Temporal Difference Learning

Recap: Bellman equation (consistency)

Bellman Eq

$$V^\pi(s) = \mathbb{E}_{a\sim\pi(\cdot|s)}\left[r(s,a) + \gamma\mathbb{E}_{s'\sim P(\cdot|s,a)}V^\pi(s')\right], \forall s$$

An iterative approach for estimating V^π

$$V^{t+1} \Leftarrow R + \gamma P V^t$$

$V^\pi =$

$\gamma^t =$

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1. Need to know the transition

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An iterative approach for estimating V^{π}

$$V^{t+1} \Leftarrow \underline{R + \gamma P V^{t}}$$

1. Need to know the transition
2. Only works for discrete small MDPs

Today

Given MDP $\mathscr{M} = (S, A, r, P, \gamma)$ & a $\pi : S \mapsto \Delta(A)$,

how to estimate $V^\pi(s), \forall s$ WITHOUT knowing P
(i.e., how to learn V^π from experience)

Motivation

our opponent's strategy is unknown

hard to model the transitions of the other drivers/ pedestrians/cyclists)

Hard to model users' reactions to recommendations

Outline:

1. Simple Monte Carlo methods
2. Temporal Difference Learning

MC estimation

Setup: we have MDP $\mathcal{M} = (S, A, P, \gamma, r)$, and ***stochastic*** π, i.e., $a \sim \pi(\cdot \mid s)$

$$V^\pi(s) = \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r_h \middle| \pi, s_0 = s\right]$$

MC estimation

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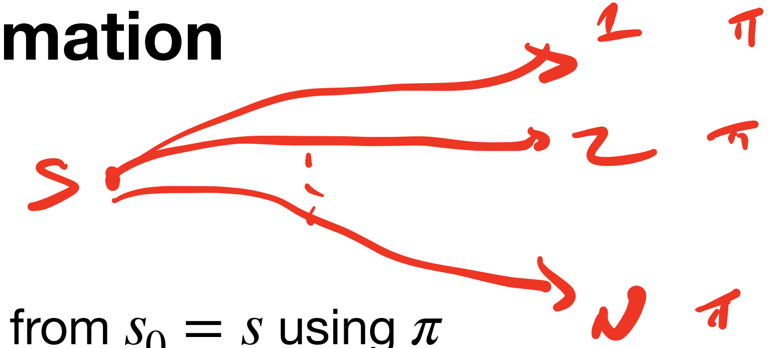
$$V^\pi(s) = \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r_h \middle| \pi, s_0 = s\right]$$

MC methods replace Expectation by Sample Average

MC estimation

For all state $s \in \mathcal{S}$:



Generate N i.i.d trajectories from $s_0 = s$ using π

MC estimation

For all state $s \in \mathcal{S}$:

Generate N i.i.d trajectories from $s_0 = s$ using π

$$\hat{V}^{\pi}(s) = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{h=0}^{T}\gamma^h r_{i,h}\right]$$

MC estimation

For all state $s \in \mathcal{S}$:

Generate N i.i.d trajectories from $s_0 = s$ using π

$$\hat{V}^\pi(s) = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{h=0}^{T}\gamma^h r_{i,h}\right]$$

Reward at time h at the i-th trajectory

MC estimation

For all state $s \in \mathcal{S}$:

Generate N i.i.d trajectories from $s_0 = s$ using π

$$\hat{V}^\pi(s) = \frac{1}{N}\sum_{i=1}^{N}\underbrace{\left[\sum_{h=0}^{T}\gamma^h r_{i,h}\right]}_{\text{Total discounted reward of the i-th trajectory}}$$

Reward at time h at the i-th trajectory

Total discounted reward of the i-th trajectory

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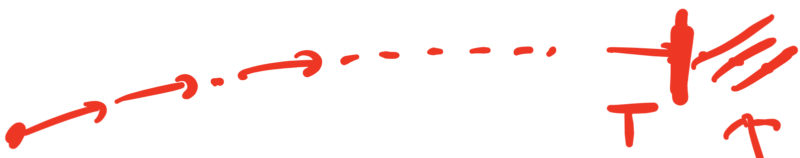
Reward at time h at the i-th trajectory

Averaging

Total discounted reward of the i-th trajectory

MC estimation



For all state $s \in \mathcal{S}$:

T needs to be large enough (i.e., $\gamma^T \approx 0$)

Generate N i.i.d trajectories from $s_0 = s$ using π

γ^T

$$\hat{V}^\pi(s) = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{h=0}^{T}\gamma^h r_{i,h}\right]$$

Reward at time h at the i-th trajectory

Averaging

Total discounted reward of the i-th trajectory

MC estimation

$T=1000$

$$\hat{V}^\pi(s) = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{h=0}^{T}\gamma^h r_{i,h}\right]$$

When $T \to \infty$ and $N \to \infty$, we have $\hat{V}^\pi(s) \to V^\pi(s)$

MC estimation

$$\hat{V}^{\pi}(s) = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{h=0}^{T}\gamma^{h} r_{i,h}\right]$$

When $T \to \infty$ and $N \to \infty$, we have $\hat{V}^{\pi}(s) \to V^{\pi}(s)$

The future after T-step has
near zero contribution

γ^{T}

MC estimation

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When $T \to \infty$ and $N \to \infty$, we have $\hat{V}^{\pi}(s) \to V^{\pi}(s)$

The future after T-step has near zero contribution

Law of large number

MC estimation with incremental update

Instead of waiting for all N trajs, we can also update the estimator incrementally every traj

Initialize $\hat{V}^{\pi}(s) = 0, \forall s$

For all state $s \in \mathcal{S}$:

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For all state $s \in \mathcal{S}$:

Generate one traj from $s_0 = s$ using π;

$$\hat{V}^\pi(s) \Leftarrow \hat{V}^\pi(s) + \eta \left[\left(\sum_{h=0}^{\infty} \gamma^h r_h \right) - \hat{V}^\pi(s) \right]$$

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total reward of one traj

$\frac{1}{n}$

learning rate

$V^{\pi}(s)$

$\sum_{h=0}^{\infty} \gamma^h \cdot r_h$

$\sum_{h=0}^{\infty} \gamma^h \cdot r_h \neq V^{\pi}(s)$

$\hat{V}(s)$

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η: learning rate

MC estimation with incremental update

The incremental update is performing
Stochastic gradient descent (SGD)

$$\hat{V}^{\pi}(s) \Leftarrow \hat{V}^{\pi}(s) + \eta \left[\left(\sum_{h=0}^{\infty} \gamma^h r_h \right) - \hat{V}^{\pi}(s) \right]$$

$\min_x \ell(x)$

GD: $x - \eta \cdot \nabla \ell(x)$

SGD: $x - \eta \cdot \widetilde{\nabla \ell}(x)$

$E[\widetilde{\nabla \ell}(x)] = \nabla \ell(x)$

MC estimation with incremental update

The incremental update is performing
Stochastic gradient descent (SGD)

$$\hat{V}^\pi(s) \Leftarrow \hat{V}^\pi(s) + \eta \left[\left(\sum_{h=0}^{\infty} \gamma^h r_h \right) - \hat{V}^\pi(s) \right]$$

$$\underbrace{\ell(\hat{V}^\pi(s))}_{\theta} := \left(\underbrace{\hat{V}^\pi(s)}_{\theta} - V^\pi(s) \right)^2$$

Estimator

True Target

MC estimation with incremental update

The incremental update is performing
Stochastic gradient descent (SGD)

$$\hat{V}^{\pi}(s) \Leftarrow \hat{V}^{\pi}(s) + \eta \left[\left(\sum_{h=0}^{\infty} \gamma^h r_h \right) - \hat{V}^{\pi}(s) \right]$$

$$\ell(\hat{V}^{\pi}(s)) := \left(\hat{V}^{\pi}(s) - V^{\pi}(s) \right)^2$$

Target

$$\nabla \ell(x) \,|_{x=\hat{V}^{\pi}(s)} = \underbrace{2 \left(\hat{V}^{\pi}(s) - V^{\pi}(s) \right)}_{\Delta}$$

MC estimation with incremental update

The incremental update is performing
Stochastic gradient descent (SGD)

$$\hat{V}^\pi(s) \Leftarrow \hat{V}^\pi(s) + \eta \left[\left(\sum_{h=0}^{\infty} \gamma^h r_h \right) - \hat{V}^\pi(s) \right]$$

$$\tilde{\nabla} \ell(x) \big|_{x=\hat{V}^\pi(s)} = 2\left(\hat{V}^\pi(s) - \sum_{h=0}^{\infty} \gamma^h r_h\right)$$

$$\hat{V}^\pi(s) - \eta \tilde{\nabla} \ell(x) \big|_{x=\hat{V}^\pi(s)}$$

$$\ell(\hat{V}^\pi(s)) := \left(\hat{V}^\pi(s) - V^\pi(s) \right)^2$$

$$\nabla \ell(x) \big|_{x=\hat{V}^\pi(s)} = 2\left(\hat{V}^\pi(s) - V^\pi(s) \right)$$

To get a stochastic gradient, replace $V^\pi(s)$ by its unbiased estimate $\sum_{h=0}^{\infty} \gamma^h r_h$

$$E\left[\sum_{h=0}^{\infty} \gamma^h r_h\right] = V^\pi(s)$$

Summary

$$\hat{V}^{\pi}(s) = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{h=0}^{T}\gamma^{h} r_{i,h}\right]$$

MC estimation is simple, and is based on well-established statistical guarantee

Summary

$$\hat{V}^{\pi}(s) = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{h=0}^{T}\gamma^h r_{i,h}\right]$$

MC estimation is simple, and is based on well-established statistical guarantee

$N \to \infty$

However

$\sum_{h=0}^{\infty} \gamma^h \cdot r_h$

It has **high variance**, i.e., total reward on an extremely long trajectory can have a lot of randomness;

Need to **wait** for completing full trajectories to update/estimate $V^{\pi}(s)$

s

Outline:

1. Simple Monte Carlo methods

2. Temporal Difference Learning

TD Learning

History: developed by Rich Sutton back in 1988

Learning to Predict by the Methods of Temporal Differences

RICHARD S. SUTTON (RICH@GTE.COM)
GTE Laboratories Incorporated, 40 Sylvan Road, Waltham, MA 02254, U.S.A.

(Received: April 22, 1987)
(Revised: February 4, 1988)

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My first research project in grad school

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Online Bellman Residual Algorithms with Predictive Error Guarantees

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TD Learning

TD relies on the idea of **Bootstrapping** — *using predictions from our estimator to update the estimator itself*

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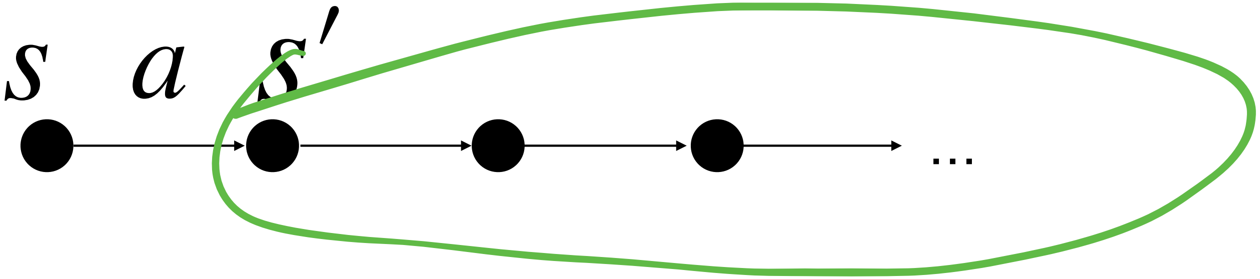

$\hat{V}^\pi \approx V^\pi$

$V^\pi(s)$

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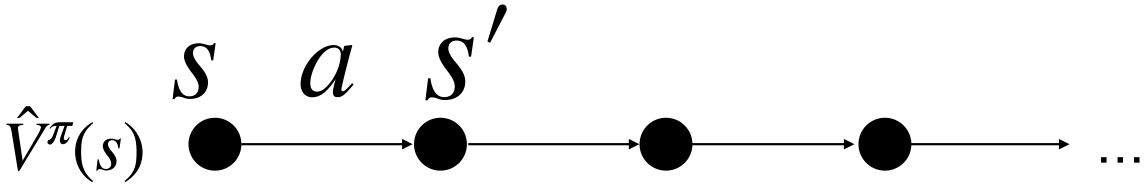

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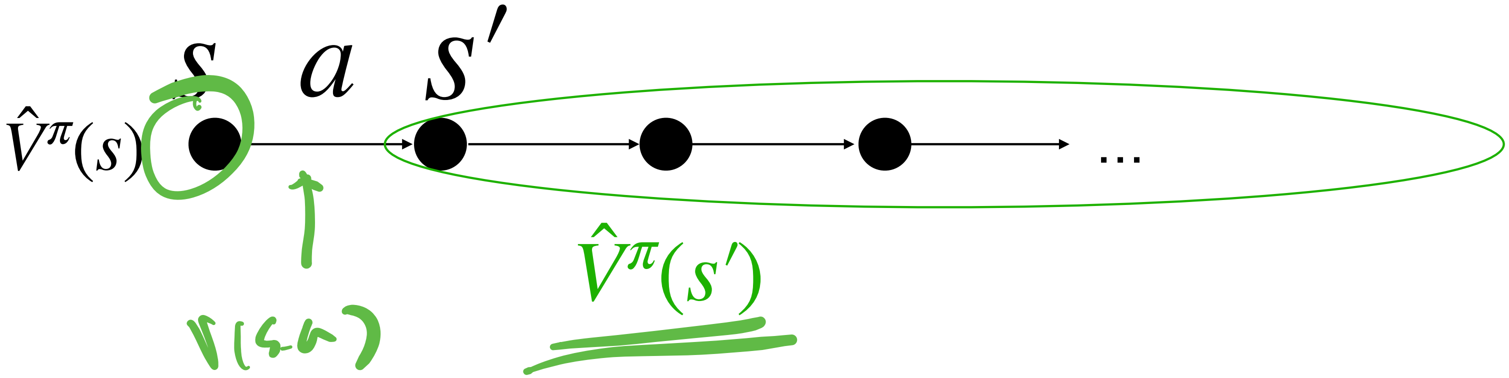

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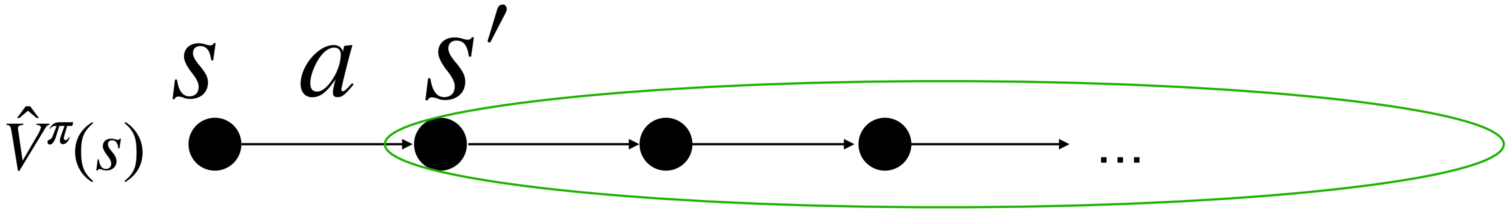

$$\underline{r(s,a)} + \gamma \, \underline{\hat{V}^\pi(s')} \approx V^\pi(s)$$

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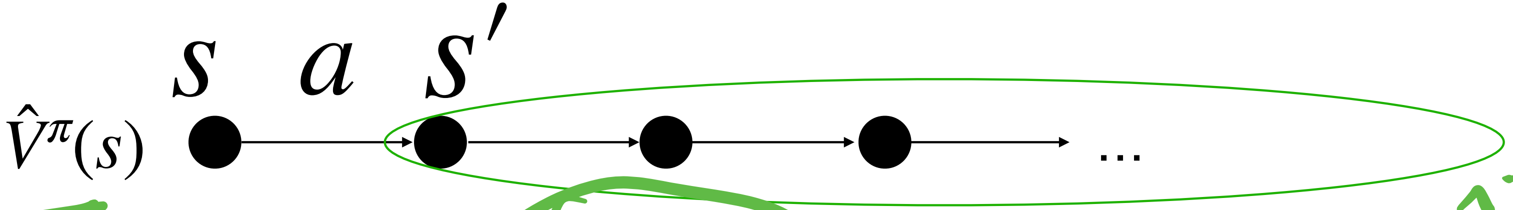

$$r(s,a) + \gamma \hat{V}^{\pi}(s')$$

New target value (bootstrapping)

TD Learning

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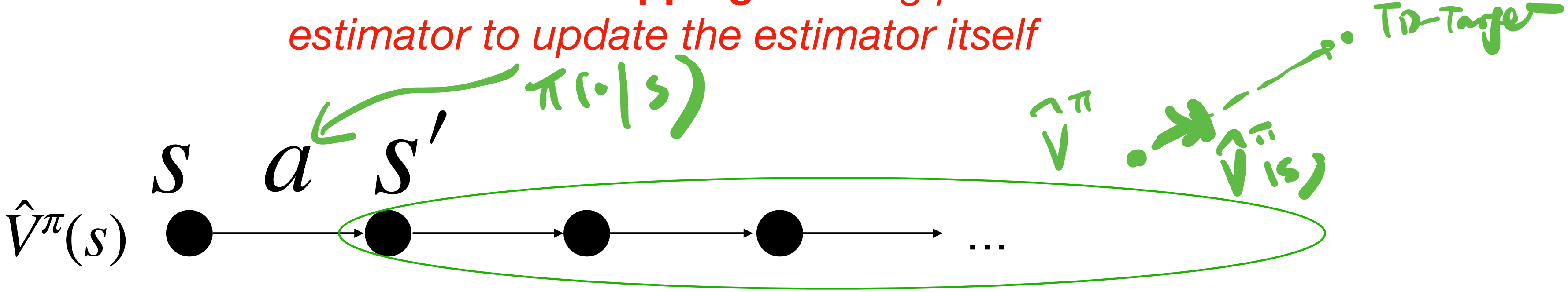

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New target value (bootstrapping)

TD-Error

TD update: $\hat{V}^{\pi}(s) \Leftarrow \hat{V}^{\pi}(s) + \eta \left(\underbrace{r(s,a) + \gamma \hat{V}^{\pi}(s')}_{\text{TD-Target}} - \hat{V}^{\pi}(s) \right)$

TD Learning

Initialize $V^\pi(s) = 0, \forall s$. Set initial state $s \in \mathcal{S}$

While True:

TD Learning

$$s \xrightarrow[r = r(s,a)]{a \sim \pi(\cdot|s)} s' \qquad s' \sim P(\cdot|s,a)$$

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Take action $a \sim \pi(\cdot \mid s)$, get reward r and next state $s' \sim P(\cdot \mid s, a)$

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Form TD target $r + \gamma \hat{V}^\pi(s')$

Bootstrapping

TD Learning

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Form TD target $r + \gamma \hat{V}^{\pi}(s')$

Update for s: $\hat{V}^{\pi}(s) \Leftarrow \hat{V}^{\pi}(s) + \eta \left(\underbrace{r + \gamma \hat{V}^{\pi}(s')}_{\text{TD-Target}} - \underbrace{\hat{V}^{\pi}(s)}_{\text{where we are}} \right)$

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Set $s \Leftarrow s'$

Remark: (1) we perform online update while interacting w/ MDP; (2) no need to wait to the end for an update (one update per step)

Interpret TD as “SGD” on TD loss

TD is not the usual SGD, i.e., it is **not** running SGD to minimize a fixed loss function

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$$\ell_{td}(\hat{V}^\pi(s)) := \underbrace{\left(\hat{V}^\pi(s) - y\right)^2}_{=0}, \text{ where } y = \mathbb{E}_{a\sim\pi(\cdot|s)}\left[r(s,a) + \gamma \mathbb{E}_{s'\sim P(\cdot|s,a)} \hat{V}^\pi(s')\right]$$

$$\hat{V}^\pi(s) = \mathbb{E}_{a\sim\pi(\cdot|s)}\left[r(s,a) + \gamma \mathbb{E}_{s'\sim P(s,a)} \hat{V}^\pi(s')\right]$$

$$= V^\pi(s)$$

Interpret TD as “SGD” on TD loss

$(\hat{V}^{\pi} - V^{\pi})^2$

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This keeps changing as we learning

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In-class exercise:

derive one-step SGD update for $\ell_{td}(\hat{V}^\pi(s))$

(Hint: how to get an unbiased estimate for y using one transition)

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$$\nabla \ell_{td}(x)\Big|_{x=\hat{V}^\pi(s)} = 2\left(\hat{V}^\pi(s) - \underset{\Delta}{y}\right)$$

$$\underline{\tilde{\nabla} \ell_{td}(x)\Big|_{x=\hat{V}^\pi(s)}} = 2\left(\hat{V}^\pi(s) - \left(r(s,a) + \gamma \hat{V}^\pi(s')\right)\right)$$

$a \sim \pi(\cdot|s)$
$s' \sim P(\cdot|s,a)$

TD: $\hat{V}^\pi(s) \leftarrow \hat{V}^\pi(s) - \eta \cdot \tilde{\nabla} \ell_{td}(x)\Big|_{x=\hat{V}^\pi(s)}$

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$$\nabla \ell_{td}(x)\,|_{x=\hat{V}^\pi(s)} := 2\left(\hat{V}^\pi(s) - y\right)$$

$$\widetilde{\nabla} \ell_{td}(x)\,|_{x=\hat{V}^\pi(s)} := 2\left(\hat{V}^\pi(s) - \left(r + \gamma \hat{V}^\pi(s')\right)\right)$$

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Unbiased estimate of y

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TD: $\hat{V}^{\pi}(s) \Leftarrow \hat{V}^{\pi}(s) - \eta\, \widetilde{\nabla}\,\ell_{td}(x)\,|_{x=\hat{V}^{\pi}(s)}$

TD Learning

[Informal] Assume π has non-trivial probability of visiting every state.

Setting learning rate η properly, we will have:

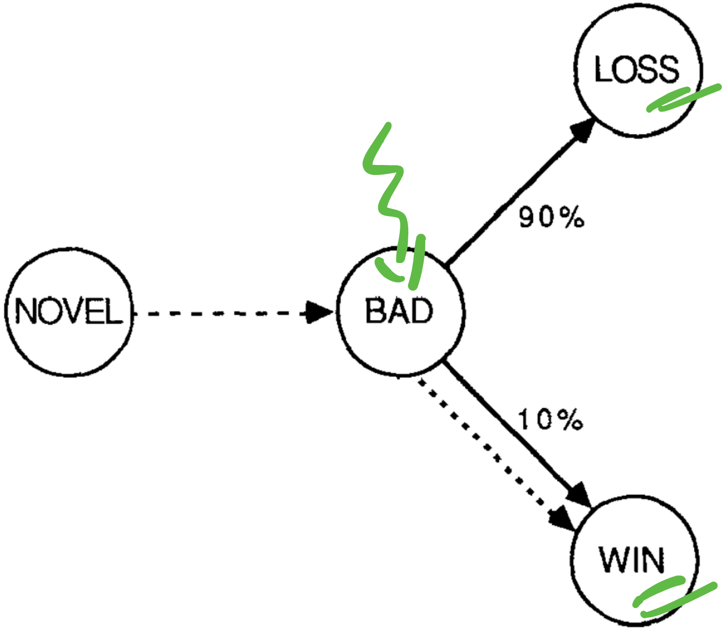
$\frac{1}{\sqrt{t}}$ $\frac{1}{t}$

$$\hat{V}^{\pi}(s) \rightarrow V^{\pi}(s), \forall s,$$

when # of interactions approaches to ∞

(concrete convergence rates are known as well)

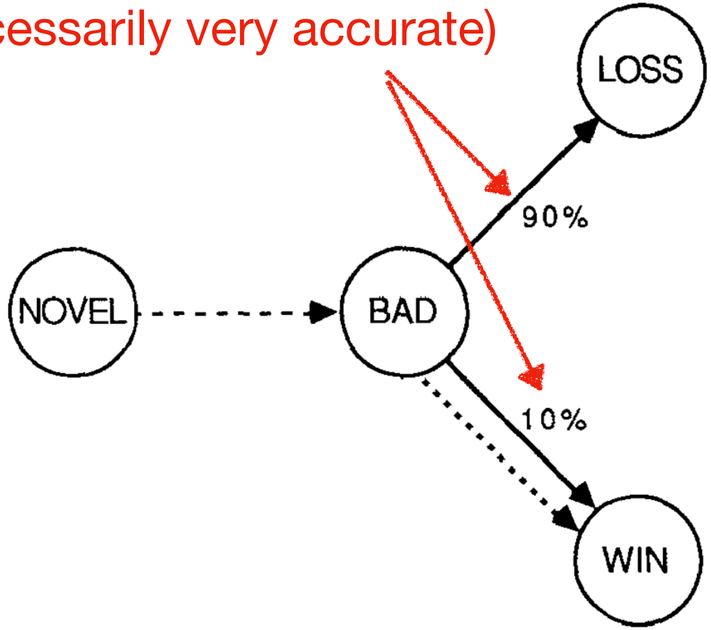
Example of faster learning with TD



[Sutton, 1988]

Example of faster learning with TD

From past game experiences
(not necessarily very accurate)



[Sutton, 1988]

Example of faster learning with TD

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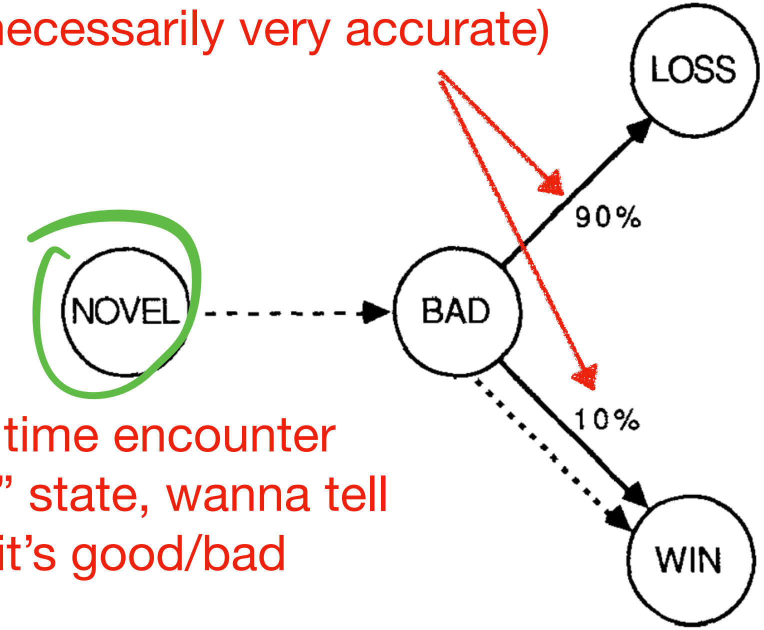

first time encounter
“novel” state, wanna tell
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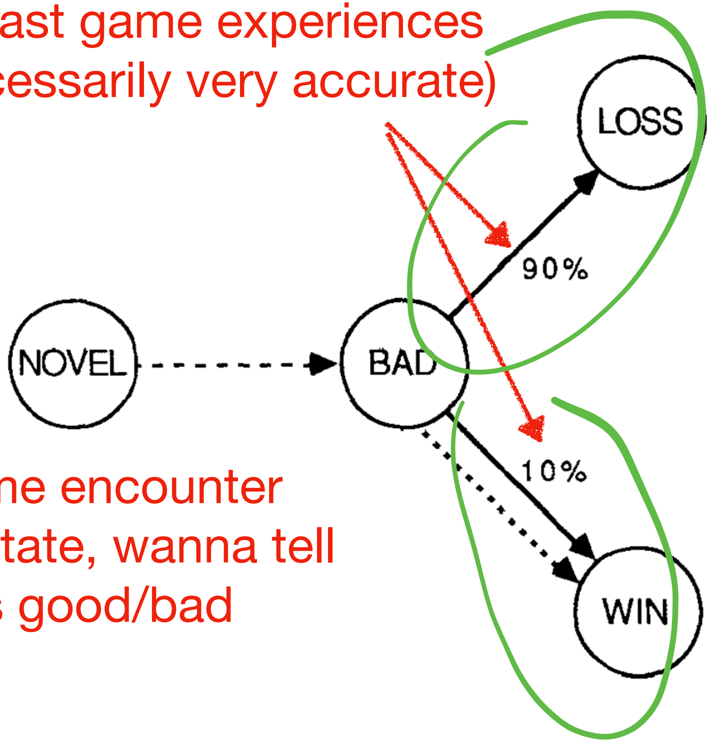

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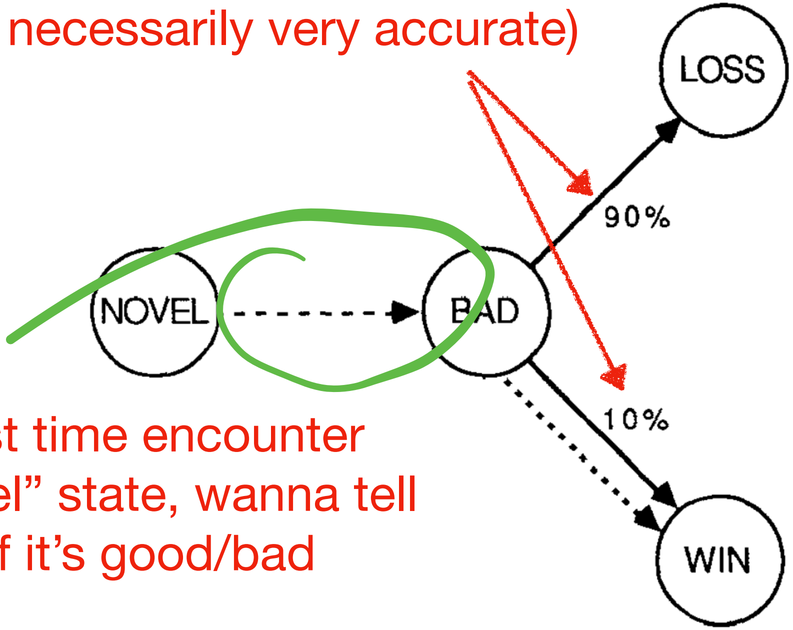

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MC: have to rollout from NOVEL many times
to get a reasonable estimate of future

TD (bootstrapping): NOVEL leading to BAD
in one-step implies NOVEL is likely bad

TD learning on CartPole

Note: Cartpole's state is continuous, so we will need TD w/ function approximation, e.g., neural network (we will get there very soon)

$\hat{V}^{\pi}(s) = NN$

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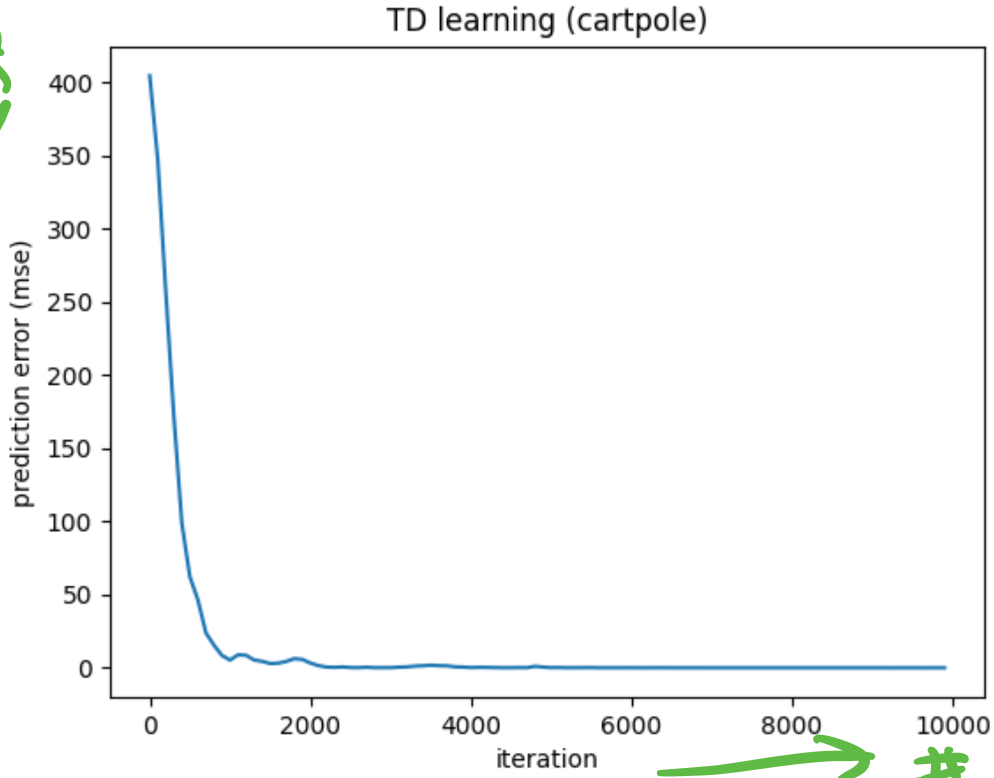
MSE



# of steps

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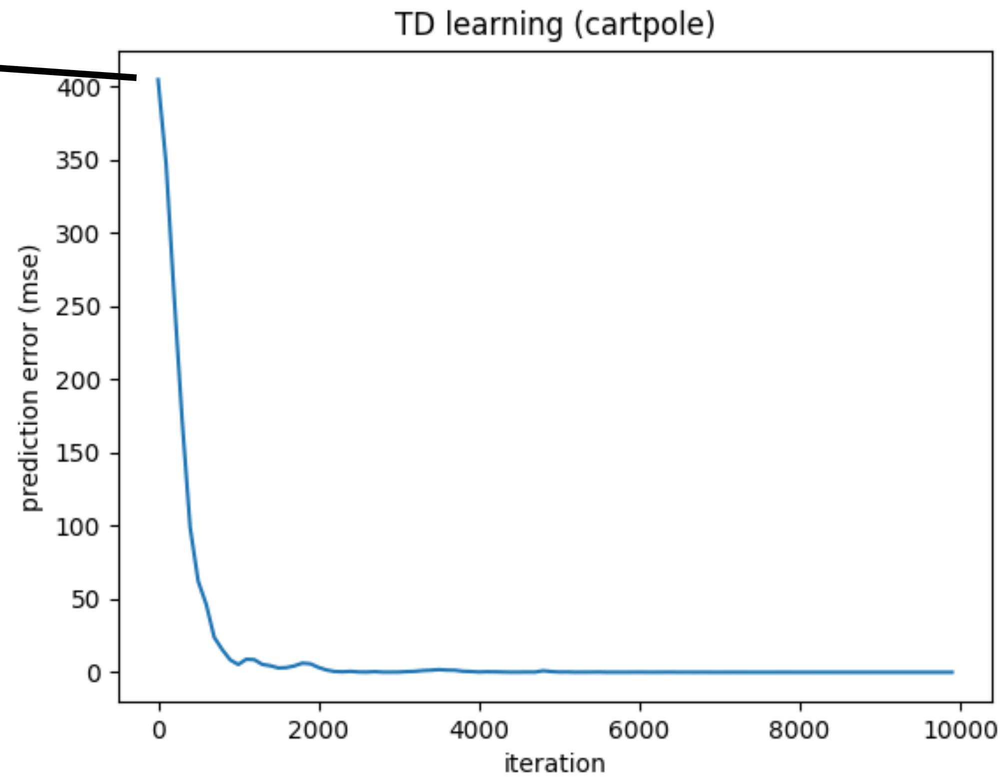
A randomly initialized NN as $\hat{V}^{\pi}(s)$ (very bad estimator)



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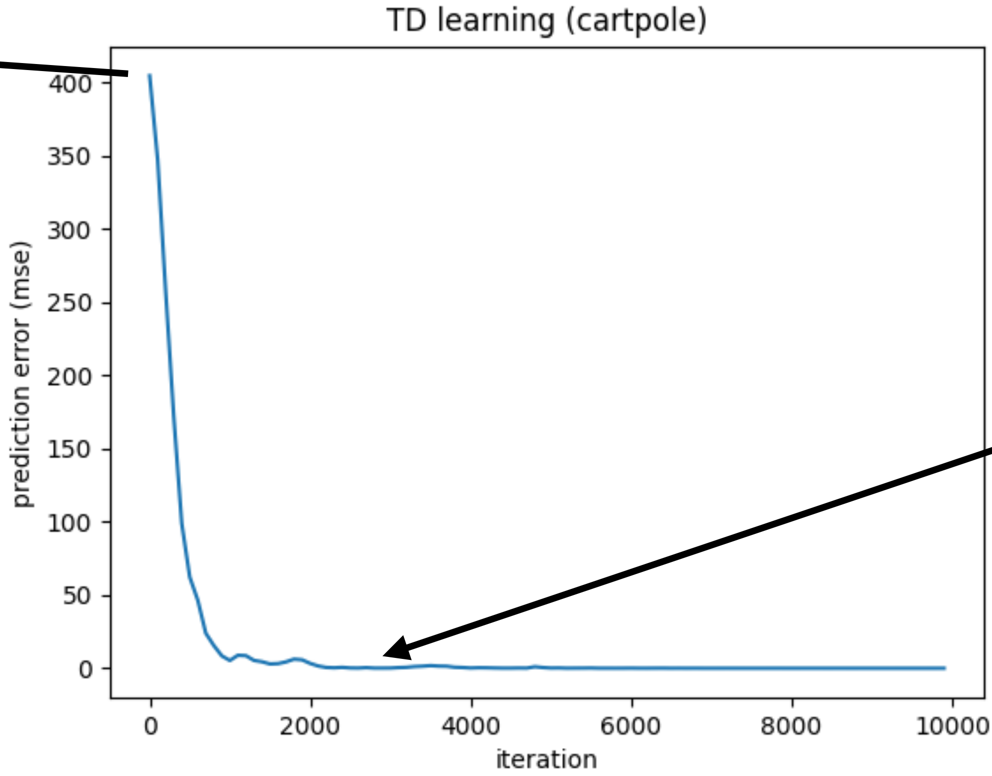

Almost perfectly estimate the true target $V^{\pi}(s)$

$T \to \infty$

TD vs MC: bias-variance tradeoff

MC is unbiased, but has higher variance (i.e., randomness over an entire traj)

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(s, a, r, s')

$$\hat{V}^{\pi}(s) \Leftarrow \hat{V}^{\pi}(s) + \eta \left(r + \gamma \hat{V}^{\pi}(s') - \hat{V}^{\pi}(s) \right)$$

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TD-Target

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Randomness in target is
from **one-step transition**

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TD can have high bias, but has lower variance

Large bias when $\hat{V}^{\pi}(s')$ is far from $V^{\pi}(s')$

$$\hat{V}^{\pi}(s) \Leftarrow \hat{V}^{\pi}(s) + \eta\left(\underline{r + \gamma \hat{V}^{\pi}(s')} - \hat{V}^{\pi}(s)\right)$$

Randomness in target is from **one-step transition**

TD(λ)

$r(s,a) + \gamma\, r(s',a') + \gamma^2 \hat{V}^{\pi}(s'')$

$s \xrightarrow{a} s' \xrightarrow{a'} s''$

Summary

TD Learning: an online algorithm for estimating V^{π} on the fly from agent's own experience

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TD target
(bootstrapping)

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