PPO and GAE

Annoucements

Will release the next reading Quiz on the PPO technical report

Will release the next programming assignment on NPG and PPO

Recap: Proximal Policy Optimization (PPO)

Policy optimization objective:

$$\hat{\ell}_{final}(\theta) = \sum_{s,a} \min \left\{ \underbrace{\frac{\pi_\theta(a\,|\,s)}{\pi_{\theta_t}(a\,|\,s)} \cdot A^{\pi_{\theta_t}}(s,a)}_{1}, \quad \text{clip}\left(\frac{\pi_\theta(a\,|\,s)}{\pi_{\theta_t}(a\,|\,s)}, 1-\epsilon, 1+\epsilon \right) \cdot A^{\pi_{\theta_t}}(s,a) \right\}$$

$$\frac{\pi_{new}(a|s)}{\pi_{old}(a|s)} \in [1-\varepsilon, 1+\varepsilon]$$

Recap: Proximal Policy Optimization (PPO)

Policy optimization objective:

w

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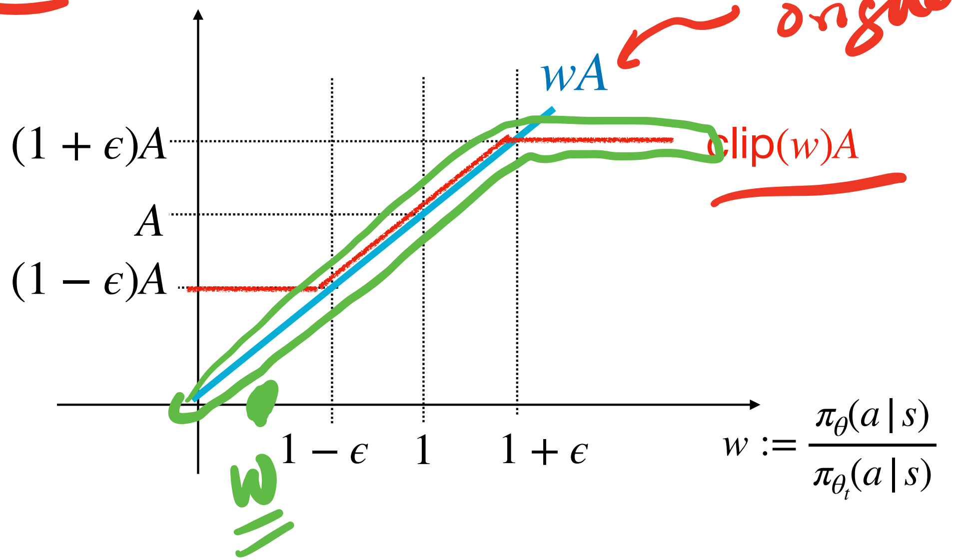
When $A^{\pi_{\theta_t}}(s,a) > 0$, we want to increase the ratio $\pi_\theta(a|s)/\pi_{\theta_t}(a|s)$



Main Question Today:

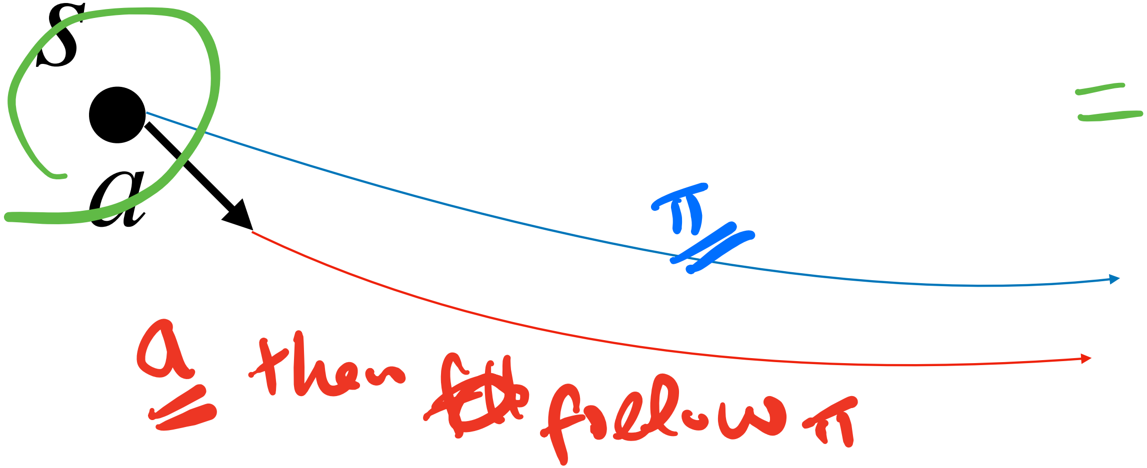
$$\hat{\ell}_{final}(\theta) = \sum_{s,a} \min \left\{ \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s,a), \quad \text{clip}\left(\frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}, 1-\epsilon, 1+\epsilon \right) \cdot A^{\pi_{\theta_t}}(s,a) \right\}$$

How to get these advantage values?

Attempt 1: Monte Carlo (MC) method

Given (s, a), estimate $A^\pi(s, a)$ via two MC rollouts

$$A^\pi(s,a) = Q^\pi(s,a) - V^\pi(s)$$



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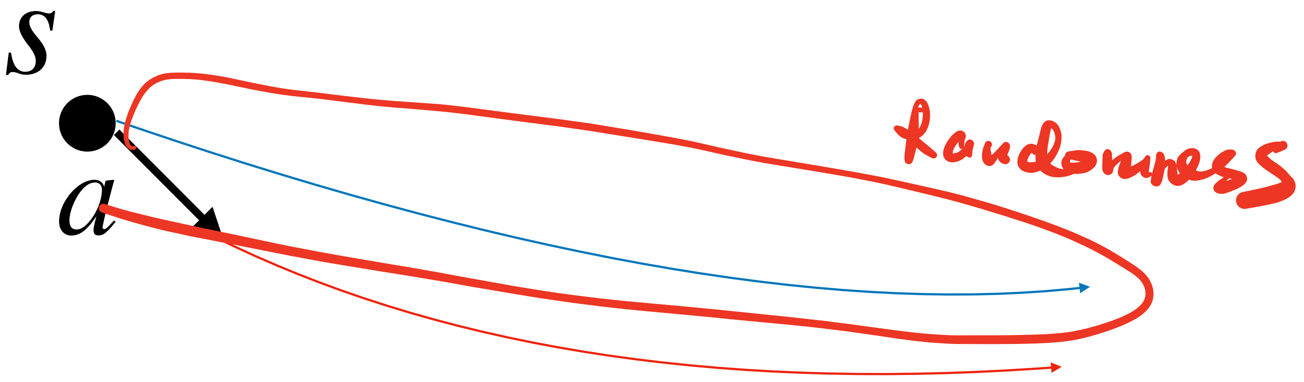

Can have high variance;

Attempt 1: Monte Carlo (MC) method

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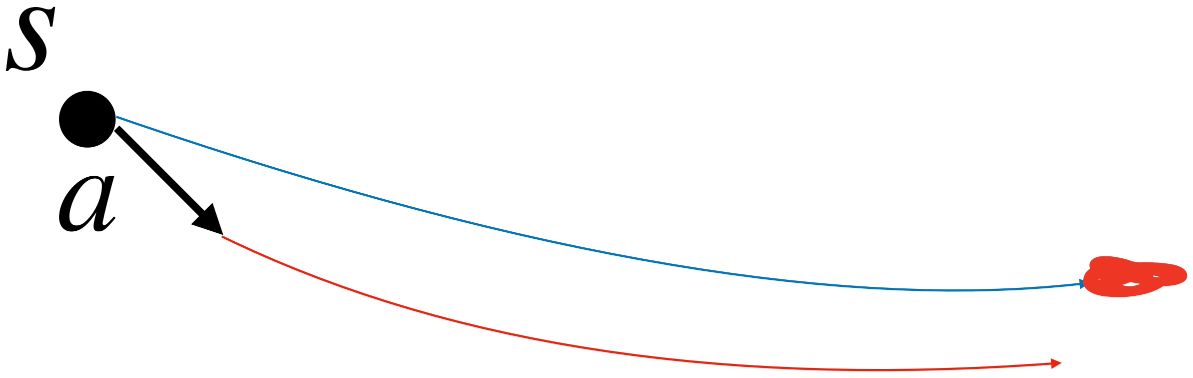

Can have high variance;

Require the ability to **reset**: i.e., go back to s again, and do one more rollout

Attempt 1: Monte Carlo (MC) method

π

Q: given (s, a), can we estimate $A^{\pi}(s, a)$ via one rollout?

$A = \underline{Q - V}$

Attempt 1: Monte Carlo (MC) method

Q: given (s, a), can we estimate $A^\pi(s, a)$ via one rollout?

$$\underline{A^\pi(s,a)/2} = \underline{(Q^\pi(s,a) - V^\pi(s))/2} = \mathbb{E}_{z \sim \text{Uniform}(\{0,1\})}(zQ^\pi(s,a) - (1-z)V^\pi)$$

$$z: \begin{cases} 0 & \frac{1}{2} \\ 1 & \frac{1}{2} \end{cases}$$

$0 \cdot Q - (1-0) \cdot V$
$= -V$

$1 \cdot Q - (1-1) \cdot V$
$= Q$

$\frac{1}{2}(-V) + \frac{1}{2} \cdot Q = (Q-V)/2$
$= A/2$

Attempt 1: Monte Carlo (MC) method

Q: given (s, a), can we estimate $A^\pi(s, a)$ via one rollout?

$$A^\pi(s,a)/2 = (Q^\pi(s,a) - V^\pi(s))/2 = \mathbb{E}_{z\sim \text{Uniform}(\{0,1\})}(zQ^\pi(s,a) - (1-z)V^\pi)$$

Can have high variance still

To further reduce variance, we will give up unbiaseness,
and trade bias for variance

Generalized Advantage Estimation

Our goal: estimate $A^{\pi}(s, a)$

We will do the following two steps:

Generalized Advantage Estimation

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1. Estimate $V^{\pi}(s)$ using function approximation (neural network, decision tree, etc)

$\hat{V} \approx V^{\pi}$

Generalized Advantage Estimation

Our goal: estimate $A^{\pi}(s, a)$

We will do the following two steps:

1. Estimate $V^{\pi}(s)$ using function approximation (neural network, decision tree, etc)

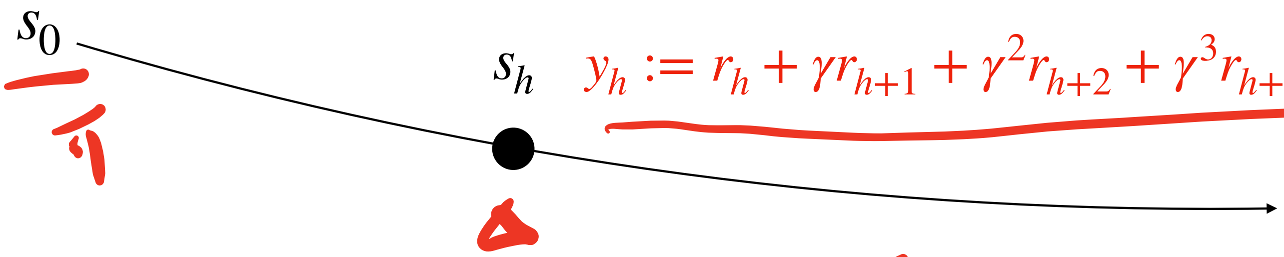
2. Form an estimate of $A^{\pi}(s, a)$ using the value function estimator V

$Q - V$

Generalized Advantage Estimation

Estimate V^π

1. sample a trajectory $\tau \sim \pi$, for all h:



$$y_h := r_h + \gamma r_{h+1} + \gamma^2 r_{h+2} + \gamma^3 r_{h+3} + \ldots$$

s_h

$V^\pi(s_h)$

$E[y_h \mid s_h]$

$= V^\pi(s_h)$

Generalized Advantage Estimation

Estimate V^π

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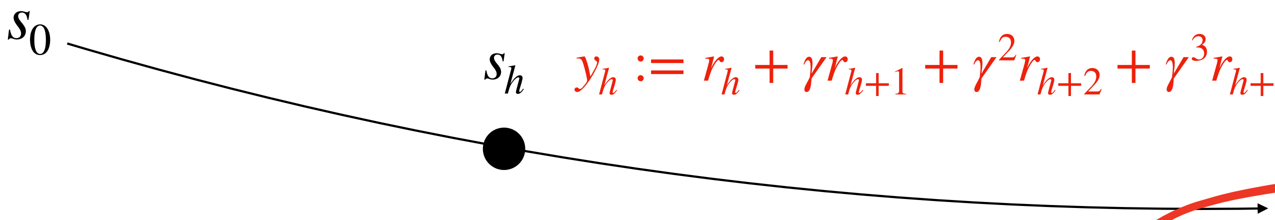

$y_h := r_h + \gamma r_{h+1} + \gamma^2 r_{h+2} + \gamma^3 r_{h+3} + \ldots$

$\{s_h, y_h\}_{h \geq 0}$

$h = 0 \ldots H$

Generalized Advantage Estimation

Estimate V^π

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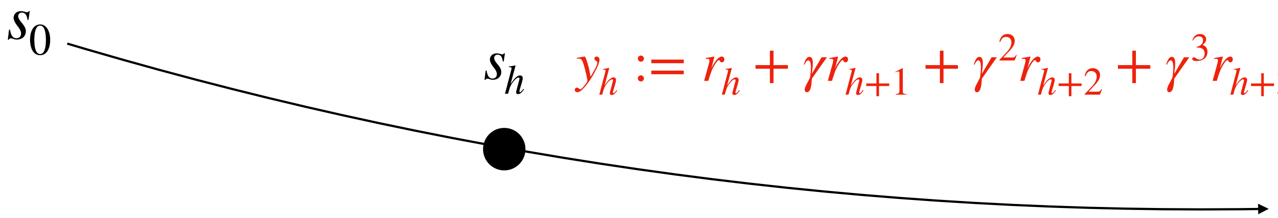

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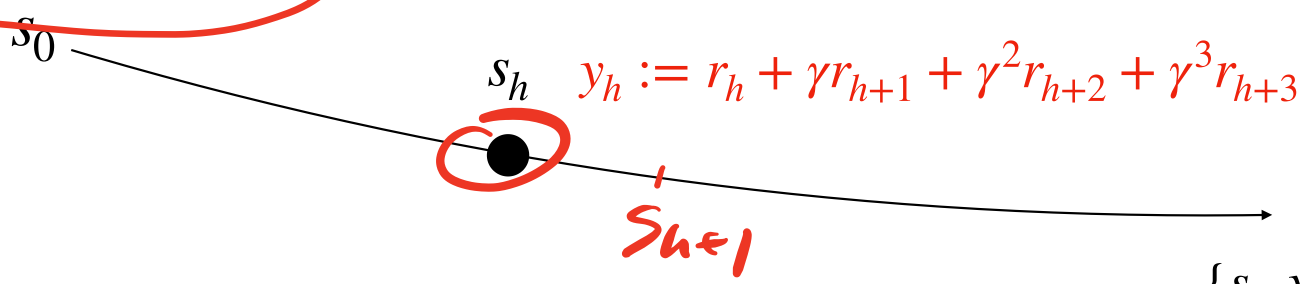
2. Repeat this for n times (i.e., n trajectories), form a regression dataset:

$\tau_1 \ \tau_2 \ .. \ \tau_n$

Generalized Advantage Estimation

Estimate V^{π}

1. sample a trajectory $\tau \sim \pi$, for all h:



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$\{s_h, y_h\}_{h \geq 0}$

2. Repeat this for n times (i.e., n trajectories), form a regression dataset:

$$\mathcal{D}_{\pi} = \{s, y\}$$

Generalized Advantage Estimation

Estimate V^π

Given $\mathscr{D}_\pi = \{s, y\}$, perform regression

MSE

$$\hat{V}^\pi = \arg\min_V \sum_{s,y \in \mathscr{D}_\pi} \left(V(s) - y\right)^2$$

$\approx V^\pi$

Generalized Advantage Estimation

Estimate V^{π}

Given $\mathscr{D}_{\pi} = \{s, y\}$, perform regression

$$\hat{V}^{\pi} = \arg\min_{V} \sum_{s,y \in \mathscr{D}_{\pi}} \left(V(s) - y\right)^2$$

$\mathbb{E}[y \mid s_0]$

$= V^{\pi}(s)$

The Bayes optimal for this regression is $\mathbb{E}[y \mid s] = V^{\pi}(s)$,
so if regression works well we can have $\hat{V}^{\pi} \approx V^{\pi}$

Generalized Advantage Estimation

Our goal: estimate $A^{\pi}(s, a)$

We will do the following two steps:

1. Estimate $V^{\pi}(s)$ using function approximation (neural network, decision tree, etc)

2. Form an estimate of $A^{\pi}(s, a)$ using the value function estimator V

Generalized Advantage Estimation

Denote V as an estimator of V^π, let's compute estimate for $A^\pi(s,a)$

$$A^\pi(s,a) = \underline{Q^\pi(s,a)} - V^\pi(s)$$

V

$$= r(s,a) + \gamma \underbrace{\mathbb{E}_{s' \sim P(s,a)}}_{} V^\pi(s') - V^\pi(s)$$

$$= \underbrace{r(s,a) + \gamma V(s')}_{\approx Q^\pi(s,a)} - V(s)$$

Generalized Advantage Estimation

Denote V as an estimator of V^π, let's compute estimate for $A^\pi(s, a)$

$$\hat{A}^{(1)}(s_h, a_h) = r_h + \gamma V(s_{h+1}) - V(s_h)$$

V^π

Q

$\approx Q^\pi$

V

$s_h \xrightarrow{a_h} s_{h+1}$

$r_h := r(s_h, a_h)$

Generalized Advantage Estimation

Denote V as an estimator of V^π, let's compute estimate for $A^\pi(s, a)$

$\hat{A}^{(1)}(s_h, a_h) = r_h + \gamma V(s_{h+1}) - V(s_h)$ (Low variance but can be highly biased)

s_h, a_h, s_{h+1}

$Q^\pi(s_h, a_h)$ $\quad h \bullet \xrightarrow{a_h} s_{h+1} \rightarrow s_{h+2}$

$V^\pi(s_{h+2})$

$(s_h, a_h, s_{h+1}, a_{h+1}, s_{h+2})$ $\quad Q^\pi(s_h, a_h)$

$$Q^\pi(s_h, a_h) - V^\pi(s_h) = E\left(r_h + \gamma r_{h+1} + \gamma^2 V^\pi(s_{h+2})\right) - V^\pi(s_h)$$

Generalized Advantage Estimation

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$$\hat{A}^{(1)}(s_h, a_h) = r_h + \gamma V(s_{h+1}) - V(s_h)$$

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$$\hat{A}^{(2)}(s_h, a_h) = \underbrace{r_h + \gamma r_{h+1} + \gamma^2 V(s_{h+2})}_{\approx Q^\pi(s_h, a_h)} - V(s_h)$$

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s_h a_h. s_{h+1} a_{h+1} s_{h+2}, a_{h+2}, s_{h+3}

Generalized Advantage Estimation

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$$\hat{A}^{(3)}(s_h, a_h) = \underbrace{\boxed{r_h + \gamma r_{h+1} + \gamma^2 r_{h+2} + \gamma^3 V(s_{h+2})}}_{\approx Q^\pi(s_h, a_h)} - V(s_h)$$

Generalized Advantage Estimation

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$$\hat{A}^{(3)}(s_h, a_h) = \underbrace{r_h + \gamma r_{h+1} + \gamma^2 r_{h+2} + \gamma^3 V(s_{h+2})}_{\approx Q^\pi(s_h,a_h)} - V(s_h)$$ (higher var but lower bias)

$\gamma^{16} = 0 \quad = 0$

Q: What is $\hat{A}^\infty(s_h, a_h)$? Would using $\hat{A}^\infty(s_h, a_h)$ in the policy gradient have any bias issue?

$$\hat{A}^\infty(s_h, a_h) = r_h + \gamma r_{h+1} + \gamma^2 r_{h+2} \cdots + \gamma^\infty \cdot V(s_{h+\infty}) - V(s_h)$$

Generalized Advantage Estimation

GAE uses an **exponential average** to combine these advantage estimators together

$$\lambda \in (0,1)$$

$$\hat{A}^{gae} = (1-\lambda)\left(\hat{A}^{(1)} + \lambda\hat{A}^{(2)} + \lambda^2\hat{A}^{(3)} + \ldots\ldots\right)$$

$\lambda^{k-1}\hat{A}^{(k)}$

Generalized Advantage Estimation

$Q^\pi(s,a)$ $V^\pi(s)$

Expressing $\hat{A}^{gae}$ using Bellman errors (Bellman residuals): $BE_\tau = r_\tau + \gamma V(s_{\tau+1}) - V(s_\tau)$

$Q - V$

$$\hat{A}^{(1)}(s_h, a_h) = r_h + \gamma V(s_{h+1}) - V(s_h) = BE_h$$

$$\hat{A}^{(2)}(s_h, a_h) = r_h + \gamma r_{h+1} + \gamma^2 V(s_{h+2}) - V(s_h)$$

$$= r_h + \gamma V(s_{h+1}) - V(s_h) \Leftarrow BE_h$$

$$+ \gamma r_{h+1} + \gamma^2 V(s_{h+2}) - \gamma V(s_{h+1})$$

$$= \gamma (BE_{h+1})$$

Generalized Advantage Estimation

Expressing $\hat{A}^{gae}$ using Bellman errors (Bellman residuals): $BE_\tau = r_\tau + \gamma V(s_{\tau+1}) - V(s_\tau)$

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$\hat{A}^{(3)}(s_h, a_h)$

Generalized Advantage Estimation

Expressing $\hat{A}^{gae}$ using Bellman errors (Bellman residuals): $BE_\tau = r_\tau + \gamma V(s_{\tau+1}) - V(s_\tau)$

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Generalized Advantage Estimation

Expressing $\hat{A}^{gae}$ using Bellman errors (Bellman residuals): $BE_\tau = r_\tau + \gamma V(s_{\tau+1}) - V(s_\tau)$

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$$\hat{A}^{(3)}(s_h, a_h) = r_h + \gamma r_{h+1} + \gamma^2 r_{h+2} + \gamma^3 V(s_{h+2}) - V(s_h) = BE_h + \gamma BE_{h+1} + \gamma^2 BE_{h+2}$$

$$\hat{A}^{(k)}(s_h, a_h) = BE_h + \gamma BE_{h+1} + \gamma^2 BE_{h+2} + \ldots \gamma^{k-1} BE_{h+k-1}$$

$$\stackrel{k}{=} (1-\lambda)\left(\hat{A}^{(1)} + \lambda \cdot \hat{A}^{(2)} + \lambda^2 \hat{A}^{(3)} + \cdots\right)$$

Generalized Advantage Estimation

Expressing $\hat{A}^{gae}$ using Bellman errors (Bellman residuals): $BE_\tau = r_\tau + \gamma V(s_{\tau+1}) - V(s_\tau)$

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$$\hat{A}^{(k)}(s_h, a_h) = BE_h + \gamma BE_{h+1} + \gamma^2 BE_{h+2} + \ldots \gamma^{k-1} BE_{h+k-1}$$

$$\underbrace{(1-\lambda)(A^{(1)} + \lambda A^{(2)} + \lambda^2 A^{(3)} + \ldots)}_{\text{GAE}} = \sum_{l=0}^{\infty} (\gamma\lambda)^l BE_{h+l}$$

$(\gamma \cdot \lambda)$ new Discount factor

Generalized Advantage Estimation

Expressing $\hat{A}^{gae}$ using Bellman errors (Bellman residuals): $BE_\tau = r_\tau + \gamma V(s_{\tau+1}) - V(s_\tau)$

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Generalized Advantage Estimation

Expressing $\hat{A}^{gae}$ using Bellman errors (Bellman residuals): $BE_\tau = r_\tau + \gamma V(s_{\tau+1}) - V(s_\tau)$

$$(1-\lambda)(A^{(1)} + \lambda A^{(2)} + \lambda^2 A^{(3)} + \ldots) = \sum_{l=0}^{\infty} (\gamma\lambda)^l BE_{h+l}$$

When $\lambda = 0$, the GAE estimate becomes

$$A^{(1)} = r_h + \gamma V(s_{h+1}) - V(s_h)$$

When $\lambda = 1$, the GAE estimate becomes

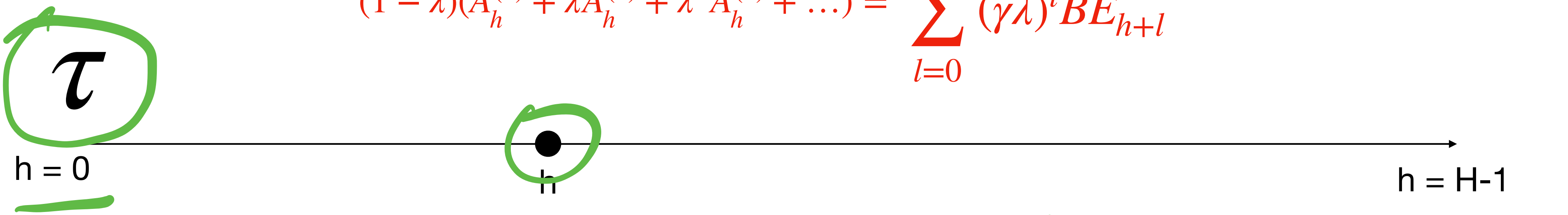
$$BE_h + \gamma \cdot BE_{h+1} + \gamma^2 \cdot BE_{h+2} + \gamma^3 \cdot BE_{h+3} + \cdots = \hat{A}^{\infty}(s_h, a_h)$$

$$= r_h + \cancel{\gamma V_{h+1}} - V_h + \gamma\left[r_{h+1} + \cancel{\gamma \cdot V_{h+2}} - \cancel{V_{h+1}}\right] + \gamma^2\left[r_{h+2} + \cancel{\gamma V_{h+3}} - \cancel{V_{h+2}}\right]$$

Generalized Advantage Estimation

In summary, given a trajectory $\tau \sim \pi$ of length H, and $V \approx V^{\pi}$, GAE copmutes $\hat{A}^{gae}(s_h, a_h)$ for all s_h, a_h on τ

$$(1-\lambda)(A_h^{(1)} + \lambda A_h^{(2)} + \lambda^2 A_h^{(3)} + \ldots) = \sum_{l=0}^{H-1} (\gamma\lambda)^l BE_{h+l}$$



$$BE_h = r_h + \gamma V_{h+1} - V_h$$

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τ

h = 0   h   h = H-1

BE_0 BE_1 $\ldots$ BE_h BE_{h+1} BE_{h+2} $\ldots$ BE_{H-1}

Generalized Advantage Estimation

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BE_0 BE_1 $\ldots$ BE_h BE_{h+1} BE_{h+2} $\ldots$ BE_{H-1}

$$\forall h : A_h^{gae} = BE_h + (\gamma\lambda)BE_{h+1} + (\gamma\lambda)^2 BE_{h+2} \ldots \ldots (\gamma\lambda)^{H-h-1} BE_{H-1}$$

Generalized Advantage Estimation

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$$(1-\lambda)(A_h^{(1)} + \lambda A_h^{(2)} + \lambda^2 A_h^{(3)} + \ldots) = \sum_{l=0}^{H-1} (\gamma\lambda)^l BE_{h+l}$$

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h = 0 h h = H-1

BE_0 BE_1 $\ldots$ BE_h BE_{h+1} BE_{h+2} $\ldots$ BE_{H-1}

$$\forall h: A_h^{gae} = BE_h + (\gamma\lambda)BE_{h+1} + (\gamma\lambda)^2 BE_{h+2} \ldots \ldots (\gamma\lambda)^{H-h-1} BE_{H-1}$$

Q: can you think about how to compute A_h^{gae} recursively using A_{h+1}^{gae} in a backward fashion?

Put everything together: PPO w/ GAE:

Initialize π_{θ_0} for the policy and V_{ω_0} for value function

For $t = 0 \rightarrow T$:

Run π_{θ_t} to collect multiple trajectories $\tau^1, \ldots, \tau^n$

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$E[y \mid s] = V^{\pi_{\theta_t}}(s)$

Form the regression dataset $\mathcal{D} = \{s, y\}$ using the trajectories

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Form A^{gae}: for each (s, a) in each trajectory, compute $A^{gae}(s, a)$ using V_{ω_t}

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Construct policy loss: $\ell_\pi(\theta) = -\sum_{s,a} \min \left\{ \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{gae}(s, a), \quad \text{clip}\left(\frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}, 1 - \epsilon, 1 + \epsilon \right) \cdot A^{gae}(s, a) \right\}$

Put everything together: PPO w/ GAE:

Initialize π_{θ_0} for the policy and V_{ω_0} for value function

For $t = 0 \rightarrow T$:

Run π_{θ_t} to collect multiple trajectories $\tau^1, \ldots, \tau^n$

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Construct V loss: $\ell_V(\omega) = \sum_{s,y} (V_\omega(s) - y)^2$ ← Estimating $\underline{\underline{V^\pi}}$

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Update π and V via a few gradient updates on the combined loss $\ell_\pi(\theta) + \ell_V(\omega)$

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Need to get your hands dirty and try it out in practice!