NPG and PPO

Annoucements

1. We will release HW3 w/ solution — it is optional, but do take a look

2. Prelim scope: first lecture to (and include) next Monday's lecture

3. We released a prelim from last year (but don't overfit to it)

Recap on NPG:

At iteration t:

$$\max_{\pi_\theta} \mathbb{E}_{s\sim d_\mu^{\pi_{\theta_t}}} \left[\mathbb{E}_{a\sim\pi_\theta(s)} A^{\pi_{\theta_t}}(s,a) \right]$$

$$\text{s.t., } KL\left(\rho_{\pi_{\theta_t}} | \rho_{\pi_\theta}\right) \le \delta$$

Intuition: maximize local adv subject to being incremental (in KL);

Recap on NPG:

At iteration t:

$$\max_{\pi_\theta} \mathbb{E}_{s\sim d_\mu^{\pi_{\theta_t}}}\left[\mathbb{E}_{a\sim\pi_\theta(s)} A^{\pi_{\theta_t}}(s,a)\right] \longrightarrow$$ First-order Taylor expansion at θ_t

$$\text{s.t., } KL\left(\rho_{\pi_{\theta_t}} | \rho_{\pi_\theta}\right) \leq \delta \longrightarrow$$ second-order Taylor expansion at θ_t

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At iteration t:

$$\max_{\pi_\theta} \mathbb{E}_{s\sim d_\mu^{\pi_{\theta_t}}} \left[\mathbb{E}_{a\sim\pi_\theta(s)} A^{\pi_{\theta_t}}(s,a) \right] \longrightarrow \text{First-order Taylor expansion at } \theta_t$$

$$\text{s.t., } KL\left(\rho_{\pi_{\theta_t}} | \rho_{\pi_\theta}\right) \le \delta \longrightarrow \text{second-order Taylor expansion at } \theta_t$$

Intuition: maximize local adv subject to being incremental (in KL);

linear

$$\max_{\theta} \nabla_\theta J(\pi_{\theta_t})^\top (\theta - \theta_t)$$

$$\text{s.t. } \underbrace{(\theta-\theta_t)^\top F_{\theta_t} (\theta - \theta_t)}_{\text{quadratic convex}} \le \delta$$

Recap on NPG:

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$$\max_{\pi_\theta} \mathbb{E}_{s\sim d_\mu^{\pi_{\theta_t}}}\left[\mathbb{E}_{a\sim\pi_\theta(s)} A^{\pi_{\theta_t}}(s,a)\right] \longrightarrow \text{First-order Taylor expansion at } \theta_t$$

$$\text{s.t., } KL\left(\rho_{\pi_{\theta_t}} | \rho_{\pi_\theta}\right) \le \delta \longrightarrow \text{second-order Taylor expansion at } \theta_t$$

Intuition: maximize local adv subject to being incremental (in KL);

determined δ =

$$\theta_{t+1} = \boxed{\theta_t + \eta F_{\theta_t}^{-1} \nabla_\theta J(\pi_{\theta_t})} \longleftarrow \begin{array}{l} \max_\theta \nabla_\theta J(\pi_{\theta_t})^\top (\theta - \theta_t) \\ \text{s.t. } (\theta-\theta_t)^\top F_{\theta_t}(\theta-\theta_t) \le \delta \end{array}$$

NPG

Recap on NPG:

At iteration t:

$$\max_{\pi_\theta} \mathbb{E}_{s\sim d_\mu^{\pi_{\theta_t}}}\left[\mathbb{E}_{a\sim\pi_\theta(s)} A^{\pi_{\theta_t}}(s,a)\right]$$ ⟶ First-order Taylor expansion at θ_t

s.t., $KL\left(\rho_{\pi_{\theta_t}} | \rho_{\pi_\theta}\right) \le \delta$ ⟶ second-order Taylor expansion at θ_t

Intuition: maximize local adv subject to being incremental (in KL);

$$\max_{\theta} \nabla_\theta J(\pi_{\theta_t})^\top(\theta - \theta_t)$$

s.t. $(\theta-\theta_t)^\top F_{\theta_t}(\theta-\theta_t) \le \delta$

$$\theta_{t+1} = \theta_t + \eta F_{\theta_t}^{-1}\nabla_\theta J(\pi_{\theta_t})$$ ⟵

NPG

$$F_{\theta_t} := \mathbb{E}_{s,a\sim d_\mu^{\pi_{\theta_t}}}\left[\nabla_\theta \ln \pi_{\theta_t}(a|s)\left(\nabla_\theta \ln \pi_{\theta_t}(a|s)\right)^\top\right] \in \mathbb{R}^{dim_\theta \times dim_\theta}$$

Outline for Today:

1. More Explanation of Natural (Policy) Gradient

2. Proximal Policy Optimization (PPO)

NPG update: $\theta_1 = \theta_0 + \eta F_{\theta_0}^{-1} \nabla_{\theta_0}$

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$$KL\left(\rho_{\pi_{\theta_0}} | \rho_{\pi_\theta}\right) \leq \delta \Rightarrow \boxed{(\theta - \theta_0)^\top F_{\theta_0}(\theta - \theta_0)} \leq \delta$$

$$= (\theta-\theta_0)^\top F_{\theta_0} (\theta-\theta_0)$$

Hessian

$$KL(\rho_{\pi_{\theta_0}} \| \rho_{\pi_\theta}) = \ell(\theta)$$

second taylor:

$$\ell(\theta_0) + \nabla \ell(\theta)\big|_{\theta=\theta_0}^\top (\theta-\theta_0) + \frac{1}{2}(\theta-\theta_0)^\top \underline{H(\theta_0)}(\theta-\theta_0)$$

0

0

NPG update: $\theta_1 = \theta_0 + \eta F_{\theta_0}^{-1} \nabla_{\theta_0}$

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Our goal is to make sure two distributions do not change to much,
but parameters θ could potential change a lot!

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Consider special case where F_{θ_0} is a diagonal matrix: $F_{\theta_0} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$

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$$\forall i: \; \theta_1[i] = \theta_0[i] + \underbrace{\left(\eta \sigma_i^{-1}\right)}_{\text{learning Rate}} \nabla_{\theta_0}[i]$$

$F_{\theta_0}^{-1} \cdot \nabla_{\theta_0}$

NPG update: $\theta_1 = \theta_0 + \eta F_{\theta_0}^{-1} \nabla_{\theta_0}$

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$$\forall i: \ \ \theta_1[i] = \theta_0[i] + \left(\eta \sigma_i^{-1}\right) \nabla_{\theta_0}[i]$$

For tiny σ_i, we indeed have a **huge** learning rate, i.e., $\eta \sigma_i^{-1}$, at coordinate i !

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For tiny σ_i, we indeed have a **huge** learning rate, i.e., $\eta \sigma_i^{-1}$, at coordinate i !

In other words, NPG **allows a big jump** on some coordinates which do not affect KL-div too much

Example of Natural Gradient on 1-d problem:

θ

$$p_\theta = \left(\frac{\exp(\theta)}{1+\exp(\theta)}, \frac{1}{1+\exp(\theta)} \right)$$ ← Distribution

$$g(\theta) = 100 \cdot p_\theta[1] + 1 \cdot p_\theta[2]$$

$\max_\theta g(\theta)$

∇g

$g'(\theta)$

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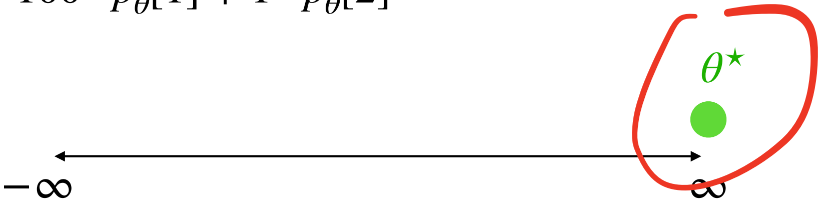

$\theta^\star = +\infty$

$\Rightarrow p_\theta(1) = 1$

$p_\theta(0) = 0$

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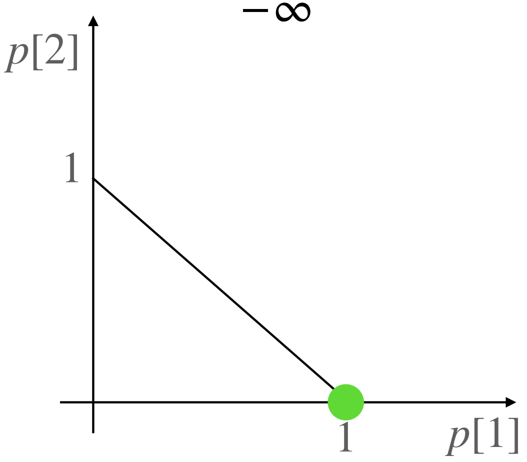

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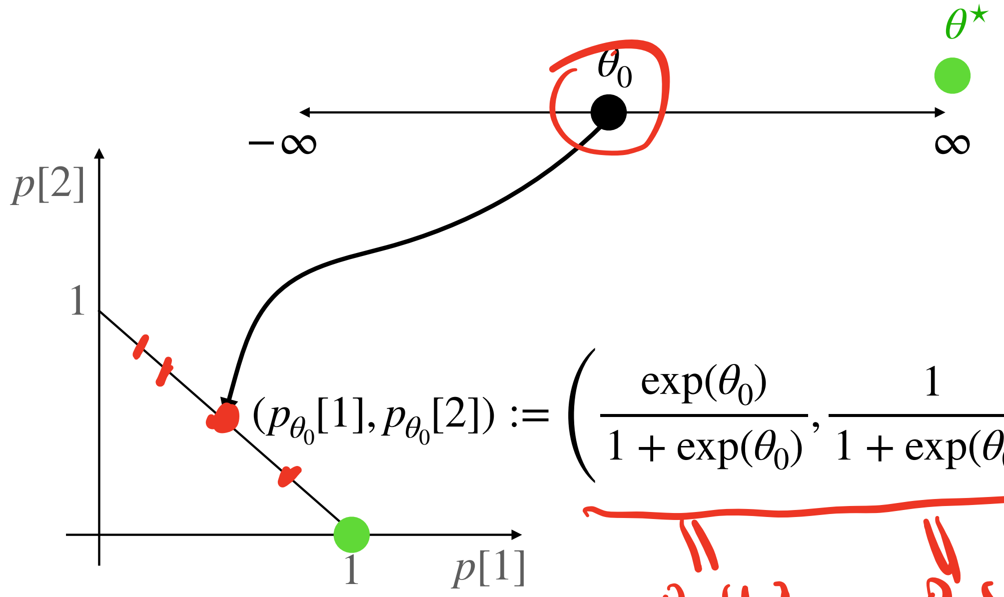
$$l(\theta) = KL(P_{\theta_0} \| P_\theta)$$

$$l''(\theta)$$



$$(p_{\theta_0}[1], p_{\theta_0}[2]) := \left(\underbrace{\frac{\exp(\theta_0)}{1+\exp(\theta_0)}}_{= p_{\theta_0}(1)}, \underbrace{\frac{1}{1+\exp(\theta_0)}}_{= p_{\theta_0}(2)} \right)$$

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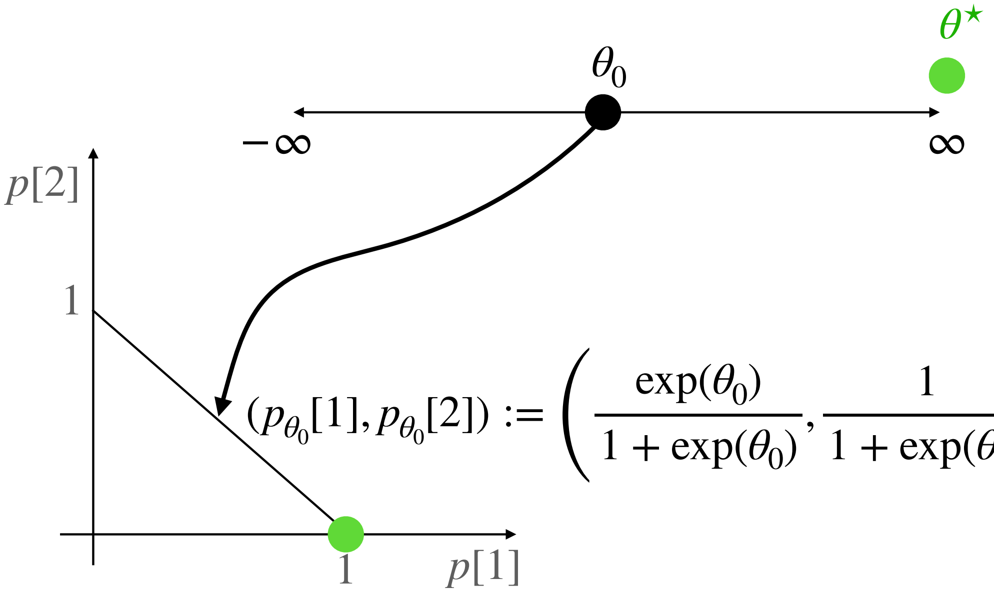
Fisher information scalar: $f_{\theta_0} = \dfrac{\exp(\theta_0)}{(1+\exp(\theta_0))^2}$



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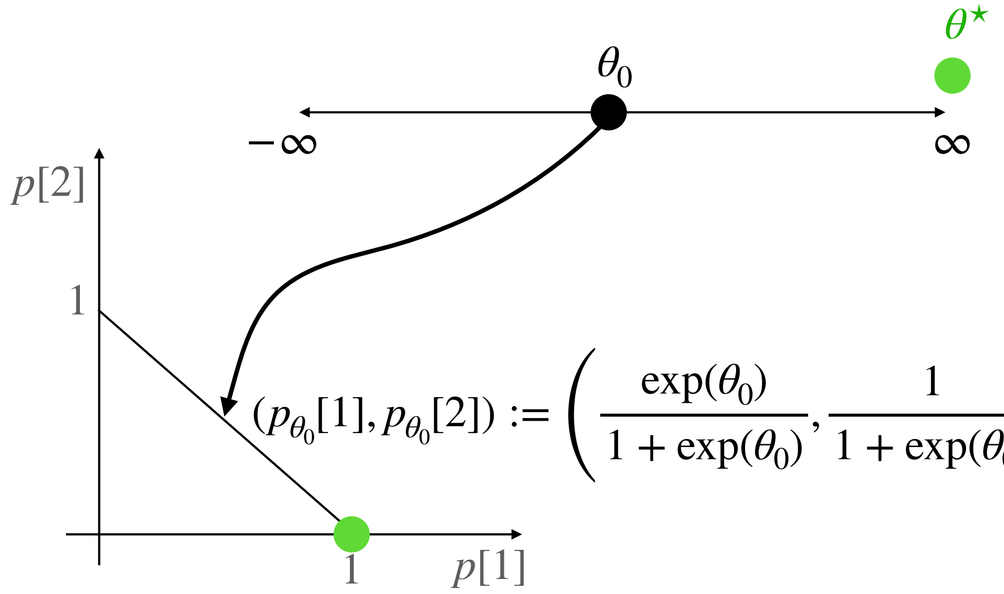
Hence: $f_{\theta_0} \to 0^+$, as $\theta_0 \to \infty$



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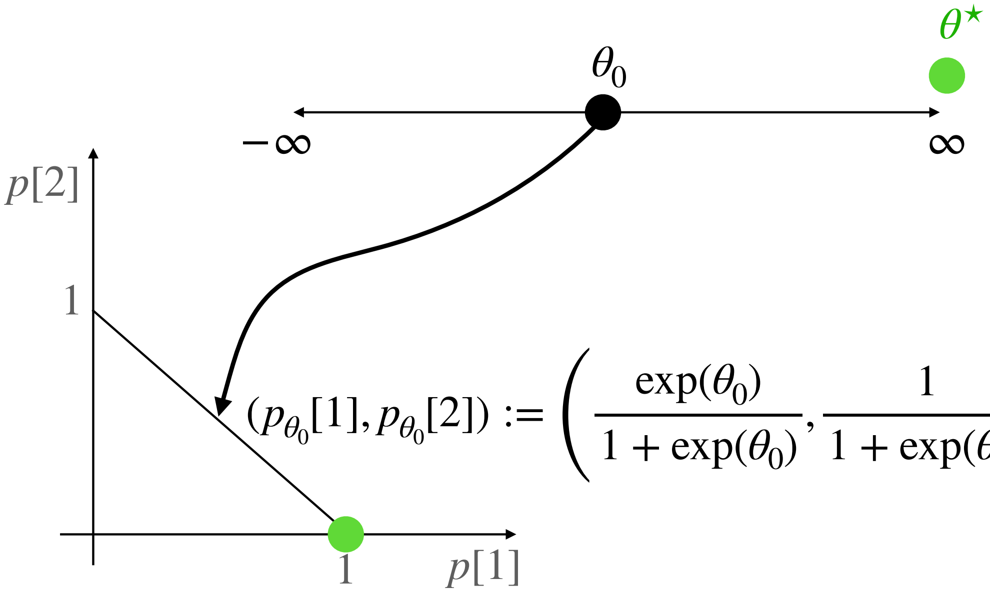

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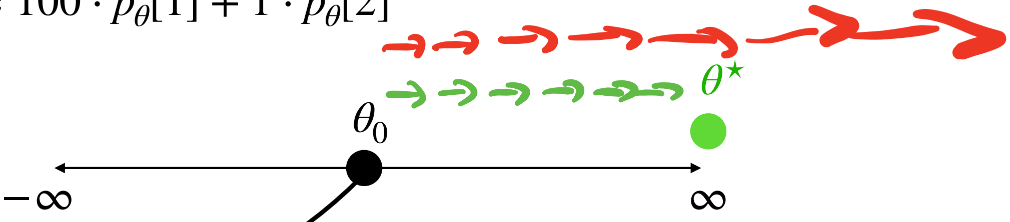

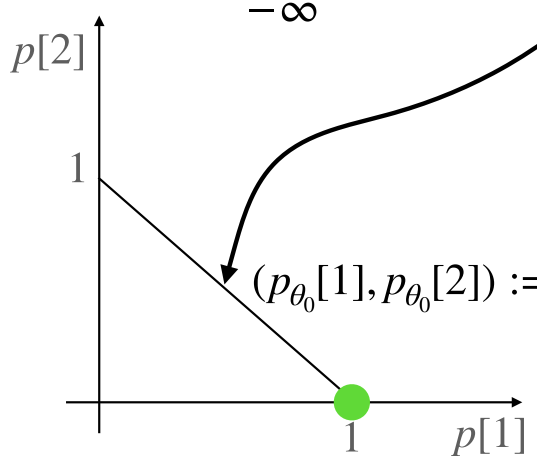

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NPG: $\theta_1 = \theta_0 + \eta \dfrac{g'(\theta_0)}{f_{\theta_0}}$ $\to 0^+$

GA: $\theta_1 = \theta_0 + \eta g'(\theta_0)$

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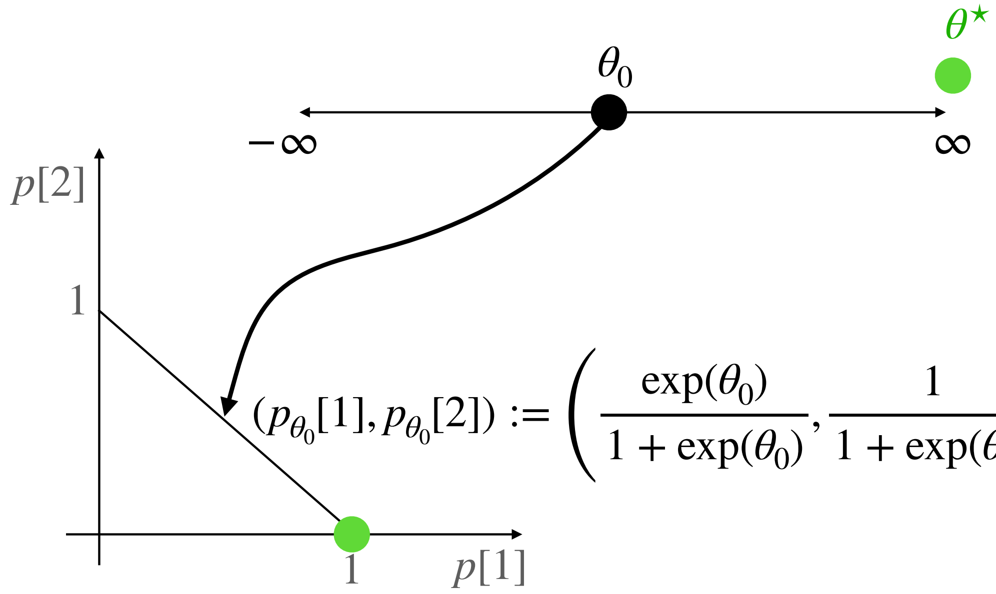

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NPG: $\theta_1 = \theta_0 + \eta \dfrac{g'(\theta_0)}{f_{\theta_0}}$

GA: $\theta_1 = \theta_0 + \eta g'(\theta_0)$

i.e., Natural GA can speed up learning when θ gets larger

Outline for Today:

1. More Explanation of Natural (Policy) Gradient

2. Proximal Policy Optimization (PPO)

Policy Gradient (e.g., REINFORCE) can unstable and slow

The potential high-variance in PG can make learning very unstable

$x = F^{-1}g$ $\boxed{\min_x \| F \cdot x - g \|_2^2} \rightarrow \arg\min_x = F^{-1}g$

Policy Gradient (e.g., REINFORCE) can unstable and slow

The potential high-variance in PG can make learning very unstable

Natural Policy gradient is computational expensive

F_θ^{-1}

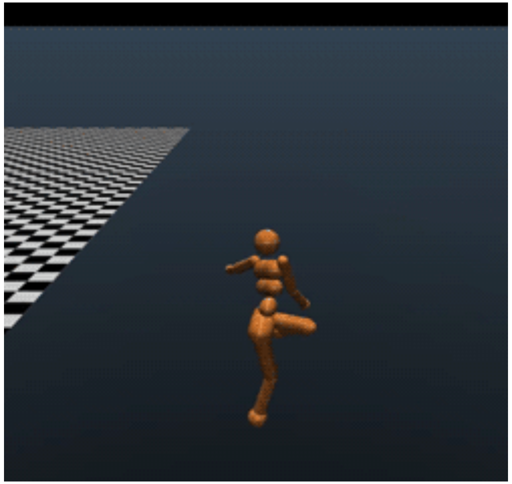
Even compute fisher information matrix is slow

$F_\theta \in \mathbb{R}^{dim_\theta \times dim_\theta}$ $(dim_\theta)^2$

These methods do not take advantage of GPUs well

$\frac{1}{n}\sum_{i=1}^{n} \nabla_\theta \log \pi(\cdots) R$

$\tau_1 \tau_2 \cdots \tau_n$



$\longleftrightarrow$

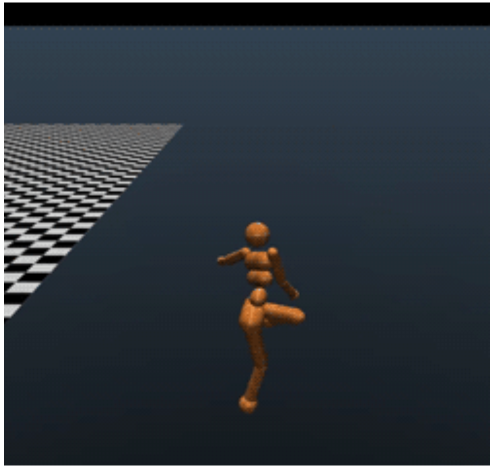
Too frequent!

GPU usage can be very low…

$\hookrightarrow$ CPU

Proximal Policy Optimization (PPO)

π_{θ_t}



Proximal Policy Optimization (PPO)

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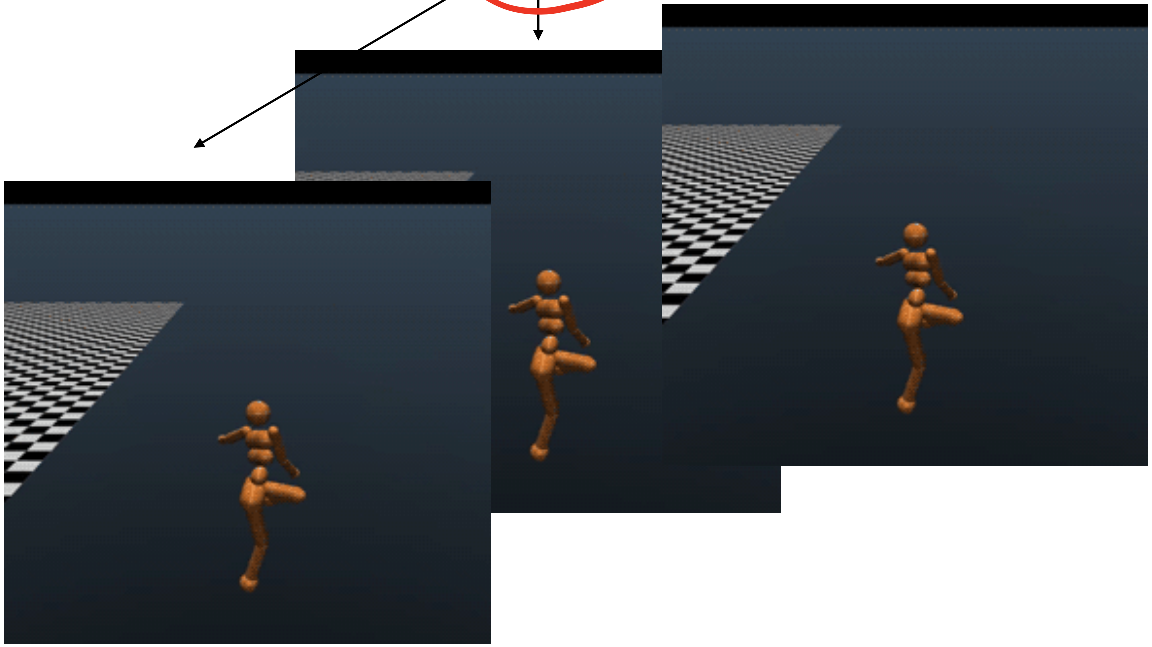
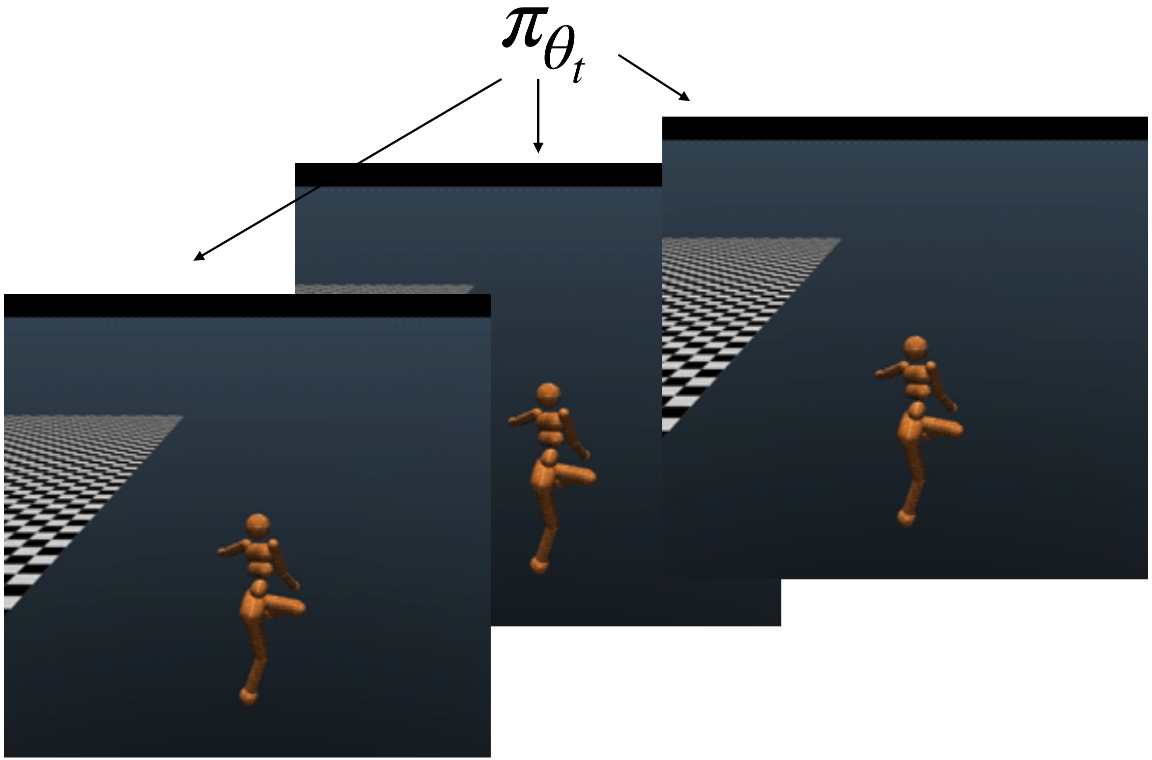


Proximal Policy Optimization (PPO)

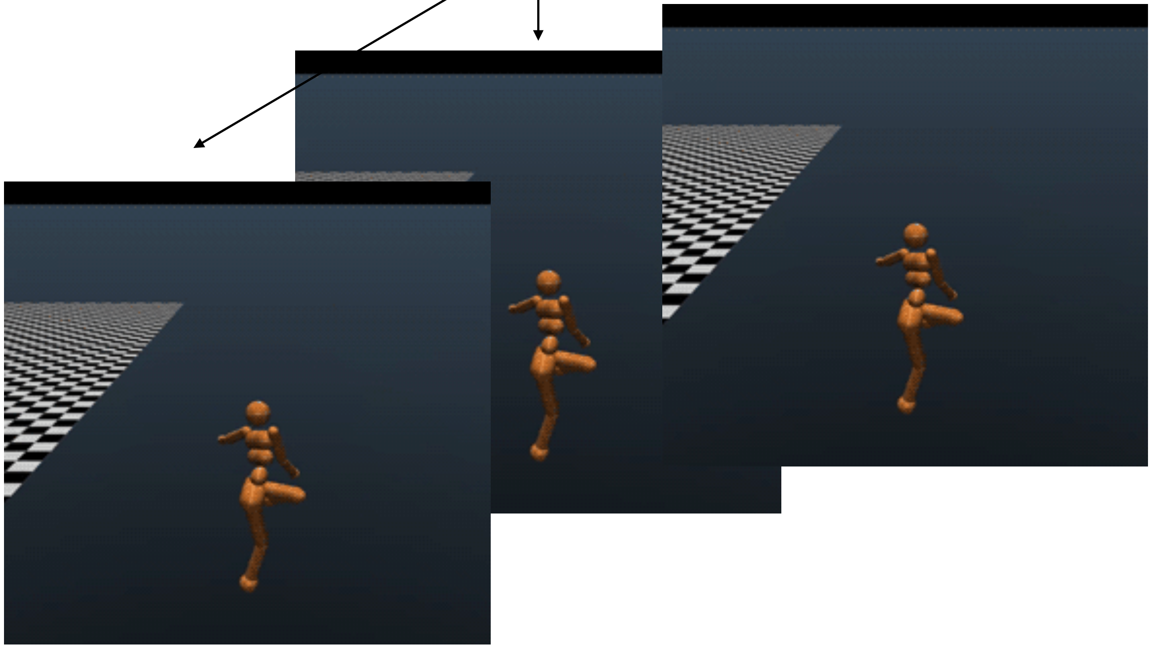


Collect a large dataset

$$\longrightarrow \mathcal{D} = \left\{ s, a, A^{\pi_{\theta_t}}(s, a) \right\}$$

Proximal Policy Optimization (PPO)

π_{θ_t}



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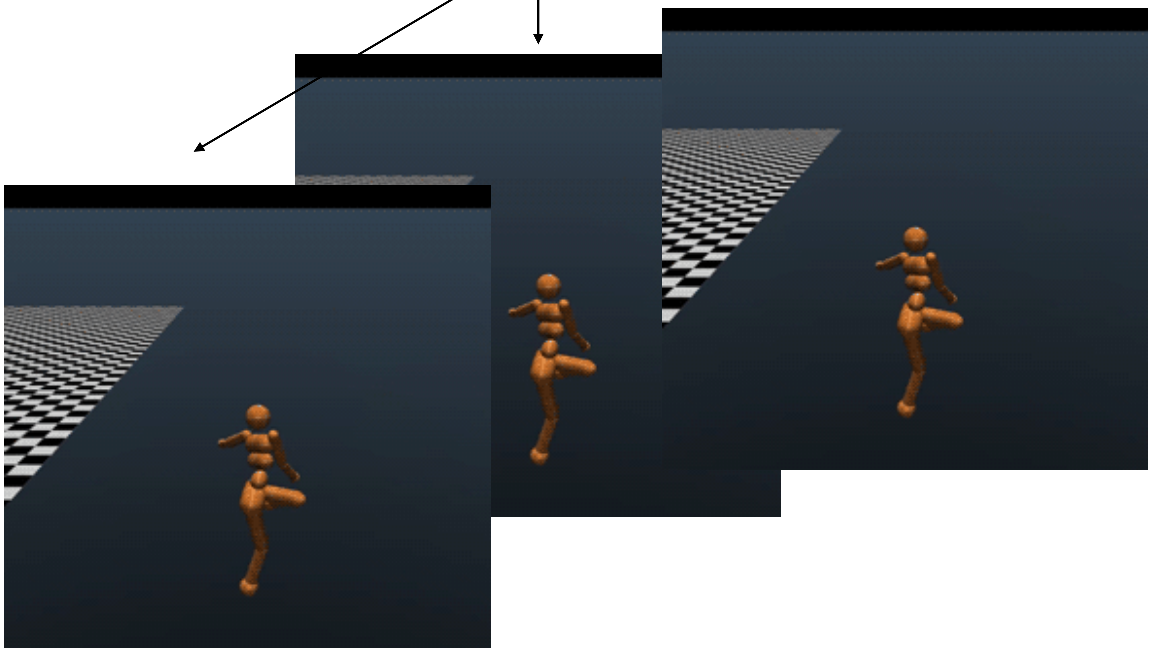

Collect a large dataset

$$\mathscr{D} = \{s, a, A^{\pi_{\theta_t}}(s, a)\}$$

Now let's do multiple epoches of mini-batch gradient update on the dataset

Proximal Policy Optimization (PPO)

Construct a **batch Supervised Learning** style objective using $\mathscr{D} = \{s, a, A^{\pi_{\theta_t}}(s, a)\}$

$$\max_{\theta} \ell(\theta) = \max_{\theta} \mathbb{E}_{s \sim d^{\pi_{\theta_t}}} \mathbb{E}_{a \sim \pi_\theta(\cdot|s)} \cdot A^{\pi_{\theta_t}}(s, a)$$

$s \sim d^{\pi_{\theta_t}}$

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IW trick $\rightarrow \mathbb{E}_{s \sim d^{\pi_{\theta_t}}} \mathbb{E}_{a \sim \pi_{\theta_t}(\cdot|s)} \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s, a)$

$$\mathbb{E}_{a \sim \pi_\theta(\cdot|s)} A(s,a) = \mathbb{E}_{a \sim \pi_{\theta_t}(\cdot|s)} \frac{\pi_\theta(a|s)}{\pi_{\theta_t}(a|s)} \cdot A(s,a)$$

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IW trick $\rightarrow \mathbb{E}_{s\sim d^{\pi_{\theta_t}}}\mathbb{E}_{a\sim\pi_{\theta_t}(\cdot|s)} \dfrac{\pi_\theta(a\,|\,s)}{\pi_{\theta_t}(a\,|\,s)} \cdot A^{\pi_{\theta_t}}(s,a)$

$$\approx \frac{1}{N} \sum_{s,a} \frac{\pi_\theta(a\,|\,s)}{\pi_{\theta_t}(a\,|\,s)} \cdot A^{\pi_{\theta_t}}(s,a)$$

$s \sim d^{\pi_{\theta_t}}$

$a \sim \pi_{\theta_t}(\cdot|s)$

Sample Avg

Proximal Policy Optimization (PPO)

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$s \sim d^{\pi_{\theta_t}}$

$a \sim \pi_{\theta_t}(\cdot \mid s)$

$(s, a) \quad \underline{A^{\pi_{\theta_t}}(s,a) > 0}$

$$\frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} A^{\pi_{\theta_t}}(s,a)$$

1

0.01

Proximal Policy Optimization (PPO)

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$$\hat{\ell}(\theta) = \sum_{s,a} \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s, a)$$

Trick 1: clipping to make sure π_θ stay close to π_{θ_t} (ensuring stability in training)

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Trick 1: clipping to make sure π_θ stay close to π_{θ_t} (ensuring stability in training)

$$\hat{\ell}_{clip}(\theta) = \sum_{s,a} \underline{\text{clip}} \left(\frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}, 1 - \epsilon, 1 + \epsilon \right) \cdot A^{\pi_{\theta_t}}(s, a)$$

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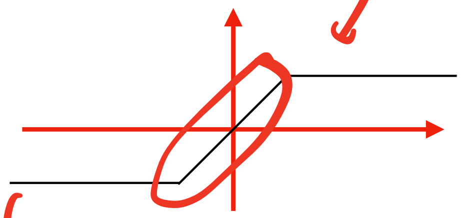
$x > 1+\varepsilon$

$$\hat{\ell}_{clip}(\theta) = \sum_{s,a} \text{clip}\left(\frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}, 1-\epsilon, 1+\epsilon \right) \cdot A^{\pi_{\theta_t}}(s, a)$$



$x < 1-\varepsilon$

clip$(x, 1-\epsilon, 1+\epsilon)$

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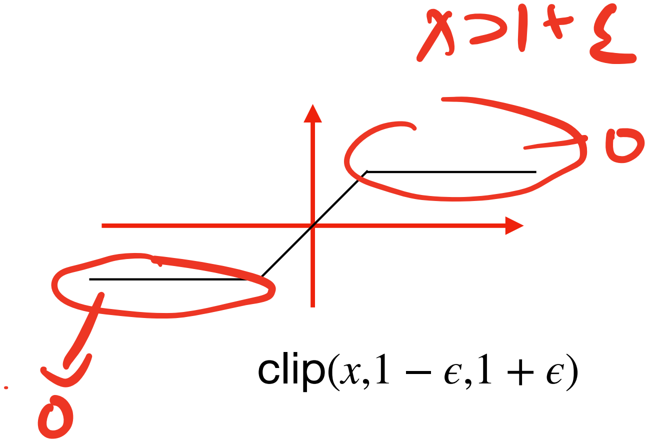
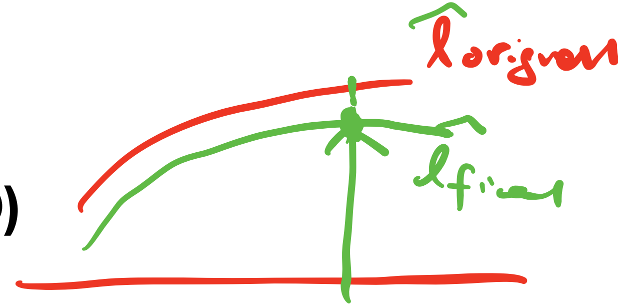
Stop updating $\pi_\theta(a \mid s)$ if it is too different from $\pi_{\theta_t}(a \mid s)$



clip$(x, 1-\epsilon, 1+\epsilon)$

Proximal Policy Optimization (PPO)



Trick 2, take the min of the clipped and uncipped (original) obj

$$\hat{\ell}_{final}(\theta) = \sum_{s,a} \min \left\{ \frac{\pi_\theta(a\,|\,s)}{\pi_{\theta_t}(a\,|\,s)} \cdot A^{\pi_{\theta_t}}(s,a), \quad \text{clip}\left(\frac{\pi_\theta(a\,|\,s)}{\pi_{\theta_t}(a\,|\,s)}, 1-\epsilon, 1+\epsilon \right) \cdot A^{\pi_{\theta_t}}(s,a) \right\}$$

Original obj

clipped obj which ensures no abrupt change in action probabilities

Proximal Policy Optimization (PPO)

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$$\hat{\ell}_{final}(\theta) = \sum_{s,a} \min \left\{ \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s,a), \quad \text{clip}\left(\frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}, 1-\epsilon, 1+\epsilon \right) \cdot A^{\pi_{\theta_t}}(s,a) \right\}$$

Just consider one term inside the summation: (s,a)

Proximal Policy Optimization (PPO)

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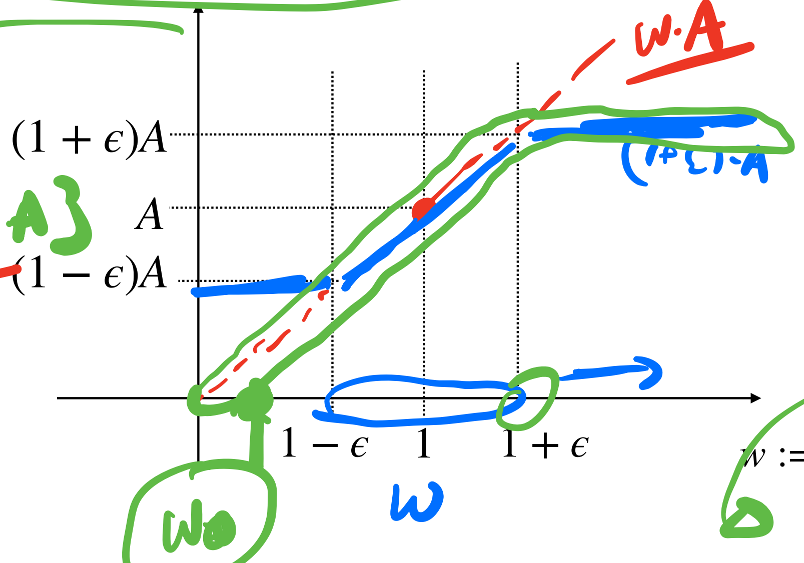
$$\hat{\ell}_{final}(\theta) = \sum_{s,a} \min \left\{ \underbrace{\frac{\pi_\theta(a|s)}{\pi_{\theta_t}(a|s)}}_{w} \cdot A^{\pi_{\theta_t}}(s,a), \quad \text{clip}\left(\underbrace{\frac{\pi_\theta(a|s)}{\pi_{\theta_t}(a|s)}}_{w}, 1-\epsilon, 1+\epsilon \right) \cdot A^{\pi_{\theta_t}}(s,a) \right\}$$

Just consider one term inside the summation:

s, a $A(s,a) > 0$

When $A^{\pi_{\theta_t}}(s,a) > 0$

$\min \{ w \cdot A, \text{clip}(w, 1-\epsilon, 1+\epsilon) \cdot A \}$



when $w > 1+\epsilon$

$\text{clip}(w, 1-\epsilon, 1+\epsilon) \cdot A \Rightarrow (1+\epsilon) \cdot A$

$$w := \frac{\pi_\theta(a|s)}{\pi_{\theta_t}(a|s)}$$

density Ratio

Proximal Policy Optimization (PPO)

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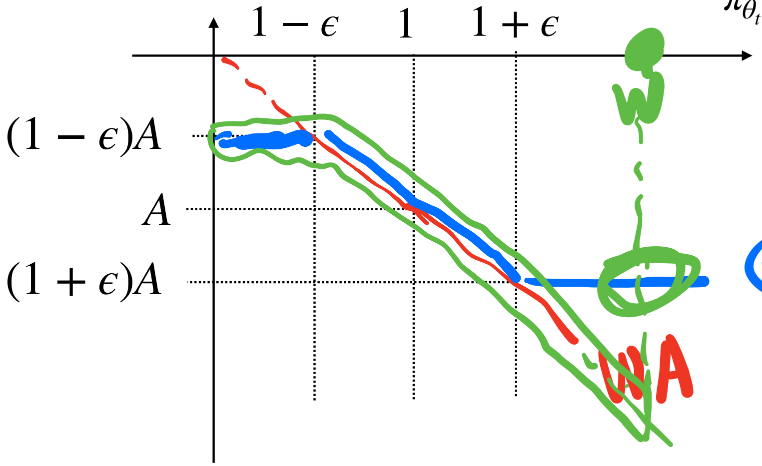
Just consider one term inside the summation:

When $A^{\pi_{\theta_t}}(s,a) < 0$

$w := \dfrac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}$

$\min\{ w \cdot A, \ \underline{\text{clip}(w, 1-\epsilon, 1+\epsilon) A} \}$

$A < 0$



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clipped obj which ensures no abrupt change in action probabilities

We compute $\theta_{t+1} \approx \arg\max_\theta \hat{\ell}_{final}(\theta)$, via performing a few epoches of minbatch SG ascent (or Adam/Adagrad) on $\hat{\ell}_{final}$

Proximal Policy Optimization (PPO)

Initialize θ_0 for the policy π_θ

For $t = 0 \to T$:

π_{θ_t} $\tau_1, \tau_2 \ldots \tau_n$

Run π_θ to collect multiple trajectories, and form the dataset $\{s, a, A^{\pi_{\theta_t}}(s, a)\}$

Proximal Policy Optimization (PPO)

Initialize θ_0 for the policy

For $t = 0 \rightarrow T$:

Run π_θ to collect multiple trajectories, and form the dataset $\{s, a, A^{\pi_{\theta_t}}(s, a)\}$

Construct the loss $\hat{\ell}_{final}(\theta)$ using the dataset

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Construct the loss $\hat{\ell}_{final}(\theta)$ using the dataset

Perform a few steps of mini-batch gradient updates on $\hat{\ell}_{final}(\theta)$ to get θ_{t+1}

Summary

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PPO is a more practical versions of NPG — making NPG really scalable while maintaing the high level idea of NPG