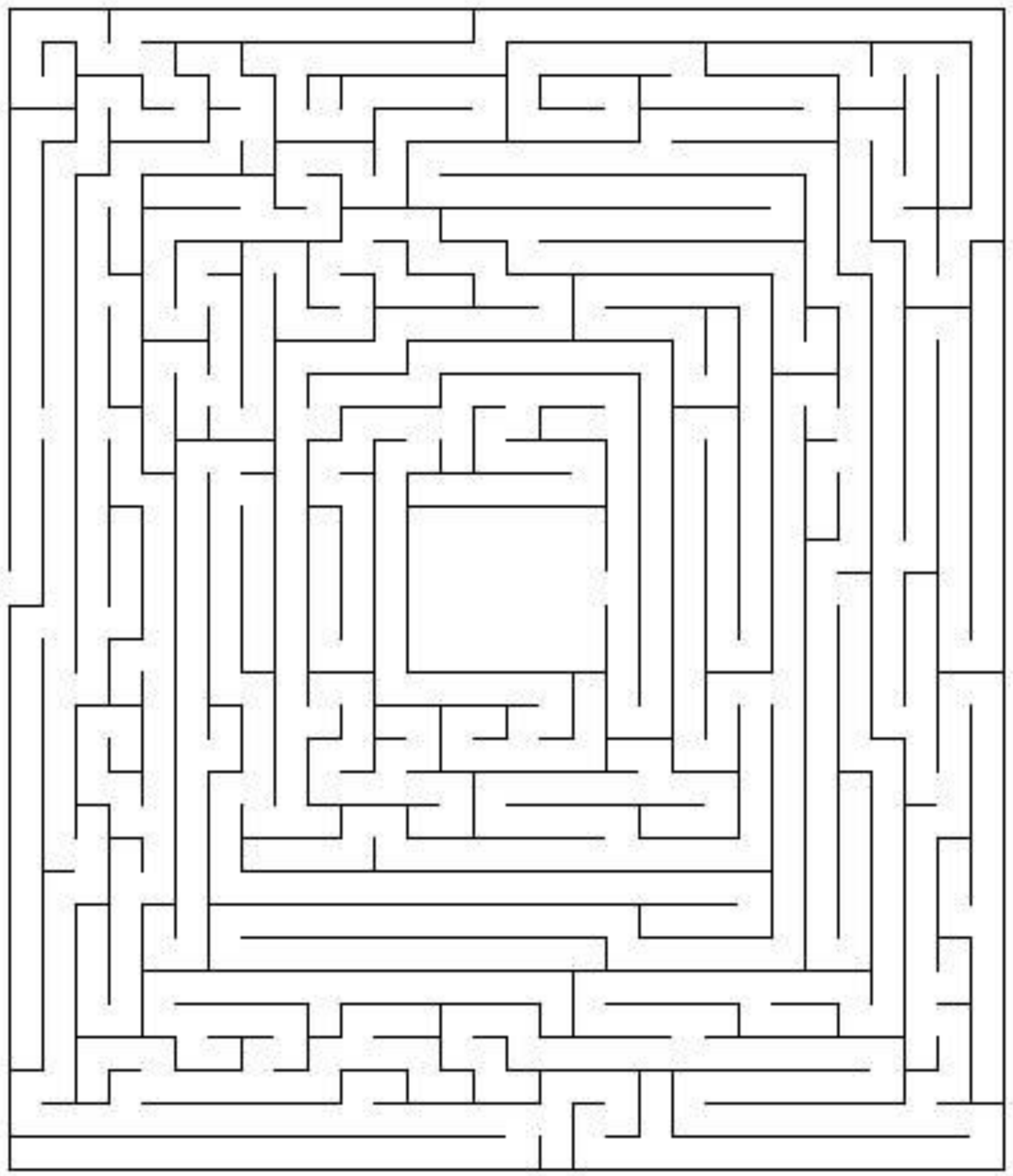

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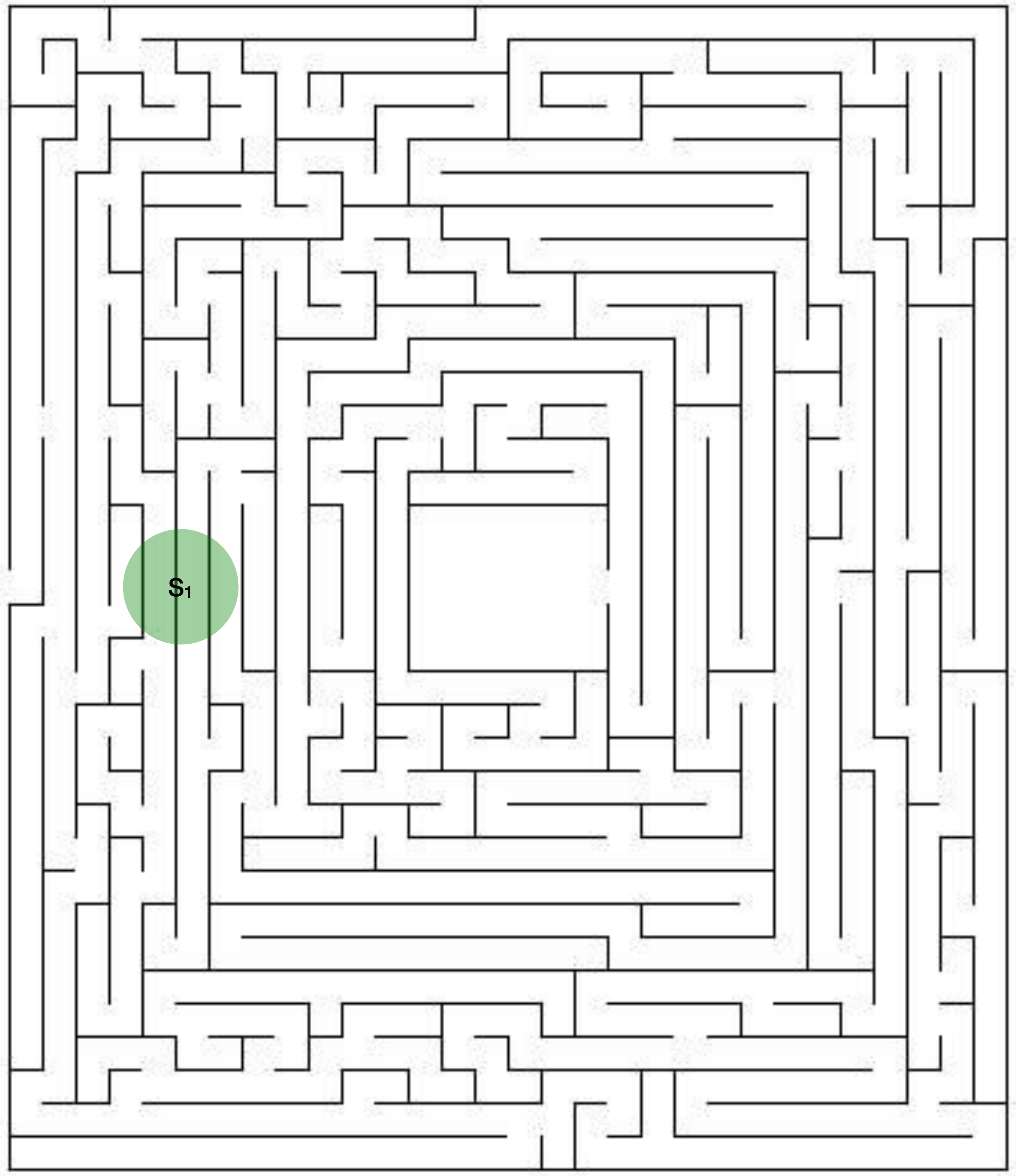

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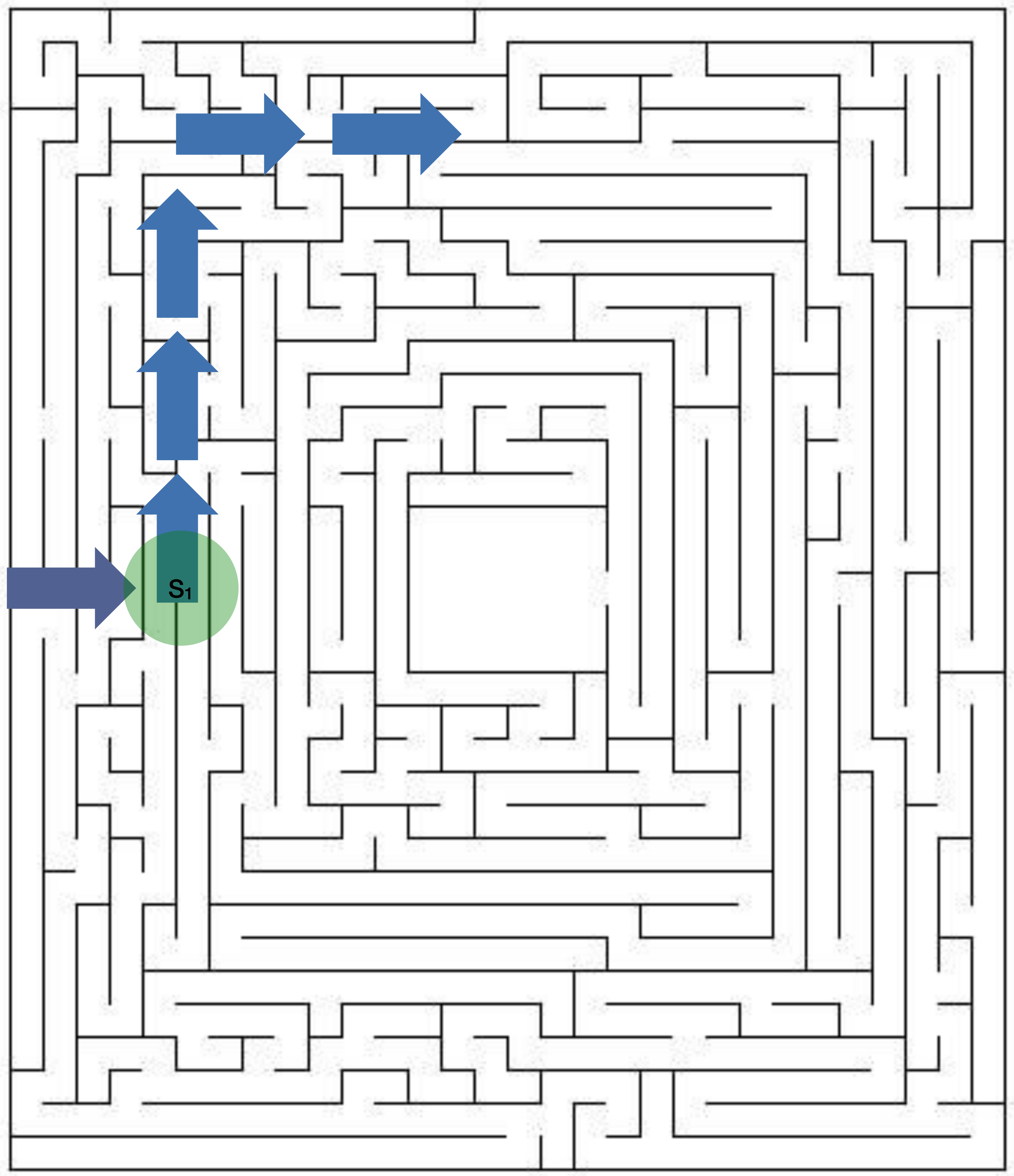

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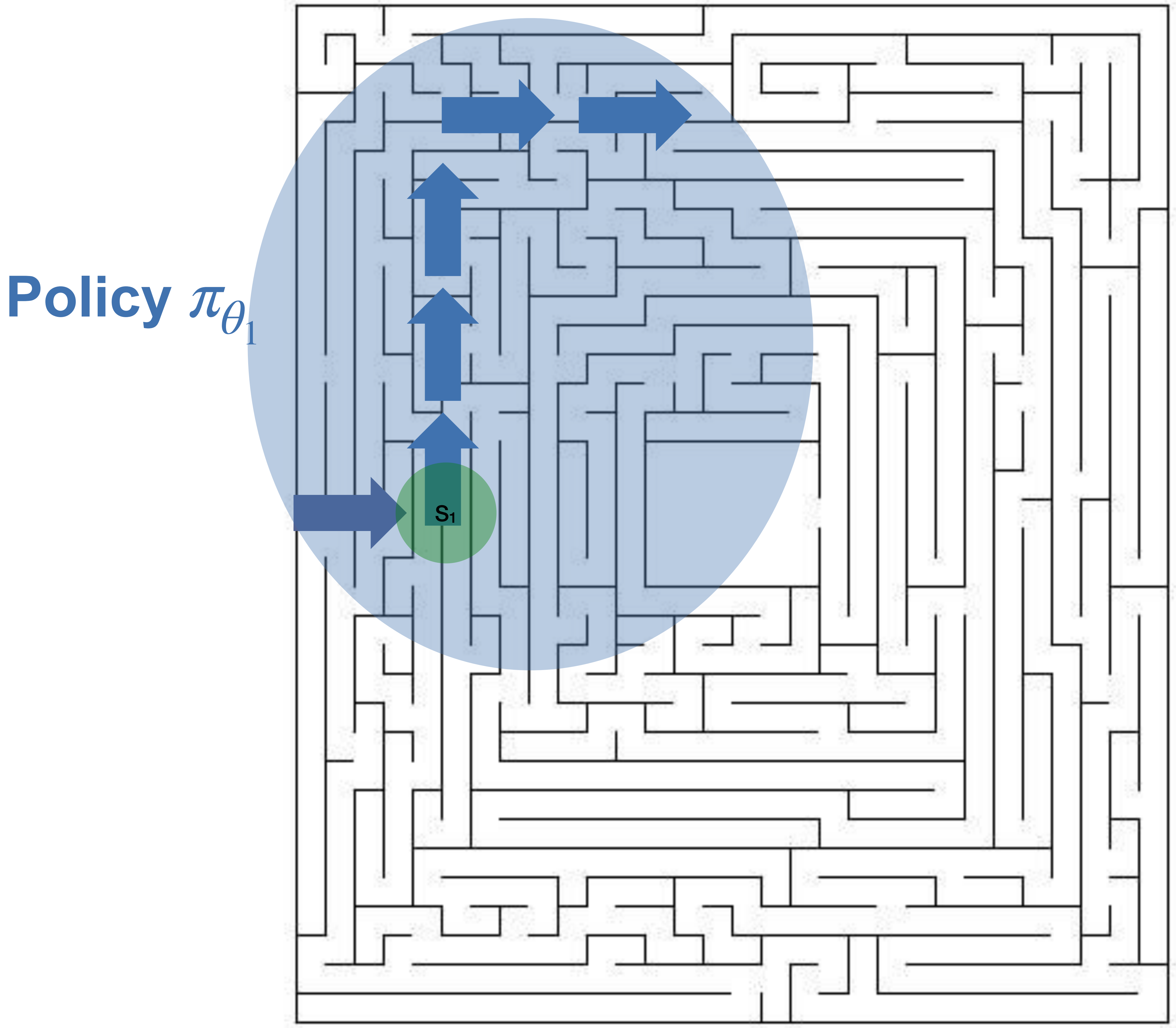

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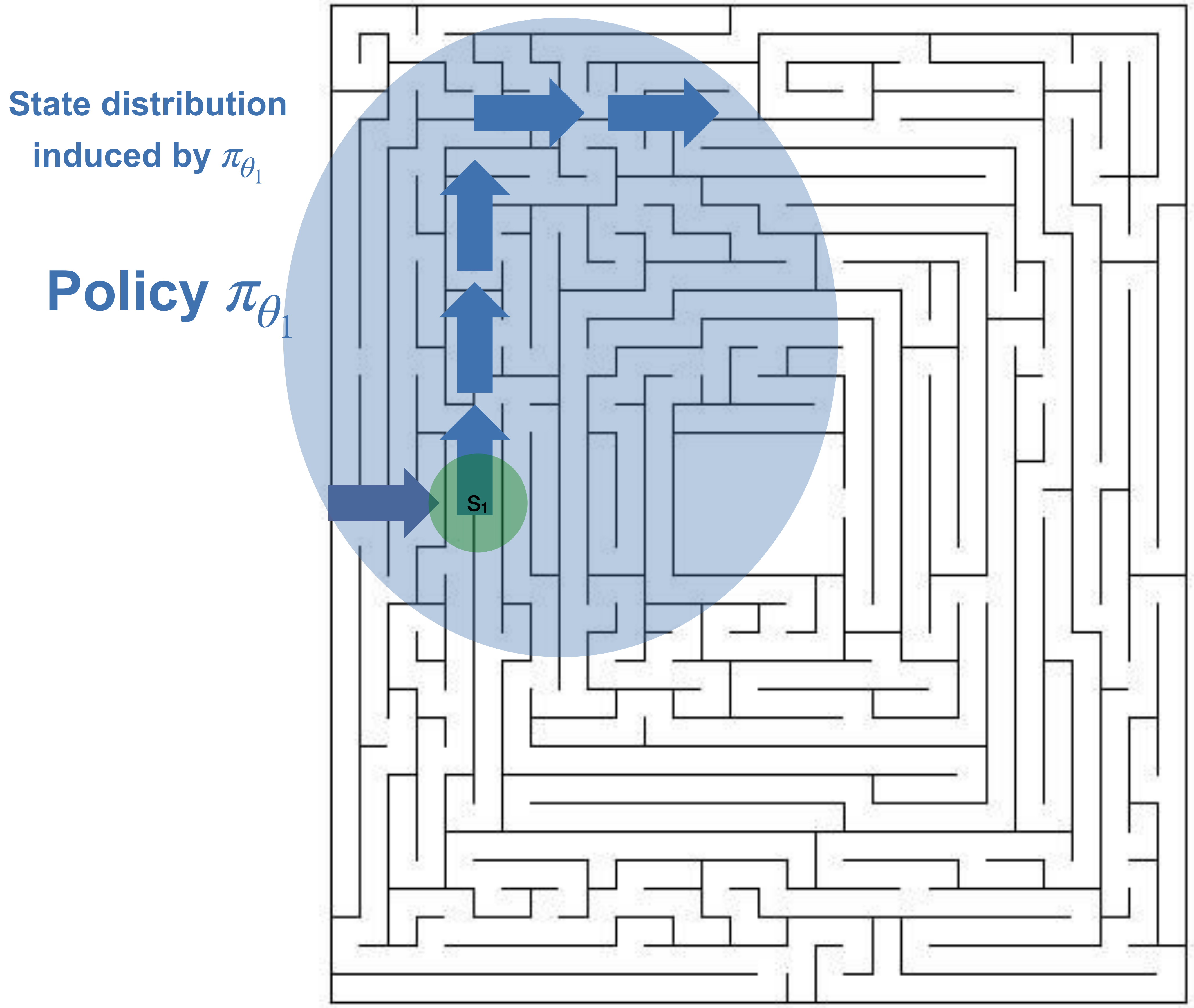

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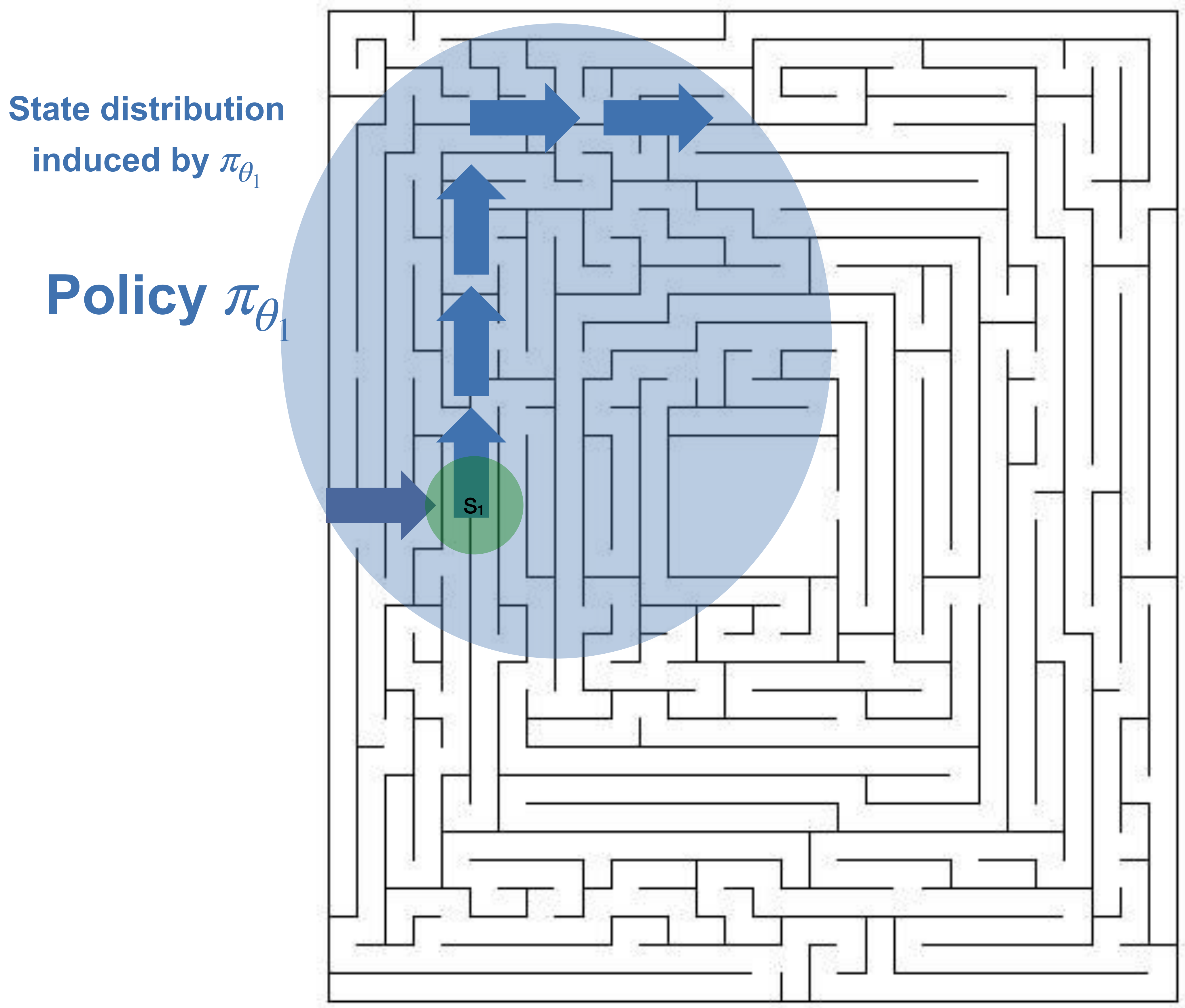

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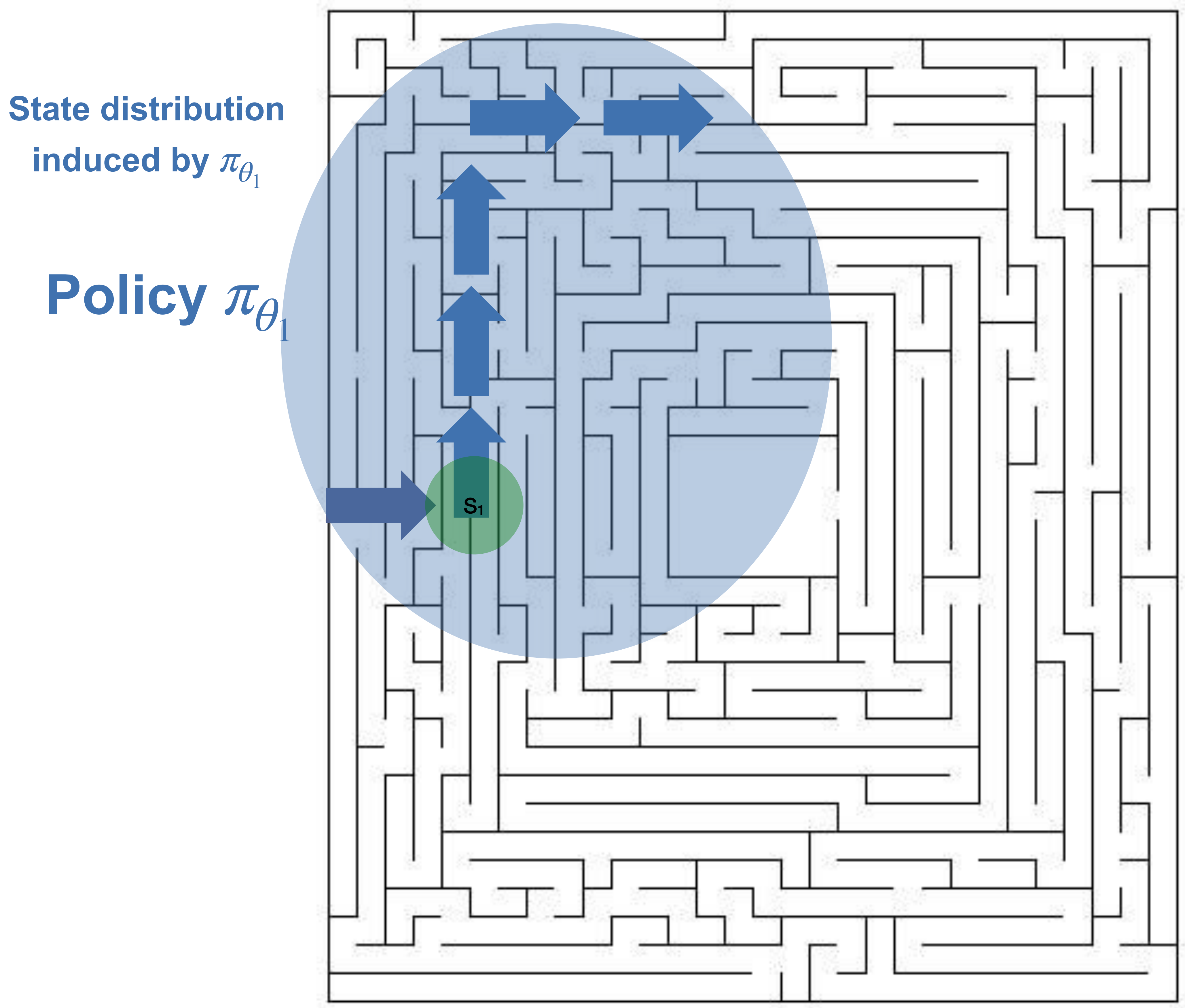

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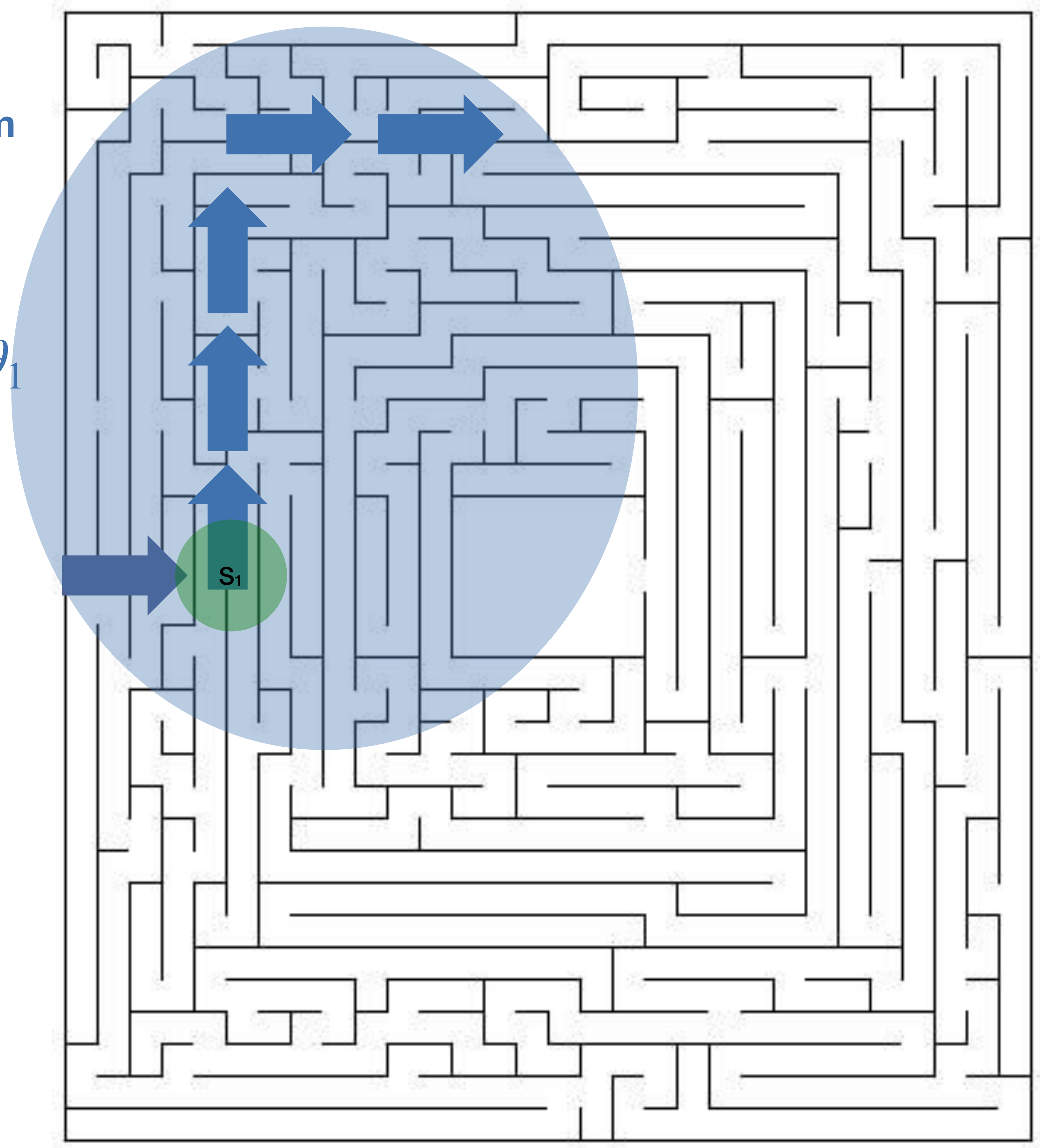
State distribution induced by π_{θ_1}

Policy π_{θ_1}



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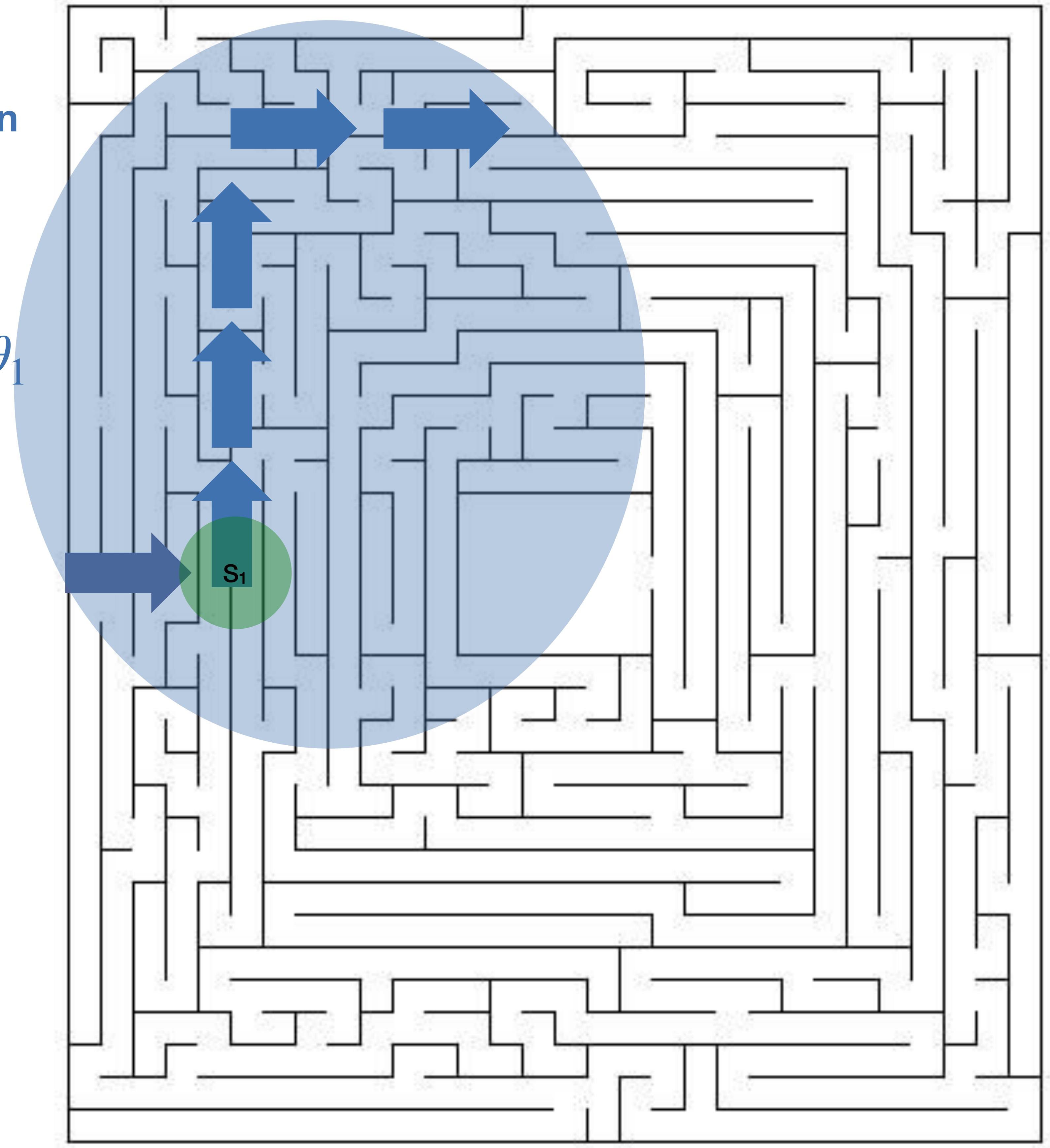

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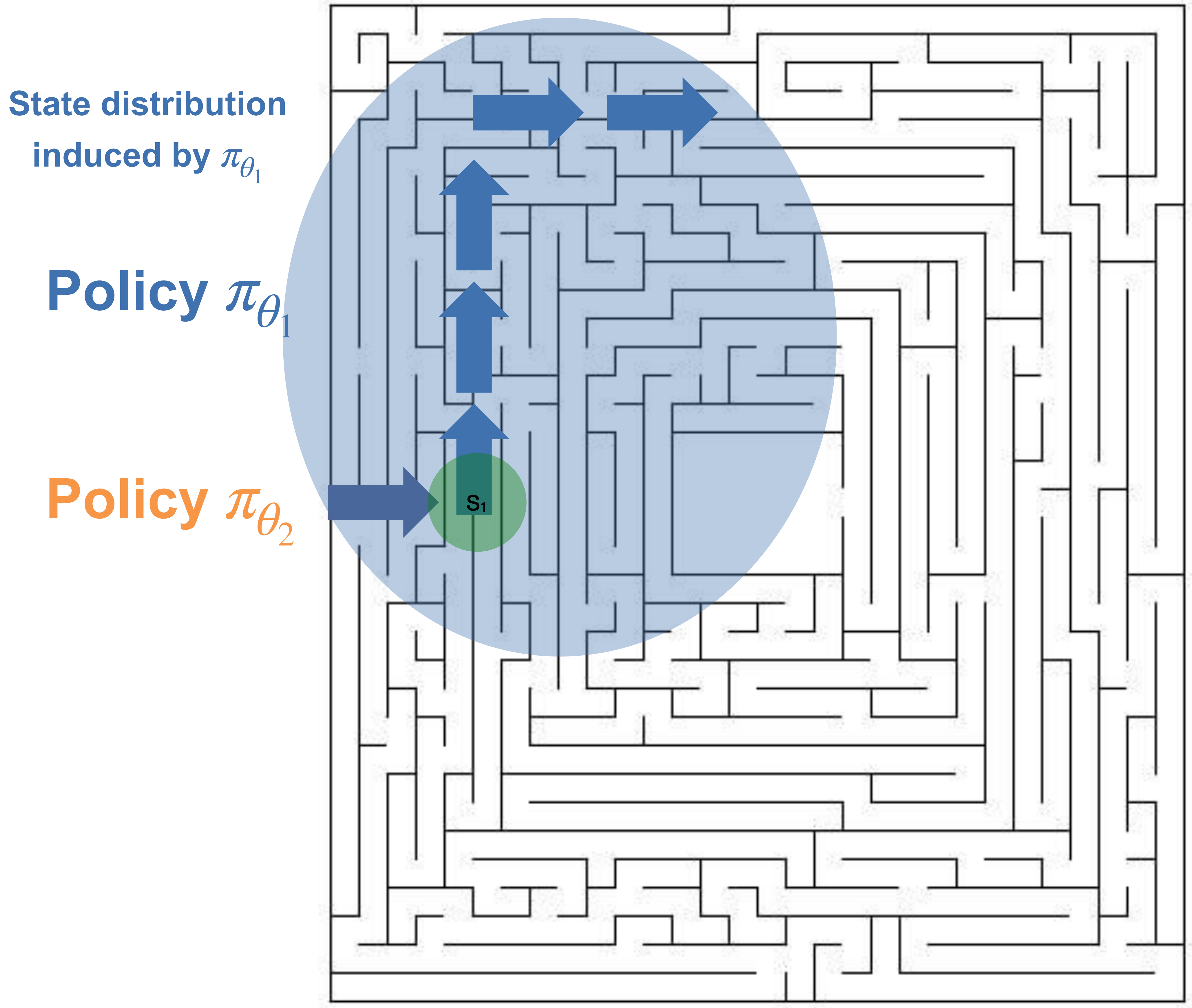

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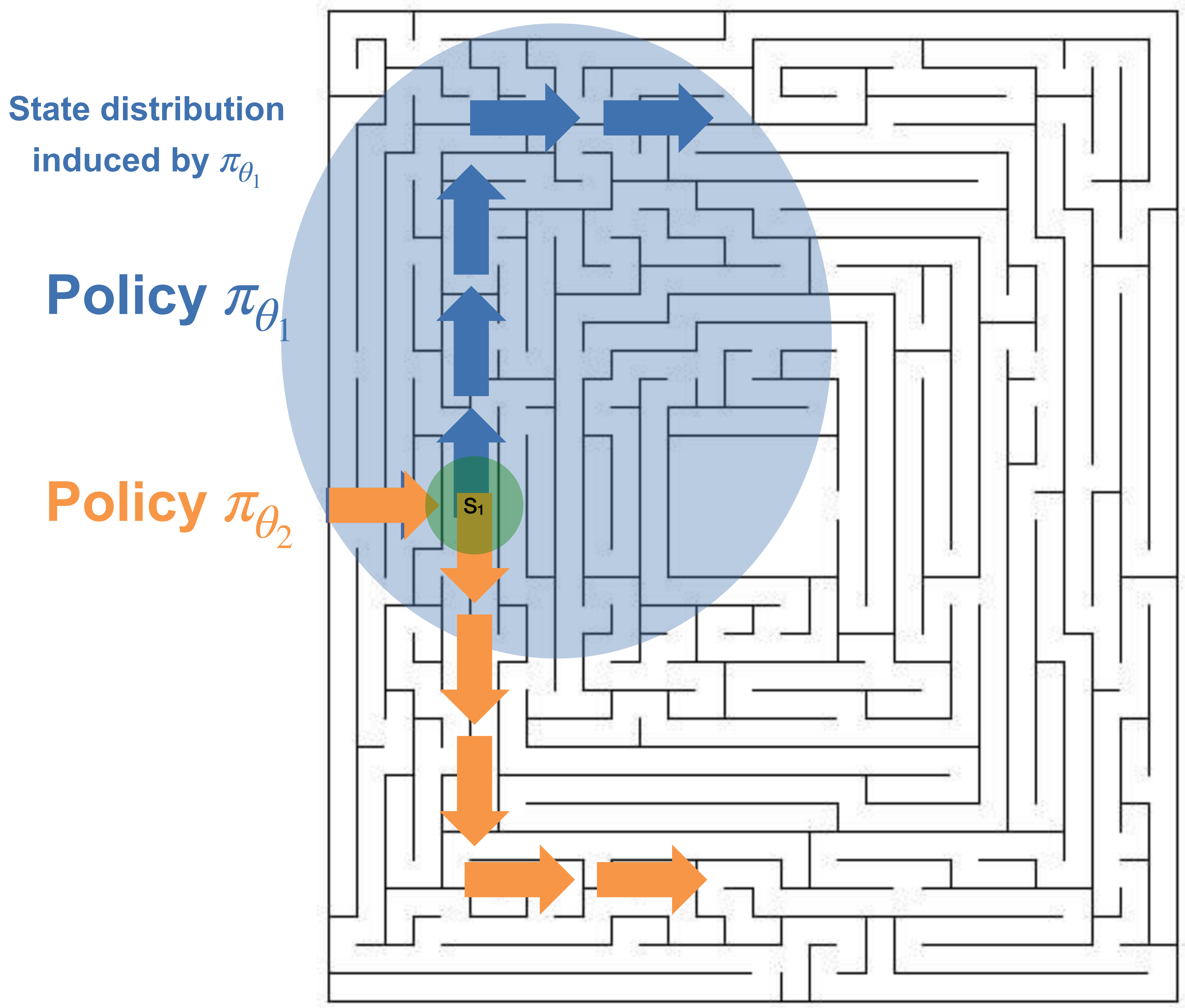

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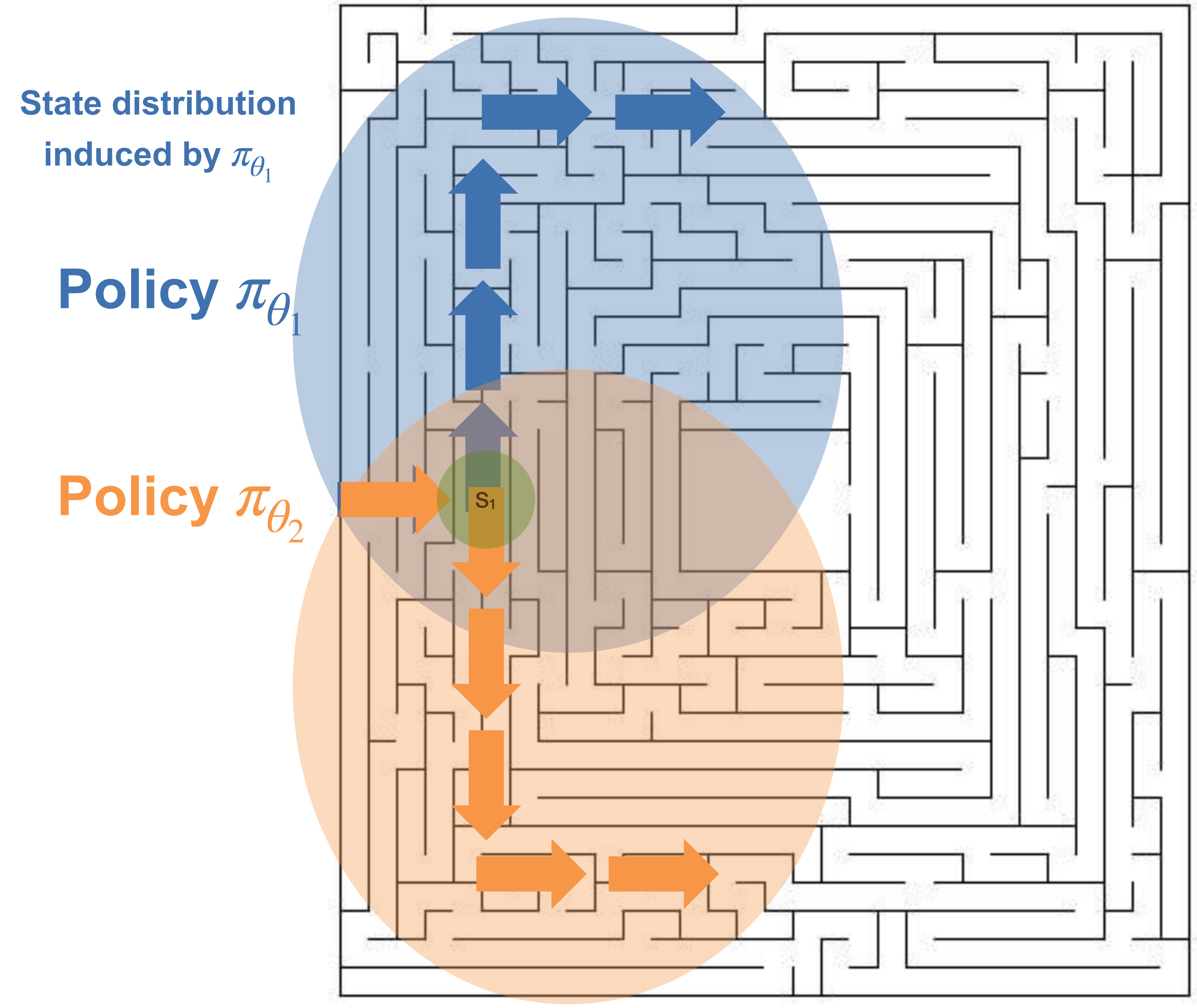

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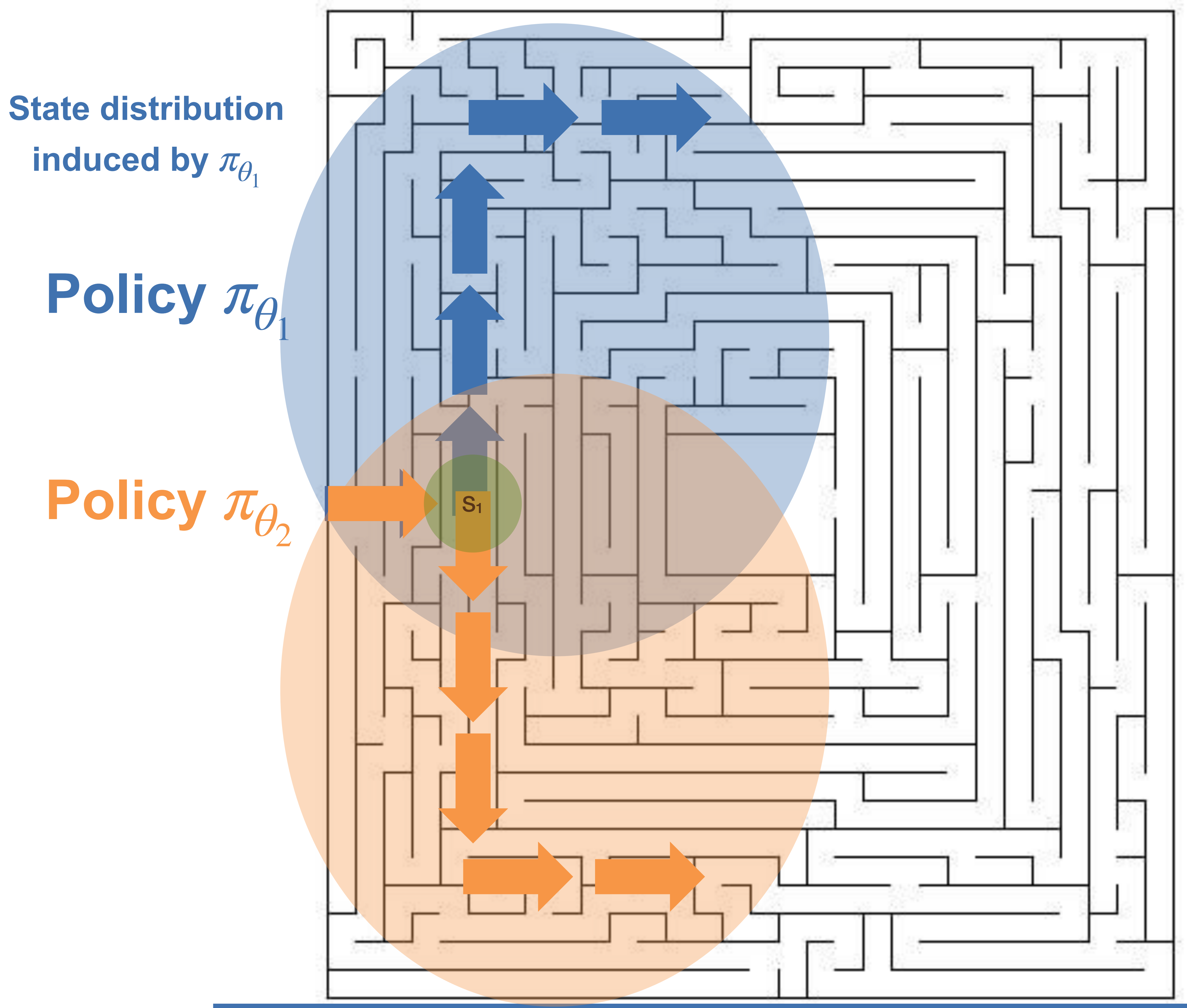

Observation: Small changes in policy's parameters can lead to *large changes in policy*

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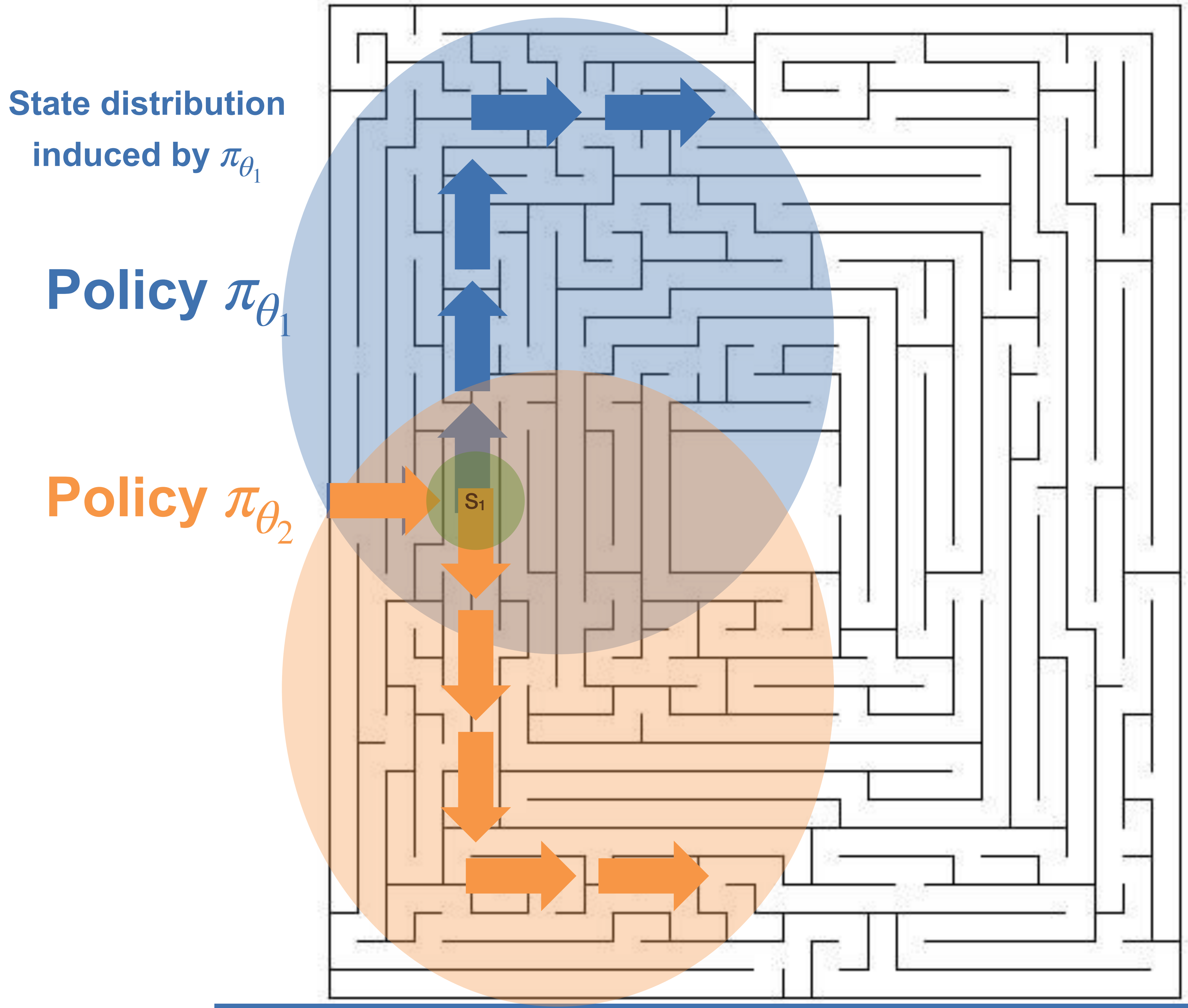

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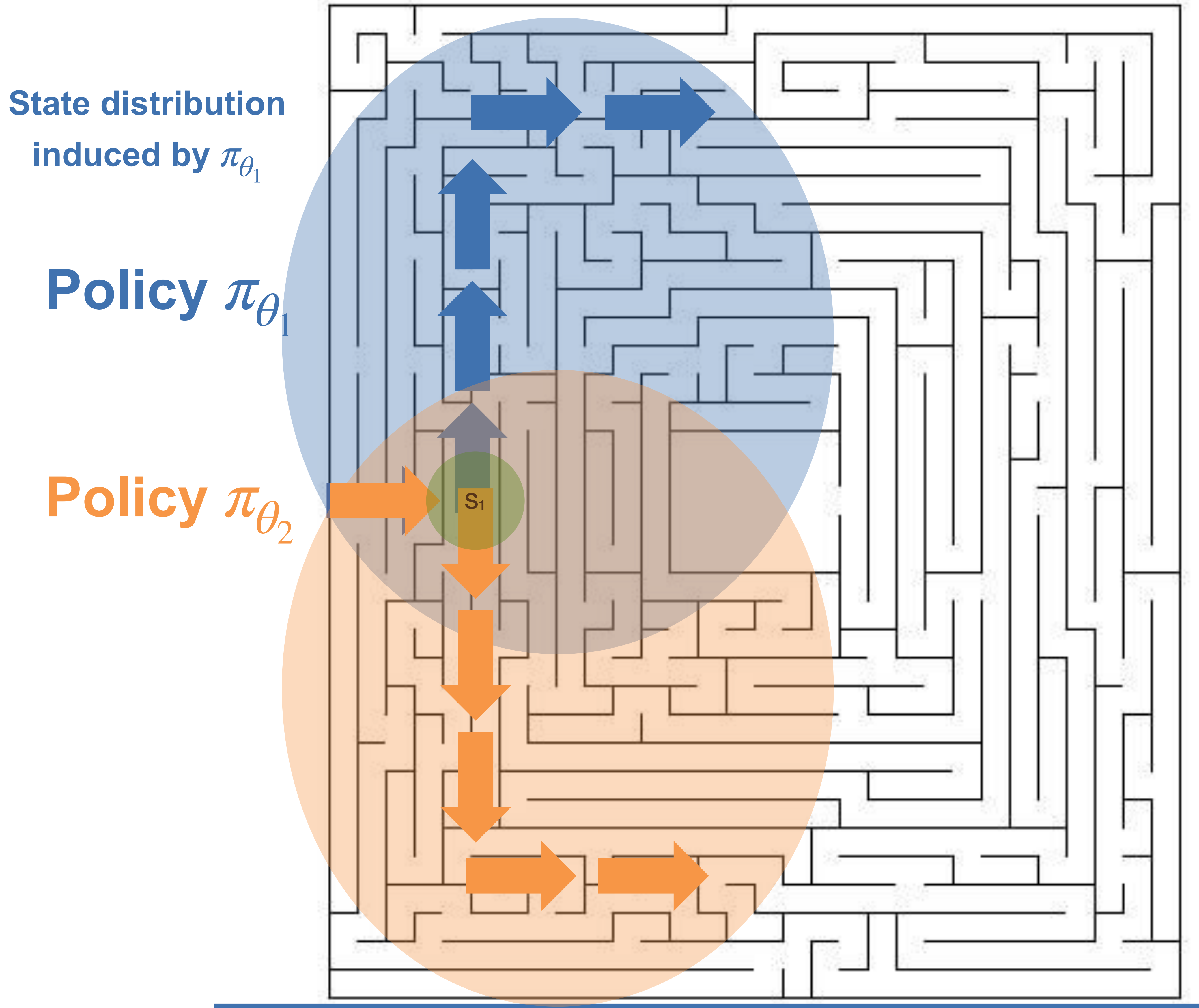

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$\pi_{\theta_2} : a_{right}$

Intuition Behind Observation #2

Observation 2: Small changes in policy’s parameters can lead to *large changes in policy*

$\mathbb{E}_{s \sim d^{\pi_{\theta_t}}}$

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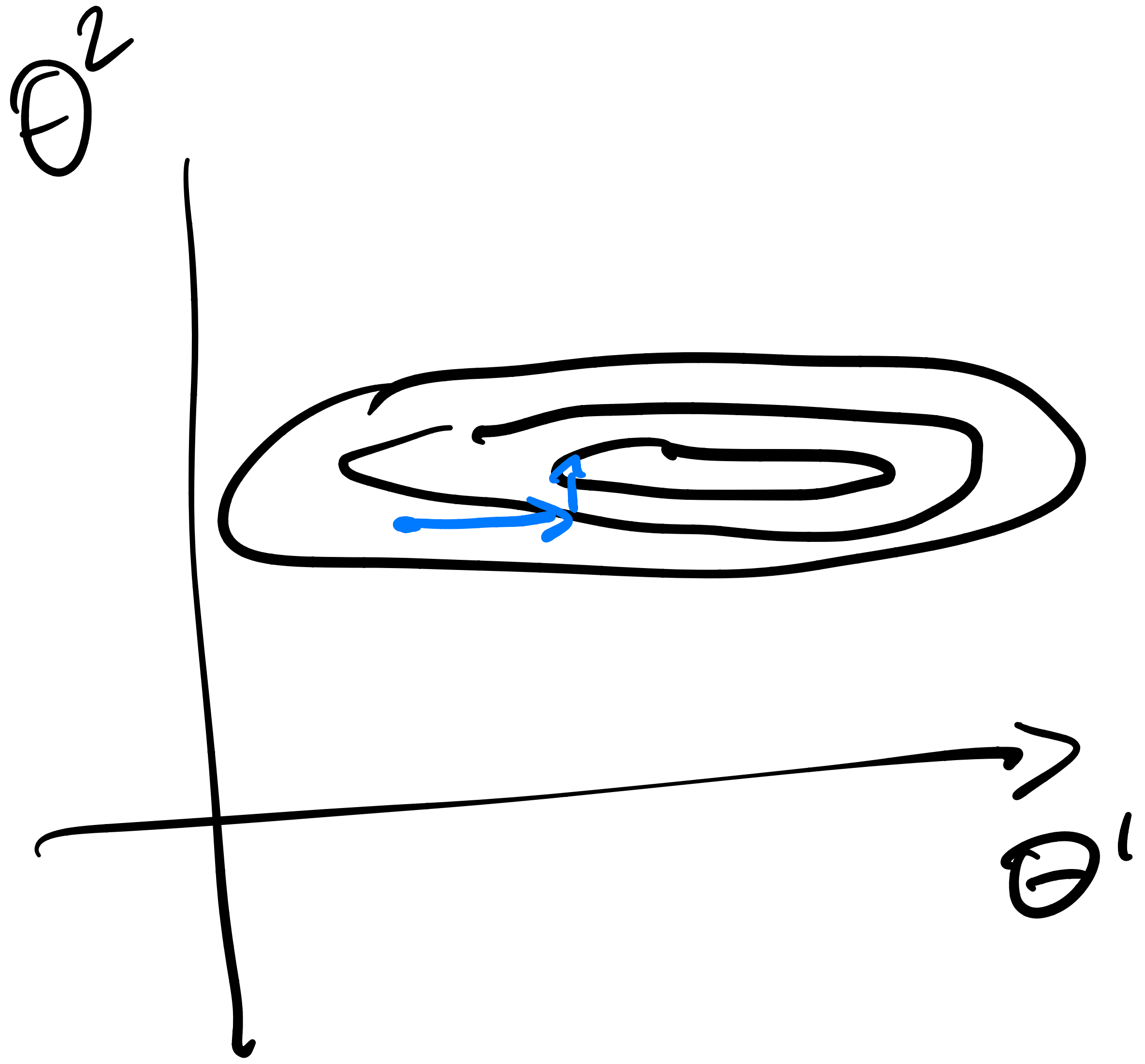

In other words…

"I don't care how big the change is to parameters (θ),
I care about the change to the policy (π_θ)"

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Implicitly, PG considers Euclidean distance in **parameter space**

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Goal of New Approach

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Perform **policy optimization**
while considering "**policy change**"

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Q: How do we measure "policy change"?

Goal of New Approach

Perform **policy optimization**
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Goal of New Approach

Perform **policy optimization**
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$$\rho_\theta(\tau) = \mu(s_0)\pi_\theta(a_0 \mid s_0)P(s_1 \mid s_0, a_0)\pi_\theta(a_1 \mid s_1)\ldots$$

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Fact:

$KL(P \mid Q) \geq 0$, and being 0 if and only if $P = Q$

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1. Simplify KL expression
2. Use Taylor expansion on KL expression

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Recall $\ell(\theta) := H\, \mathbb{E}_{s,a\sim d^{\pi_{\theta_t}}}\left[\ln \frac{\pi_{\theta_t}(a \mid s)}{\pi_\theta(a \mid s)}\right]$

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$Q^{\pi_{\theta_t}}(s,a) - V^{\pi_{\theta_t}}$

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Summary

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How can we optimize the policy's parameters while considering policy change?

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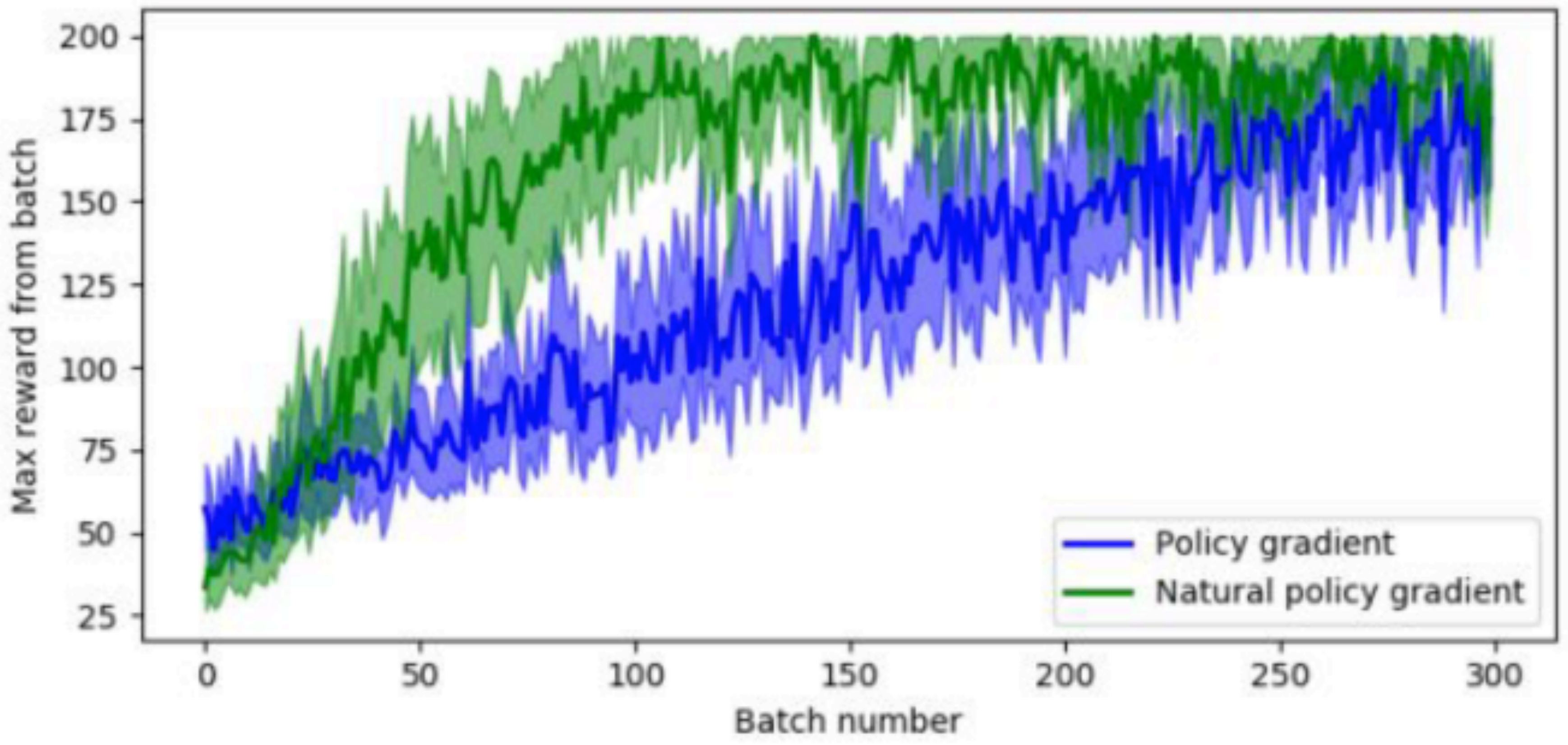
PG vs Natural PG



Review on Policy Optimization:

We have huge space space, i.e., $|S|$ might be $255^{3\times 512\times 512}$

We can only reset from initial state distribution $s_0 \sim \mu$

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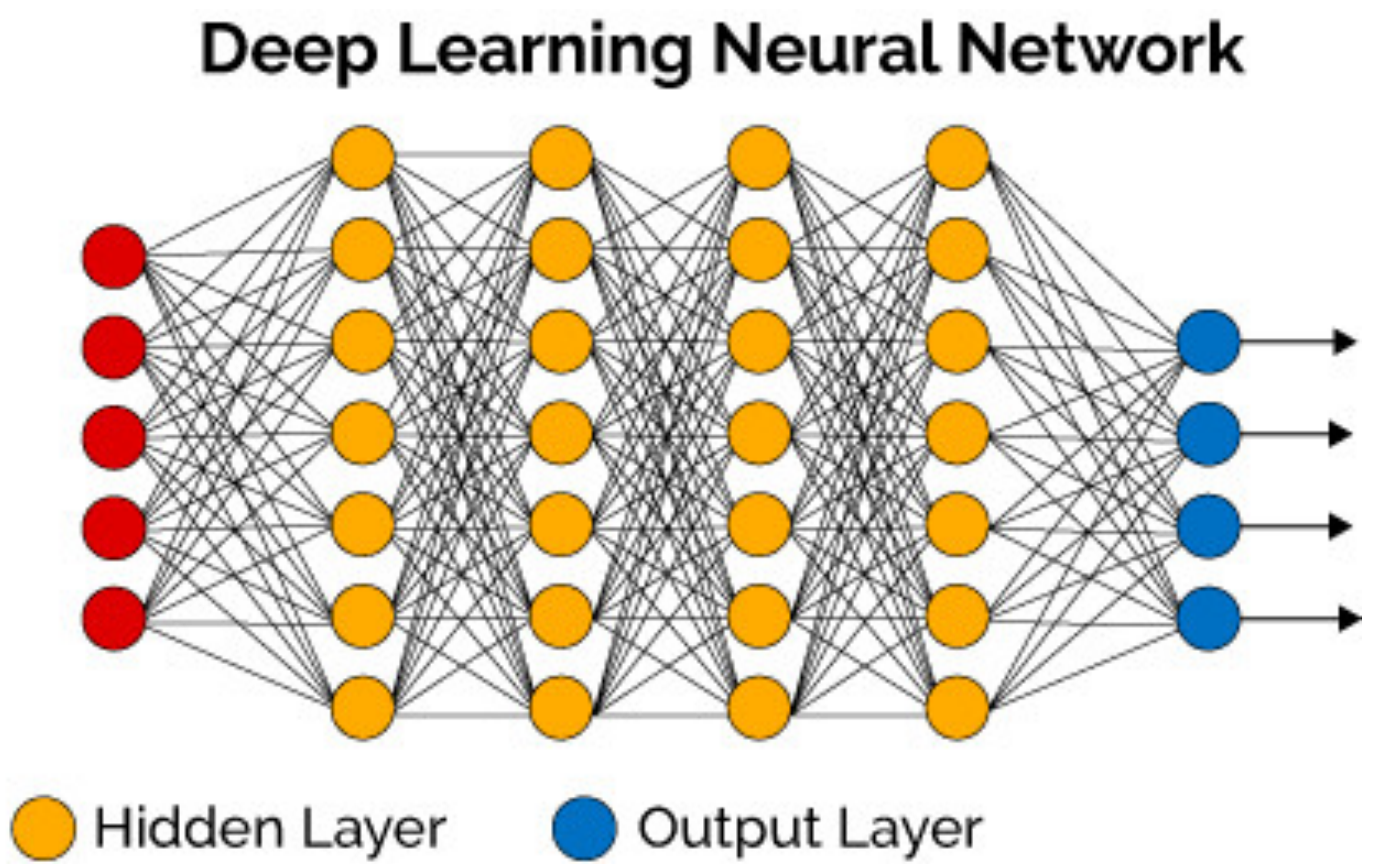

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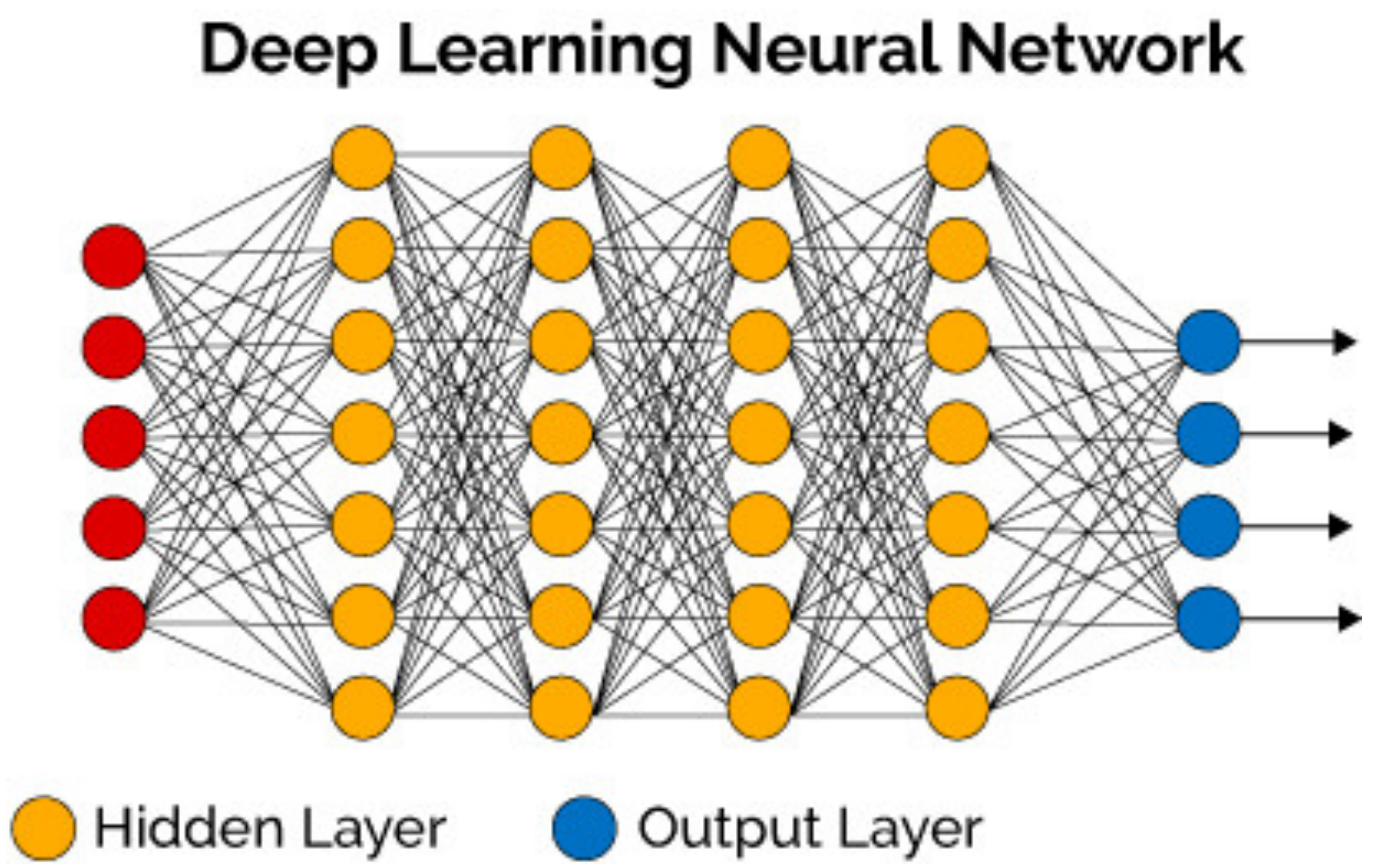

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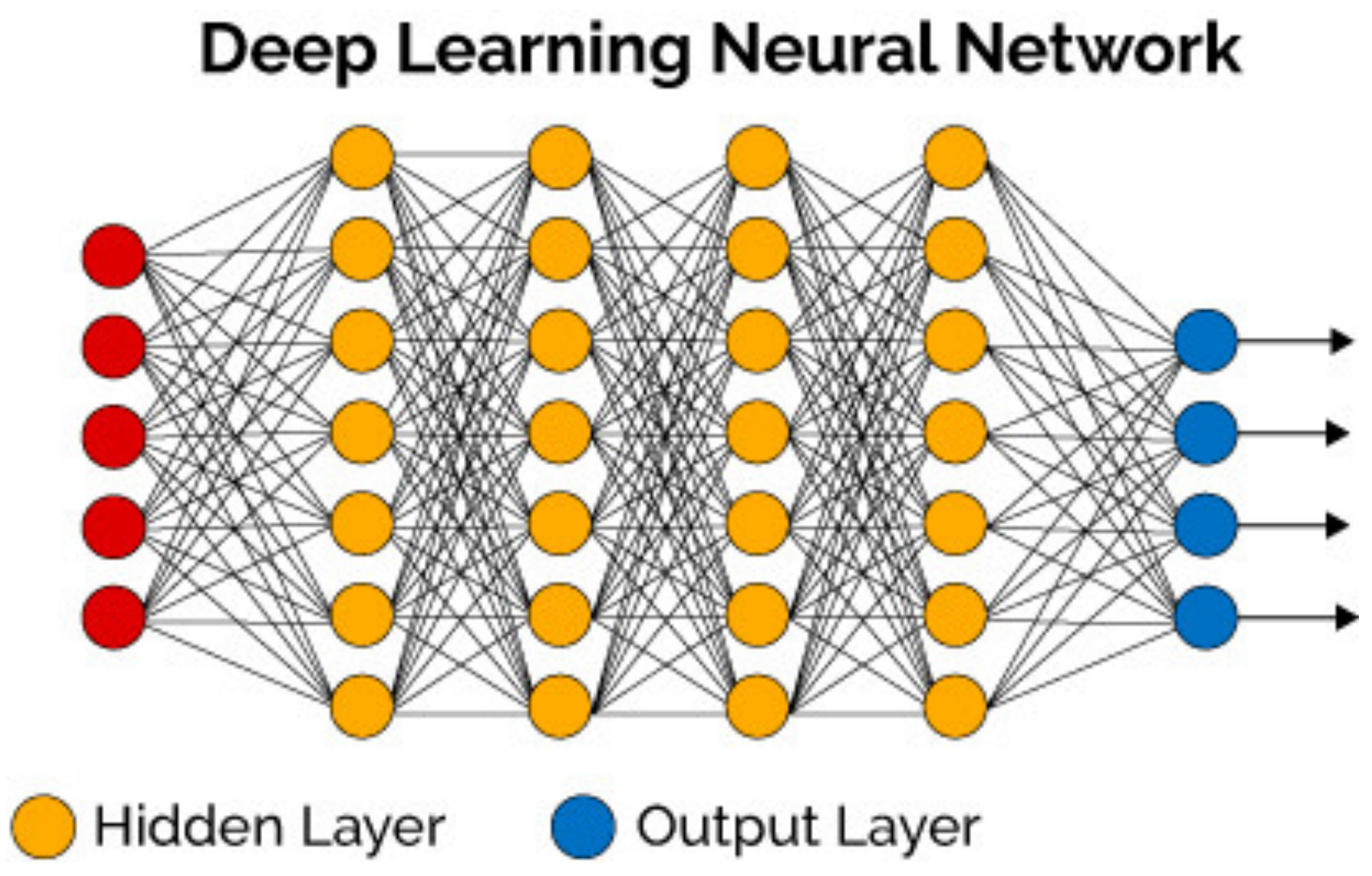

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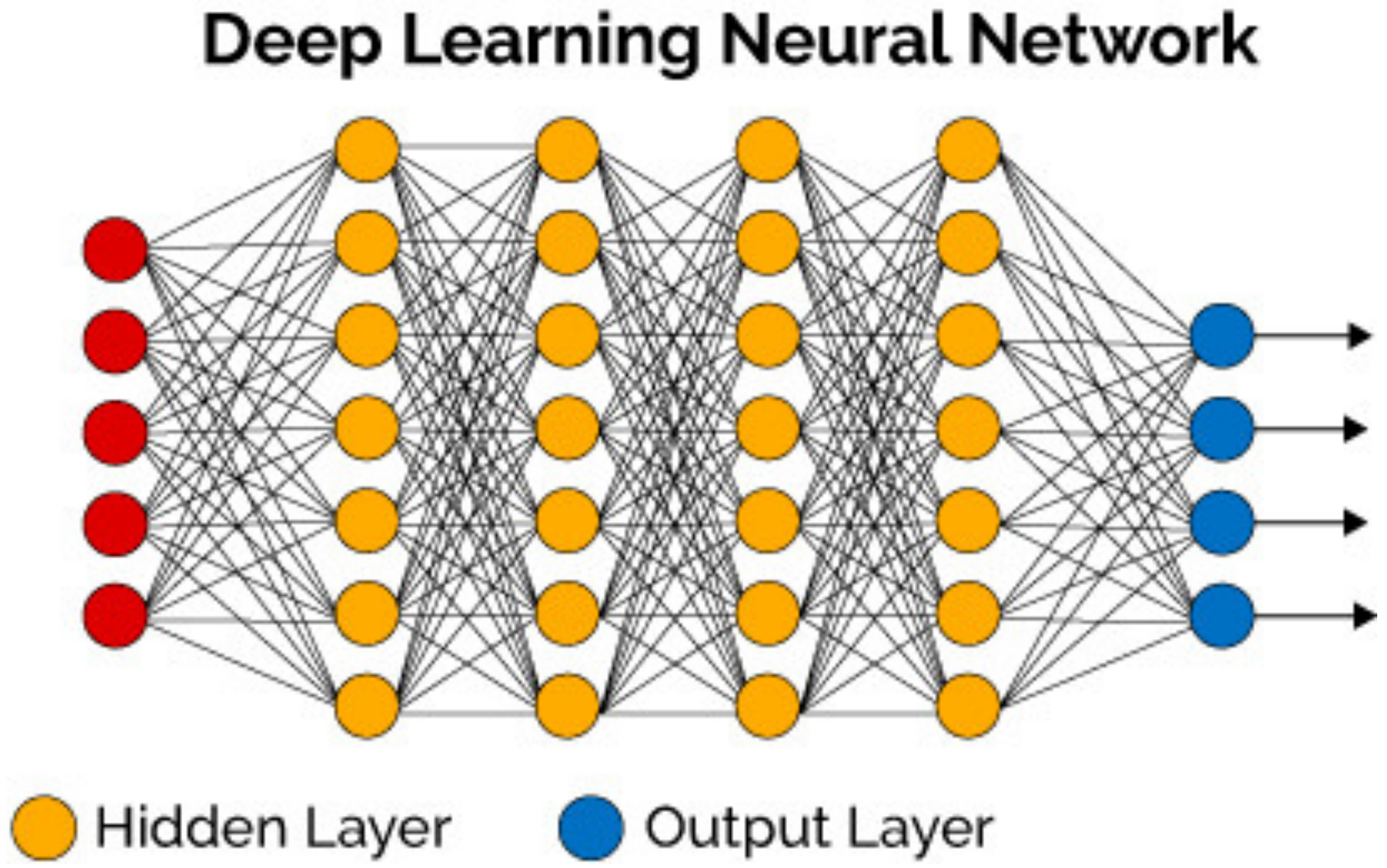

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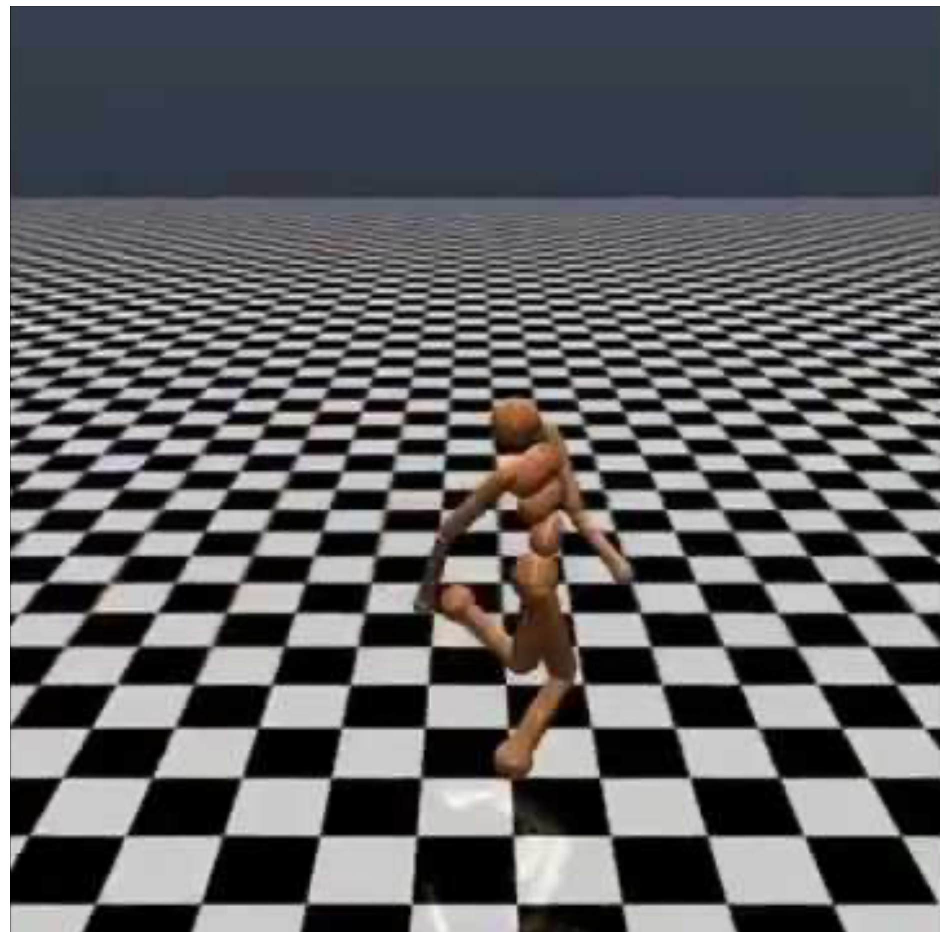

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Given an current policy π^t, we perform policy update to π^{t+1}

Third attempt: **PG on parameterized policy**

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When $\eta \to 0^+$, gradient ascent ensures
we improve the objective function

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$$\max_{\theta} \nabla_\theta J(\pi_{\theta_t})^\top (\theta - \theta_t), \ \text{s.t.,} \ (\theta - \theta_t)^\top F_{\theta_t} (\theta - \theta_t) \leq \delta$$

An extension of NPG (even faster in practice):

Given an current policy π^t, we perform policy update to π^{t+1}

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PPO: Perform a few steps of mini-batch SGA on $\ell(\theta)$ to approximate $\arg\max_\theta \ell(\theta)$