Maximum Entropy IRL (continue)

Recap on the setting for inverse RL

Finite horizon MDP $\mathscr{M} = \{S, A, H, c, P, \mu, \pi^\star\}$

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Key Assumption on cost:

$c(s,a) = \langle \theta^\star, \phi(s,a) \rangle$, **linear w.r.t feature** $\phi(s,a)$

Plan for Today:

1. MaxEnt IRL alg

2. Case study of AlphaGo

Maximum Entropy Inverse RL formulation

Matching expert's feature with an entropy regularization

$$\arg\min_{\pi} \left\| \mathbb{E}_{s,a\sim d^{\pi}}\phi(s,a) - \mathbb{E}_{s,a\sim d^{\pi^\star}}\phi(s,a) \right\|_2 - \lambda \mathbb{E}_{s\sim d^{\pi}} \text{entropy}(\pi(\cdot \mid s))$$

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≈ 0

π's expected feature — $\pi^\star$'s expected feature — Encourage diversity in π

Q: why matching experts feature is enough? (Reward the linear reward assumption..)

$$\mathbb{E}_{s,a\sim d^{\pi}} \phi(s,a) \approx \mathbb{E}_{s,a\sim d^{\pi^*}} \phi(s,a) \qquad \underline{\theta^{*T}\phi(s,a)}$$

$$\left| \mathbb{E}_{s,a\sim d^{\pi}} \theta^{*T}\phi(s,a) - \mathbb{E}_{s,a\sim d^{\pi^*}} \theta^{*T}\phi(s,a) \right| \le \|\theta^*\|_2 \left\| \mathbb{E}_{d^{\pi}} \phi(s,a) - \mathbb{E}_{\pi^*} \phi(s,a) \right\|$$

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π's expected feature | $\pi^\star$'s expected feature | Encourage diversity in π

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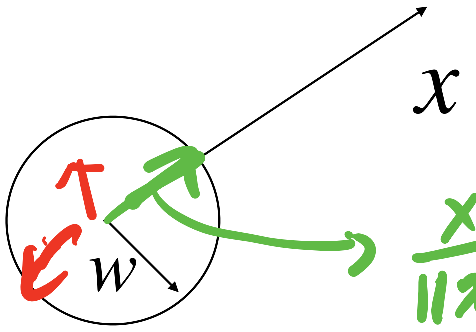
This isn't an RL problem (e.g., not maximizing some reward), seems hard to optimize π…

Maximum Entropy Inverse RL formulation

Re-write the ℓ_2 norm as an optimization problem..

$$\|x\|_2 = \max_{w:\|w\|_2 \leq 1} w^\top x$$



$$\left(\frac{x}{\|x\|_2}\right)^\top \cdot x = \frac{\|x\|_2^2}{\|x\|_2}$$

$$\frac{x}{\|x\|_2} \qquad = \|x\|_2$$

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Using this new form for $\ell_2$norm

$$\arg\min_{\pi} \left(\max_{w:\|w\|_2\le 1} \mathbb{E}_{s,a\sim d^{\pi}} w^\top \phi(s,a) - \mathbb{E}_{s,a\sim d^{\pi^\star}} w^\top \phi(s,a) \right) - \lambda \mathbb{E}_{s\sim d^{\pi}} \text{entropy}(\pi(\cdot \mid s))$$

$$= \| \mathbb{E}_{\pi} \phi - \mathbb{E}_{\pi^*} \phi \|_2$$

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2. Given w, optimize π is like an RL with cost $w^\top \phi$ and entropy reg…

Maximum Entropy Inverse RL Algorithm framework

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For $t = 0 \to T-1$ Given w^t

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Return w^T, π^T

(# Learned cost function $\phi^\top(w^T)$, and its optimal policy) $\approx \theta^{\star\top} \phi(s,a)$

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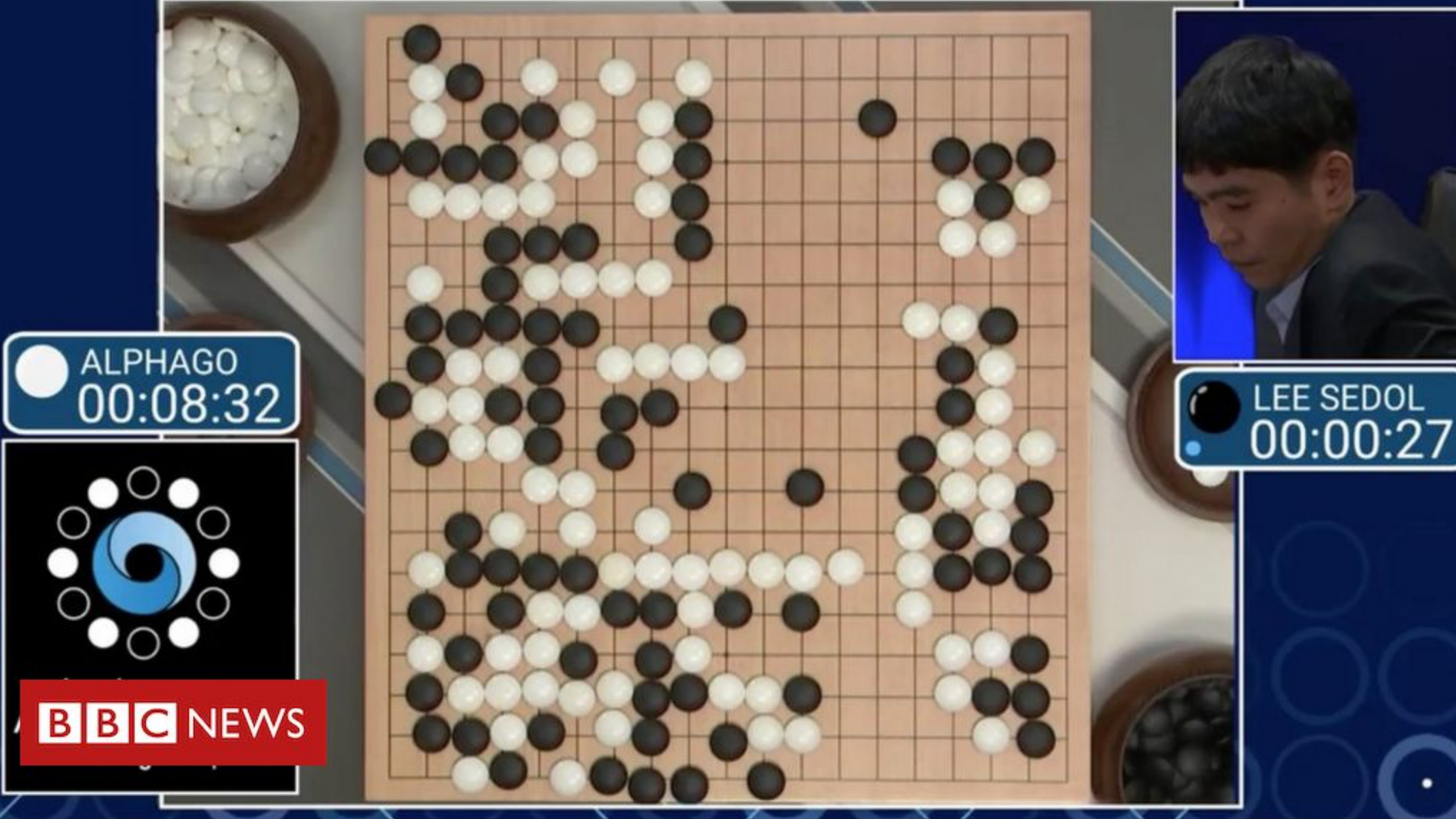
ALPHAGO
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LEE SEDOL
00:00:27
BBC NEWS

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Thus, we cannot enumerate, we must **generalize via function approximation..**

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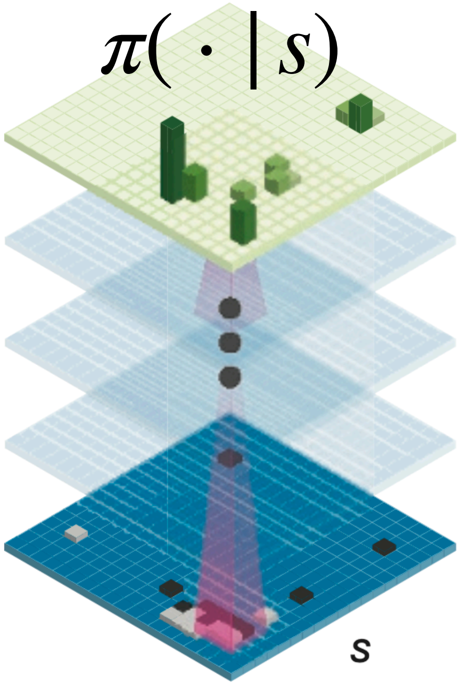

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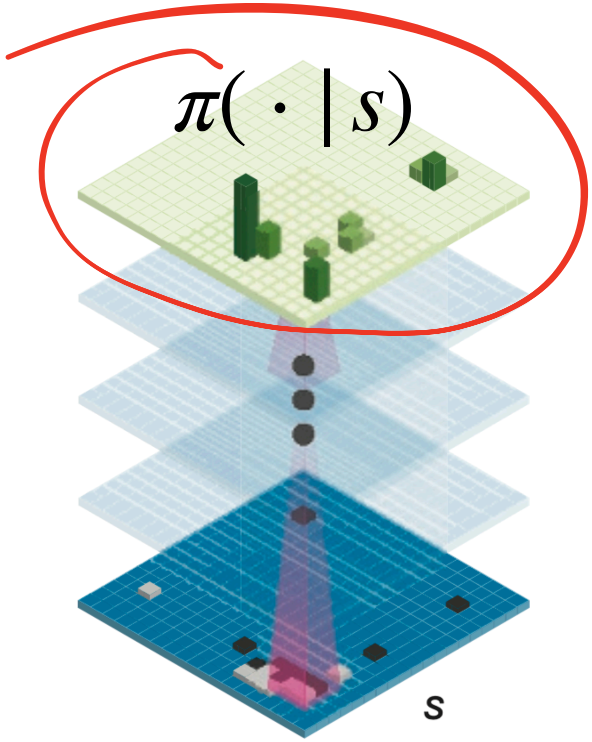
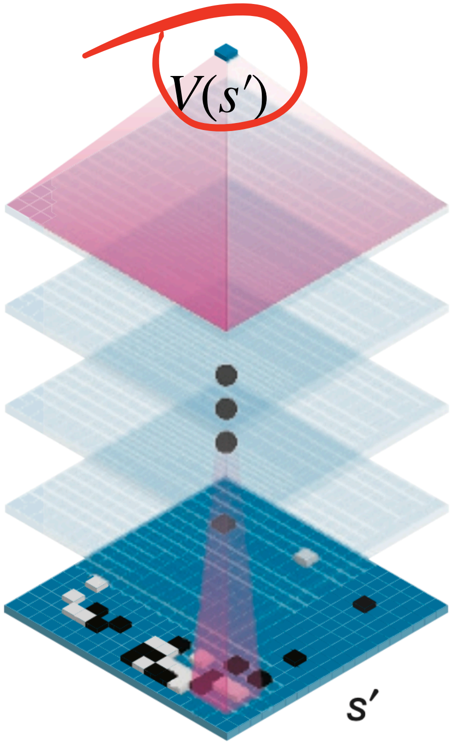


2. Value Network $\approx V^\star(s')$



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Behavior Cloning!

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But to beat human champions on Go, this is clearly not enough yet…

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Denote the PG policy as $\hat{\pi}$, we will approximate $V^{\hat{\pi}}$ instead:

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Where s is a **random state in one game play**, and z is the outcome of the play..

(We only keep one sample per game play, i.e., we are really sampling $s \sim d^{\hat{\pi}}$ i.i.d)

Final stage of training: Learn a value function $V(s) \approx V^\star$

+1, -1

Self-play 30m games, and get 30m (s, z) pairs

0

Final stage of training: Learn a value function $V(s) \approx V^\star$

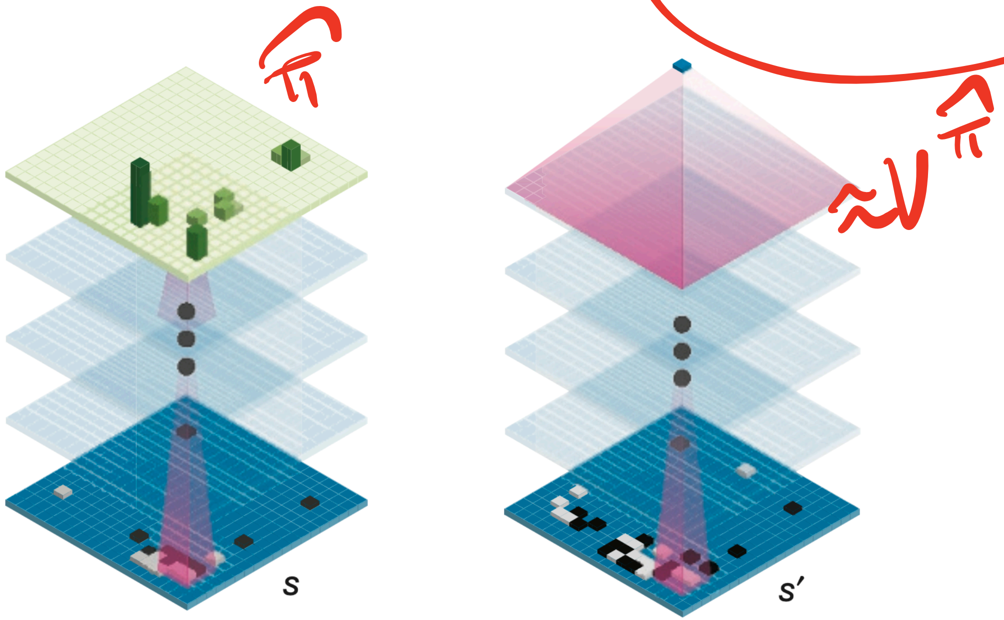
Self-play 30m games, and get 30m (s, z) pairs

Optimize least square via SGD again:

$$\beta_{t+1} = \beta_t - \eta \sum_{(s,z)\in B} (V_\beta(s) - z)\, \nabla_\beta V_\beta(s)$$

Summary so far

We have learned a policy $\hat{\pi}$ (BC+PG) and $\hat{V} \approx V^{\hat{\pi}}$



To make the program even more powerful, we combine them with a **Search Tree**

Combine with Tree Search (a naive version)

Imagine that we are at state s right now, let's simulate all possible moves into the future

AlphaGo uses Monte-Carlo Tree Search (MCTS)

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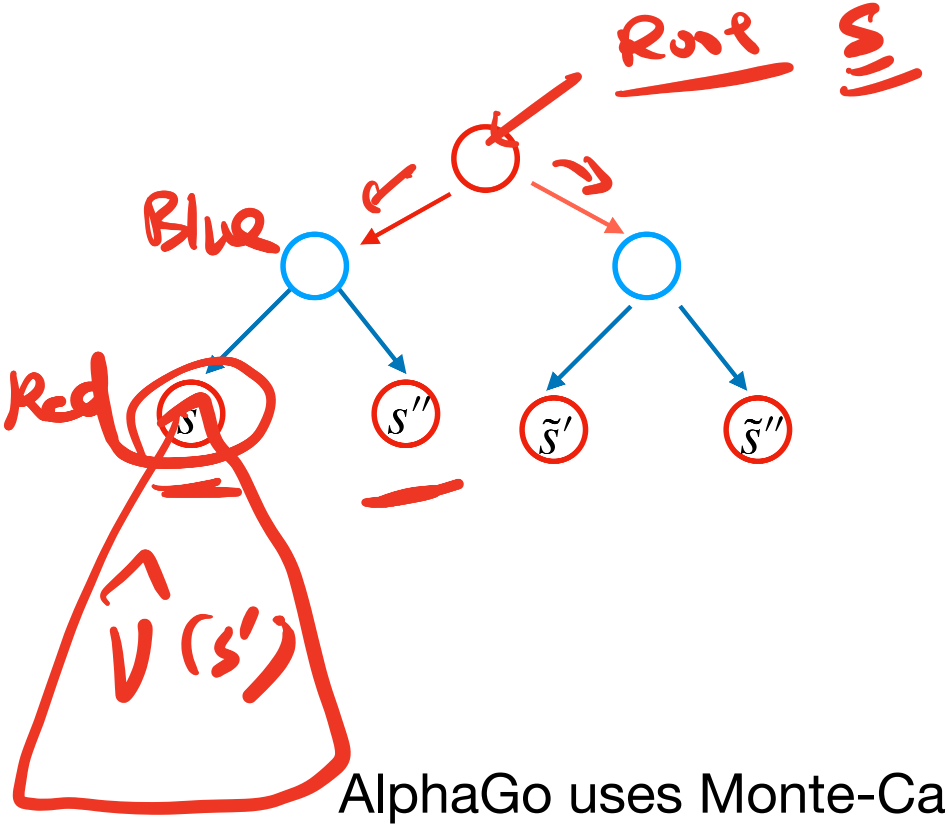

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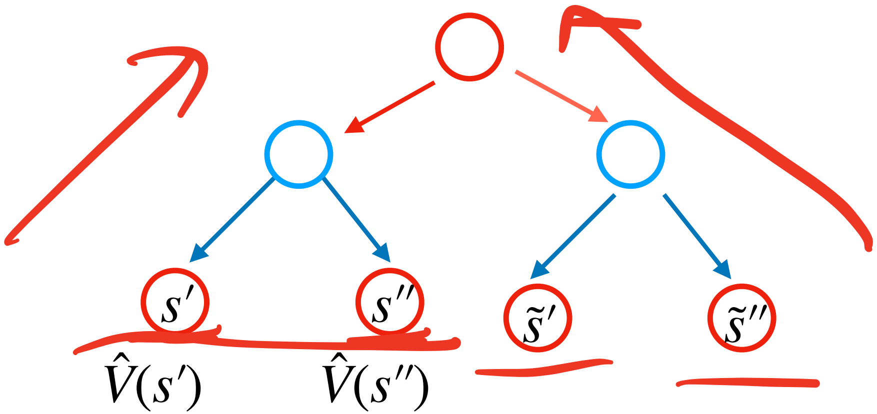

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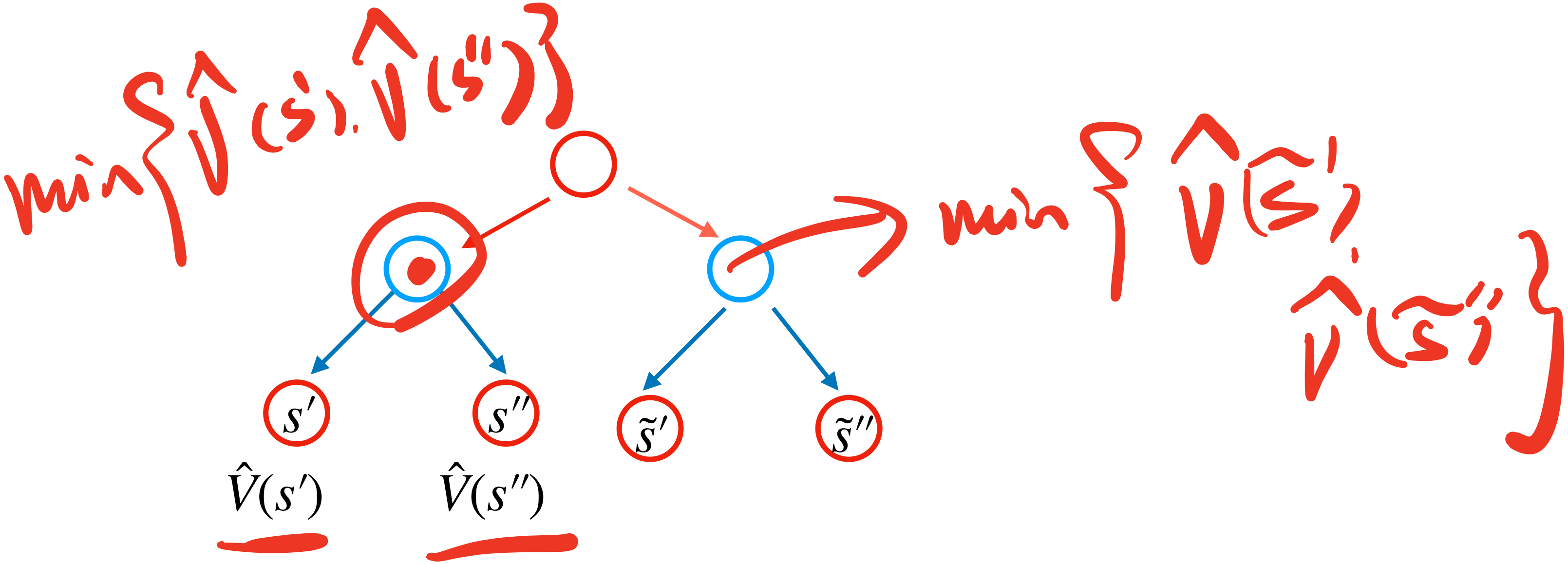

$\hat{V}(s')$: win rate of red player starting at s'

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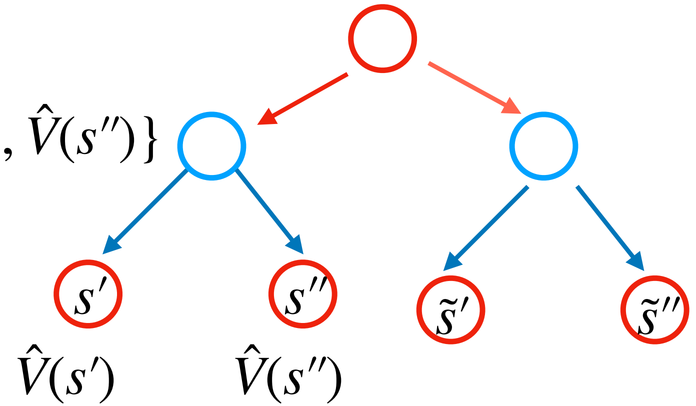
$v_L = \min\{\hat{V}(s'), \hat{V}(s'')\}$



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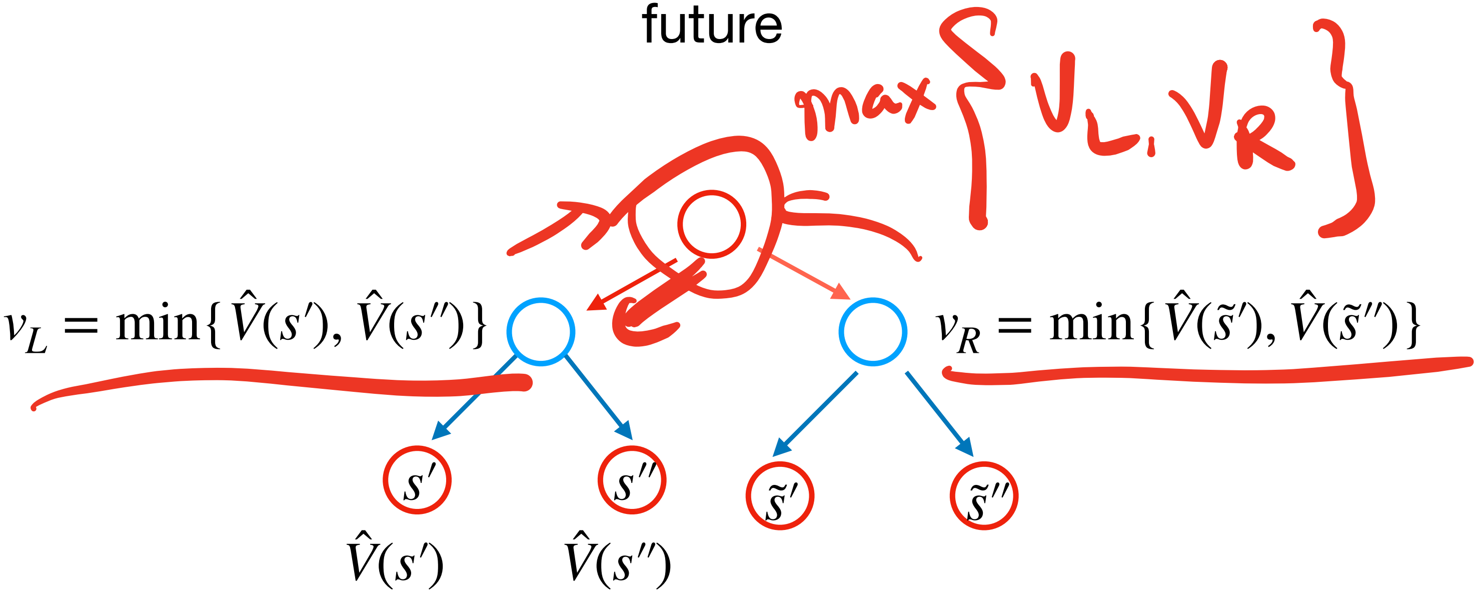

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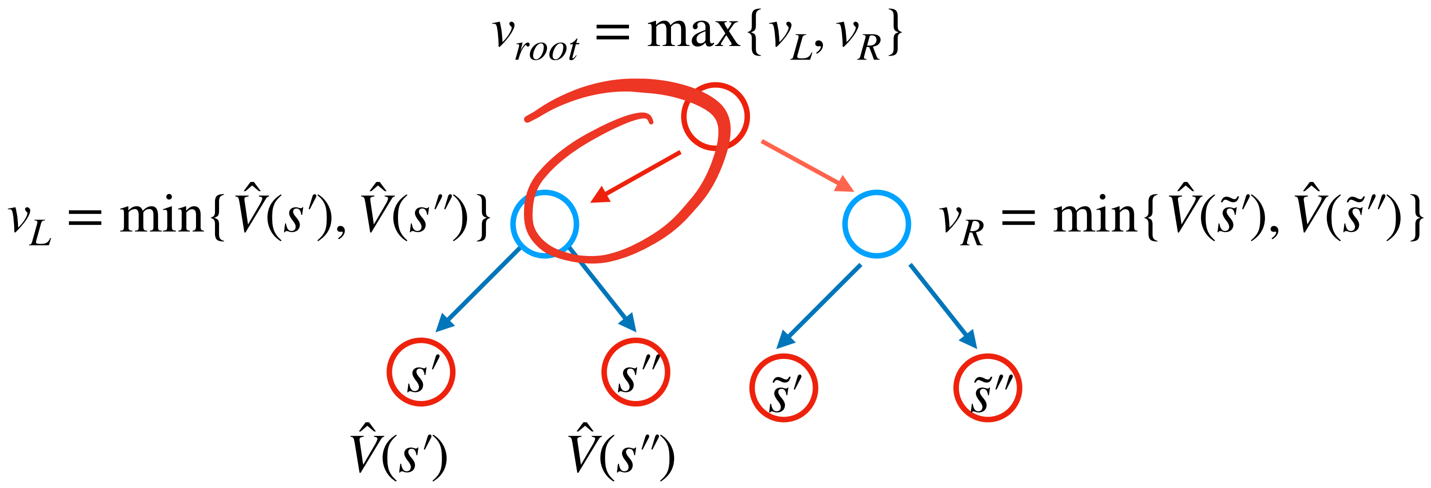

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Summary of the AlphaGo Program

1. Behavior cloning on 30m expert data samples

2. Classic Policy gradient on self-play games

3. Train a value network $\hat{V}$ to predict PG policy's outcome

4. Build search tree and use $\hat{V}$ to significantly reduce the search tree depth

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Comment: might need step 4 in generative models if we really want them to dicover new things (dicover new drugs, prove open math problems, etc)