Policy Gradient
(continue)

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update: $\theta \Leftarrow \theta + \eta g$ (or adaptive methods like Adam)

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Often require large n to reduce the variance, especially when policy π_θ is quite random

Today's Question:

How to reduce Variance in Policy Gradient?

Outline:

1. A $Q(s, a)$ based Policy Gradient

2. Variance Reduction via A Baseline

Notations

$$\mathscr{M} = \{P, r, \gamma, \mu, S, A\} \quad \text{where } s_0 \sim \mu$$

$$V^\pi(s) = \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r(s_h, a_h) \,|\, s_0 = s, a_h \sim \pi\right]$$

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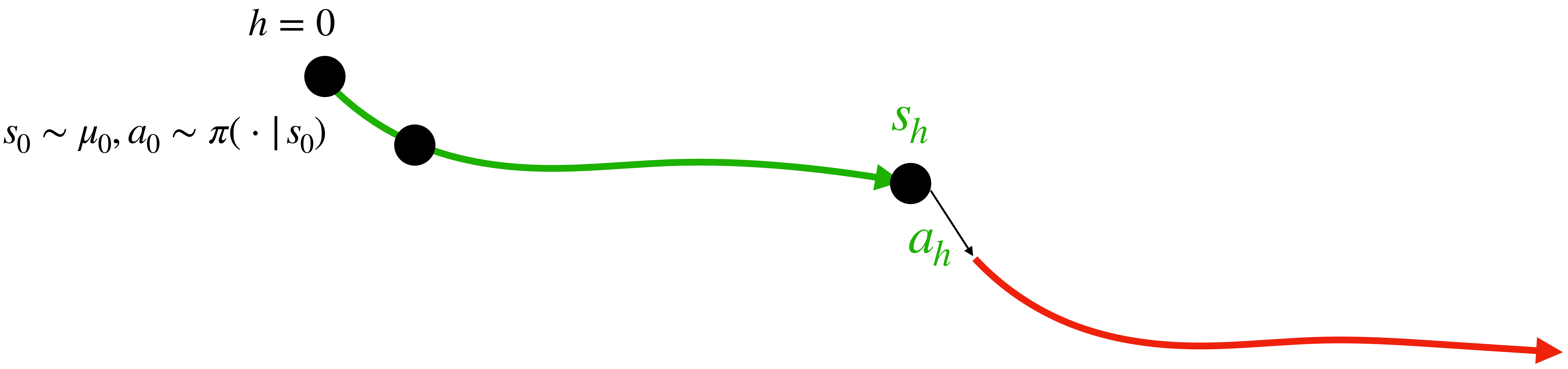

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This still contains the entire future, could still have high variance

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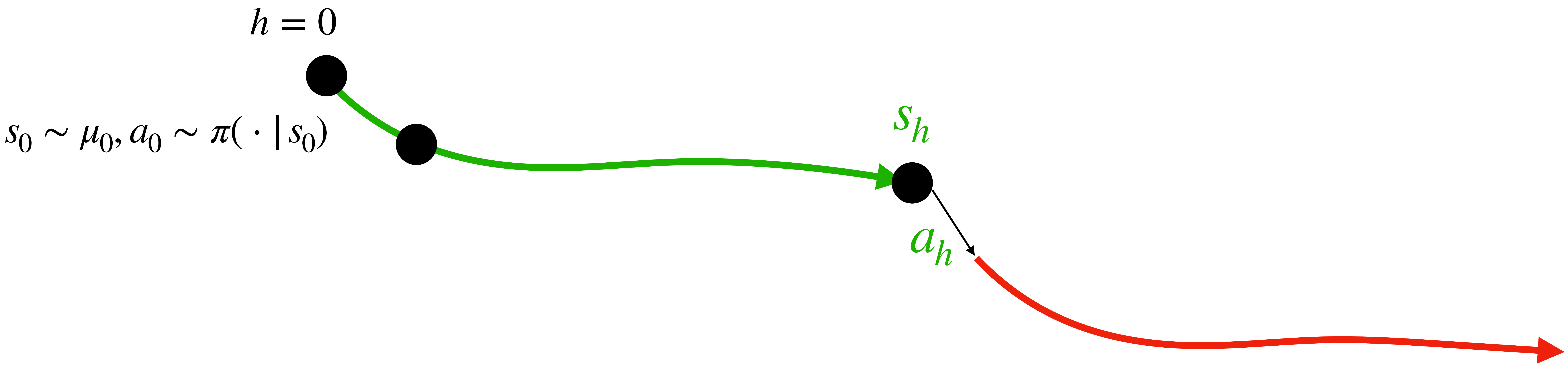

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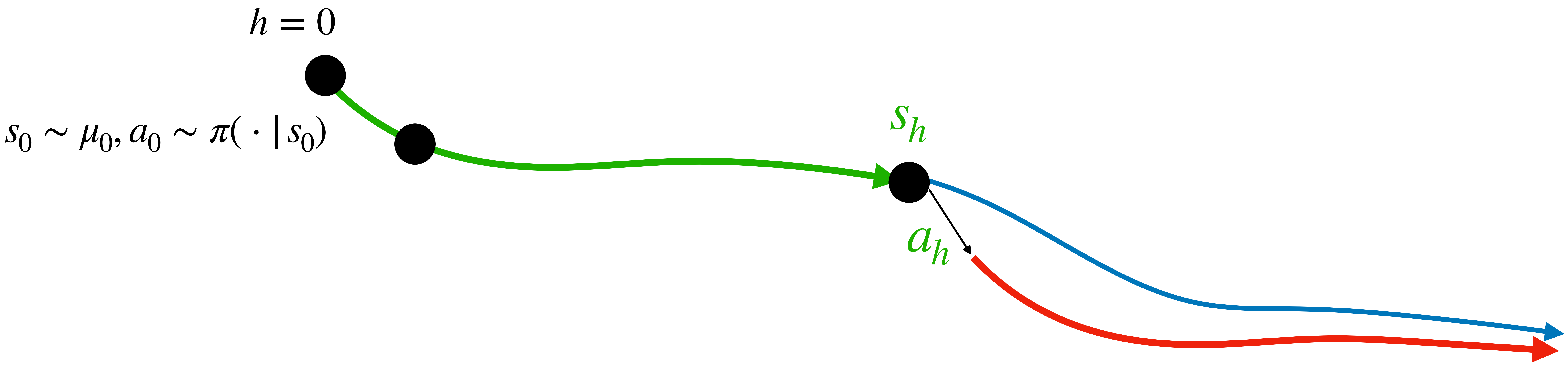

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Three common PG formulations:

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$$\nabla J(\theta) = \mathbb{E}_{\tau \sim \rho_\theta(\tau)} \left[\left(\sum_{h=0}^{\infty} \nabla_\theta \ln \pi_\theta(a_h | s_h) \right) R(\tau) \right]$$

PG w/ Q function

$$\nabla_\theta J(\theta) = \frac{1}{1-\gamma} \mathbb{E}_{s,a \sim d^{\pi_\theta}} \left[\nabla_\theta \ln \pi_\theta(a | s) \left(Q^{\pi_\theta}(s, a) \right) \right]$$

PG w/ A function (use $V^\pi(s)$ as a baseline)

$$\nabla_\theta J(\theta) = \frac{1}{1-\gamma} \mathbb{E}_{s,a \sim d^{\pi_\theta}} \left[\nabla_\theta \ln \pi_\theta(a | s) \left(A^{\pi_\theta}(s, a) \right) \right]$$