Imitation Learning: Behavior Cloning, Distribution Shift, & Distribution Matching

Sham Kakade and Wen Sun

CS 6789: Foundations of Reinforcement Learning

Announcements

HW3 is out last night and is due Nov 24th 11:59pm

(Lots of bonus questions, but try them out!)

No classes on Nov 17, 19, & 24 (university semi-final week)

Recap

Offline RL

$\mathscr{D} = \{s_i, a_i, r_i, s_i'\}_{i=1}^{n}$, where $s_i, a_i \sim \mu, r_i = r(s_i, a_i), s_i' \sim P(\cdot \mid s_i, a_i)$

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$$f_{t+1} = \arg\min_{f \in \mathcal{F}} \sum_{i=1}^{n} \left(f(s_i, a_i) - \underbrace{\left(r_i + \gamma \max_{a'} f_t(s_i', a') \right)}_{\mathcal{T} f_t(s_i, a_i)} \right)^2$$

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Performing regression from (s_i, a_i) to $\mathscr{T} f_t(s_i, a_i)$

$\sup_{\pi, s, a} \frac{d^\pi(s, a)}{\mu(s, a)} < \infty$, $\quad \forall f \in \mathscr{F}, \mathscr{T} f \in \mathscr{F}$, $\Rightarrow$ FQI learns near-optimal policy in polynomially sample complexity

Today: Imitation Learning

1. Introduction of Imitation Learning

2. Offline Imitation Learning: Behavior Cloning

3. The hybrid Setting: Statistical Benefit and Distribution Matching

An Autonomous Land Vehicle In A Neural Network *[Pomerleau, NIPS '88]*

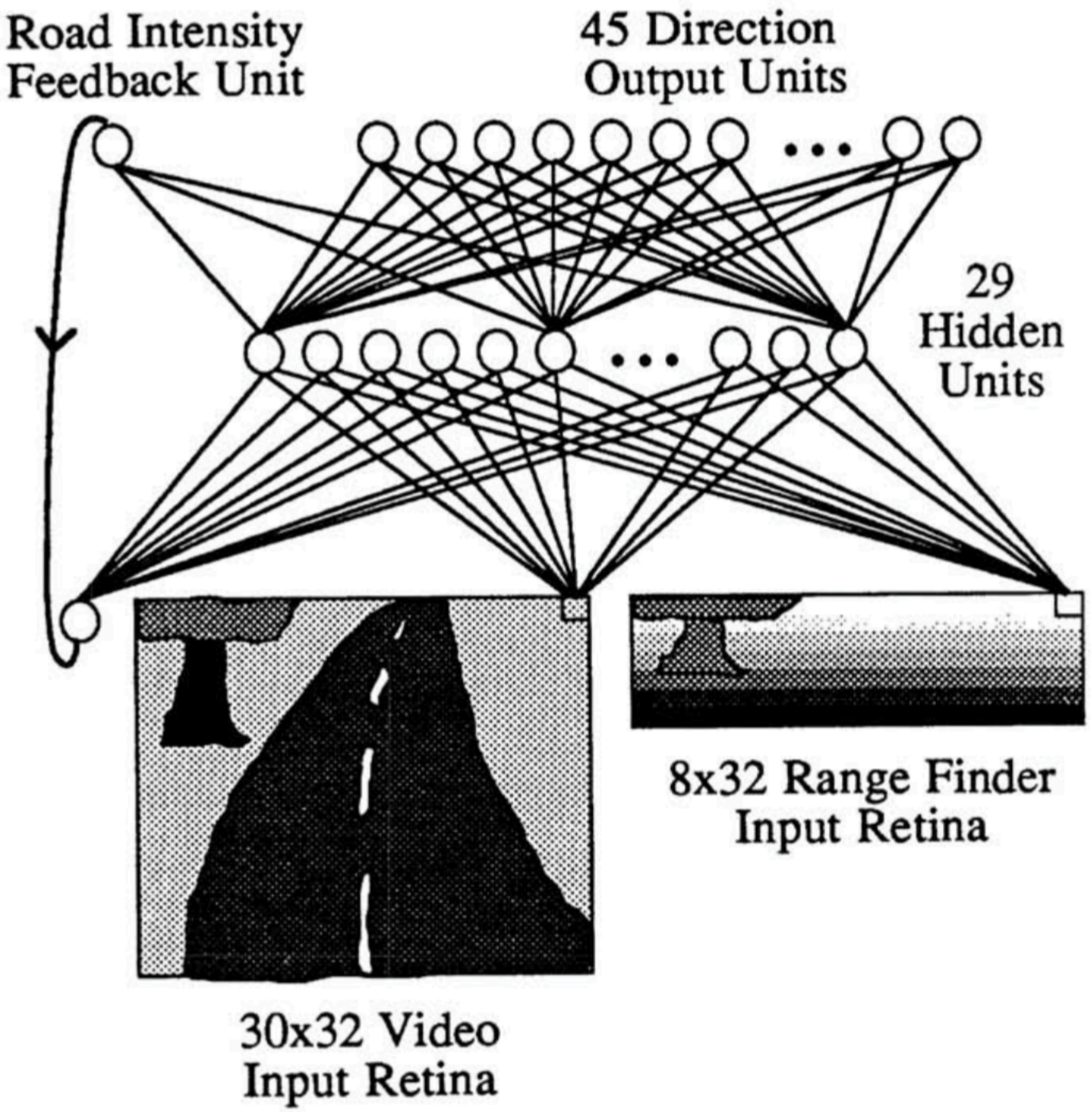
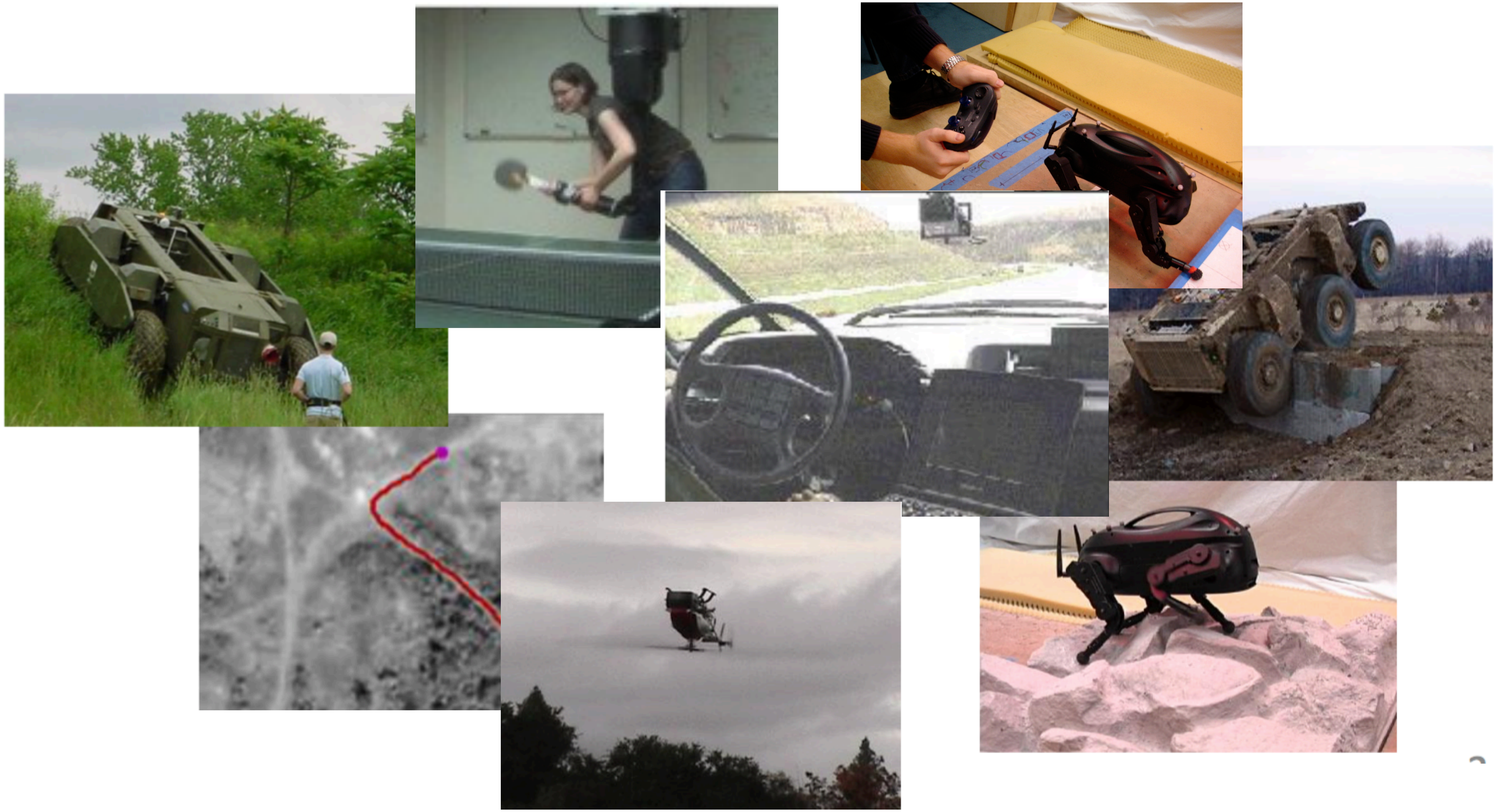


Figure 1: ALVINN Architecture

Imitation Learning



Imitation Learning

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Expert Demonstrations

Imitation Learning

Expert Demonstrations → Machine Learning Algorithm

- SVM
- Gaussian Process
- Kernel Estimator
- Deep Networks
- Random Forests
- LWR
- …

Imitation Learning

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Machine Learning Algorithm

Policy π

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Maps *states* to <u>actions</u>

Learning to Drive by Imitation

[Pomerleau89, Saxena05, Ross11a]

Input:

Camera Image

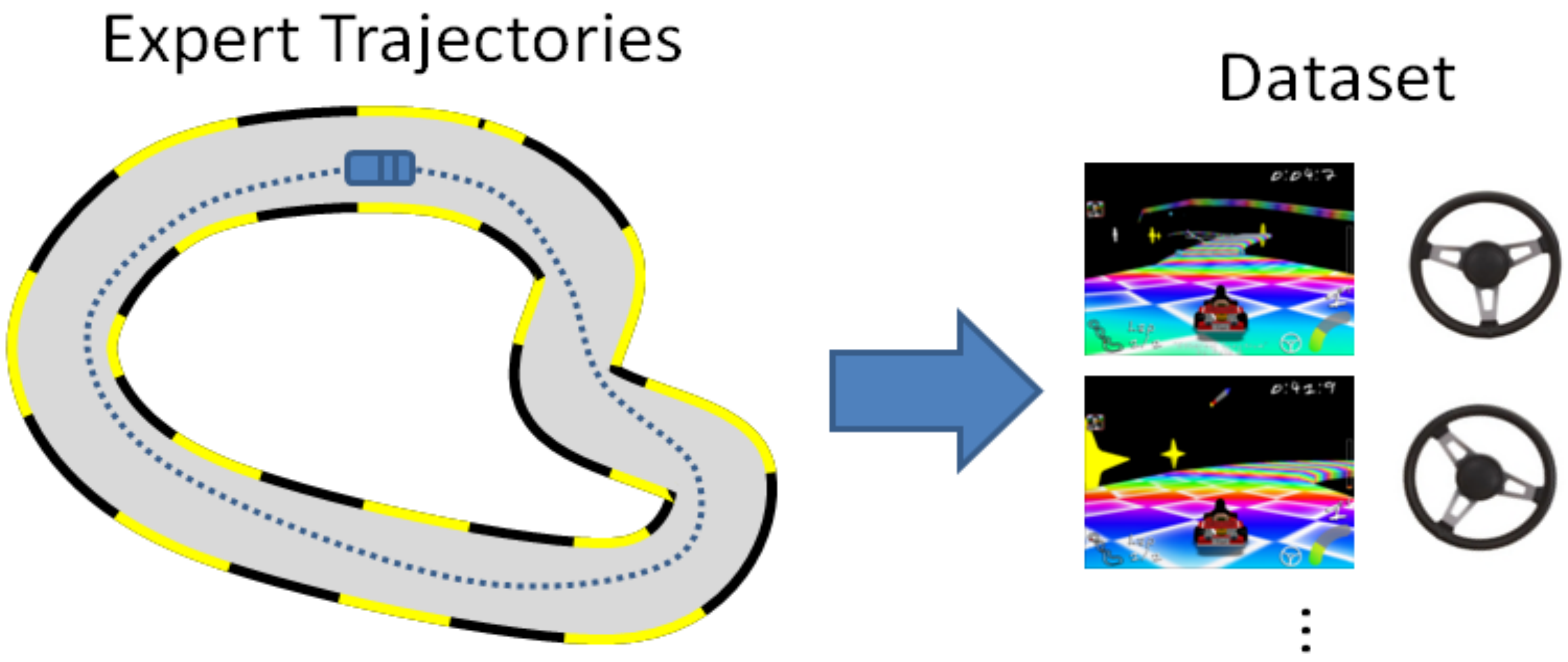
Policy →

Output:

Steering Angle
in [-1, 1]

[Widrow64,Pomerleau89]

Supervised Learning Approach: Behavior Cloning



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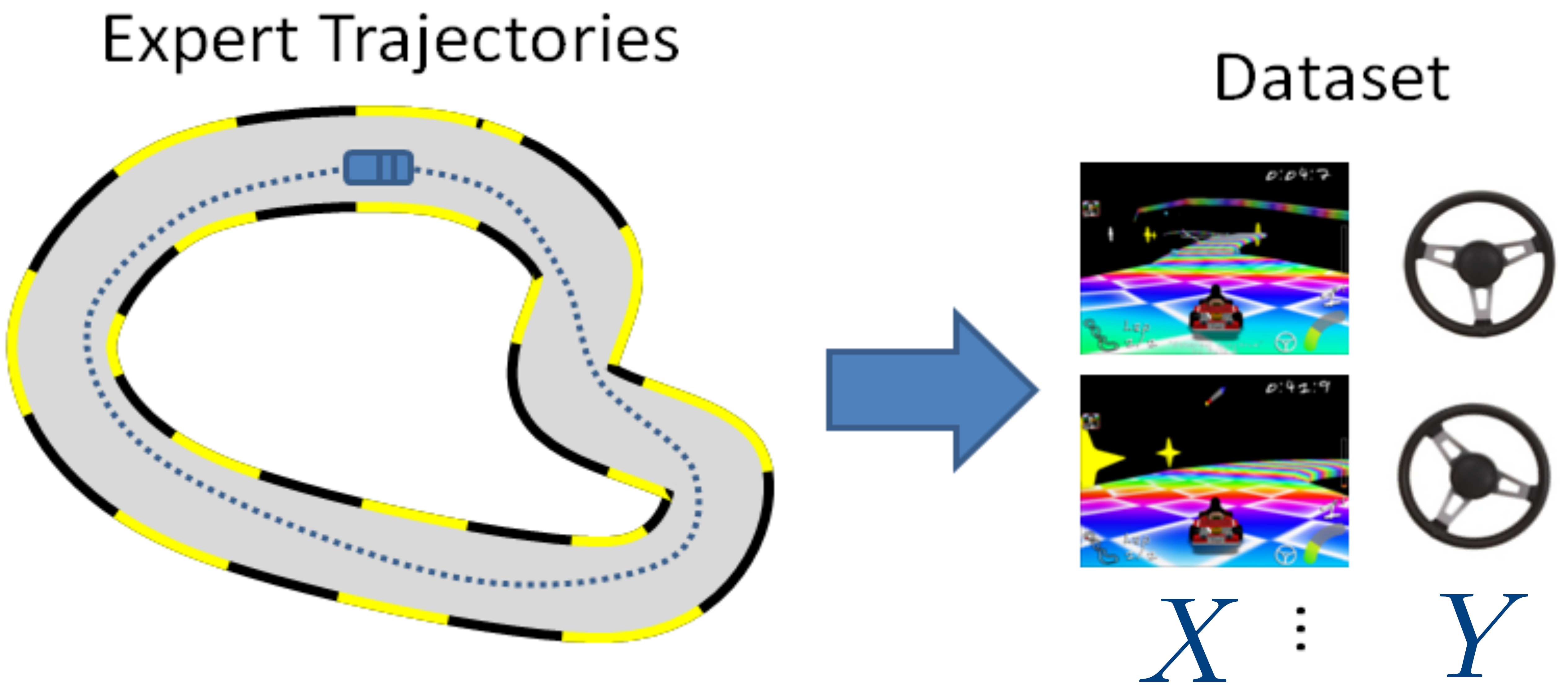

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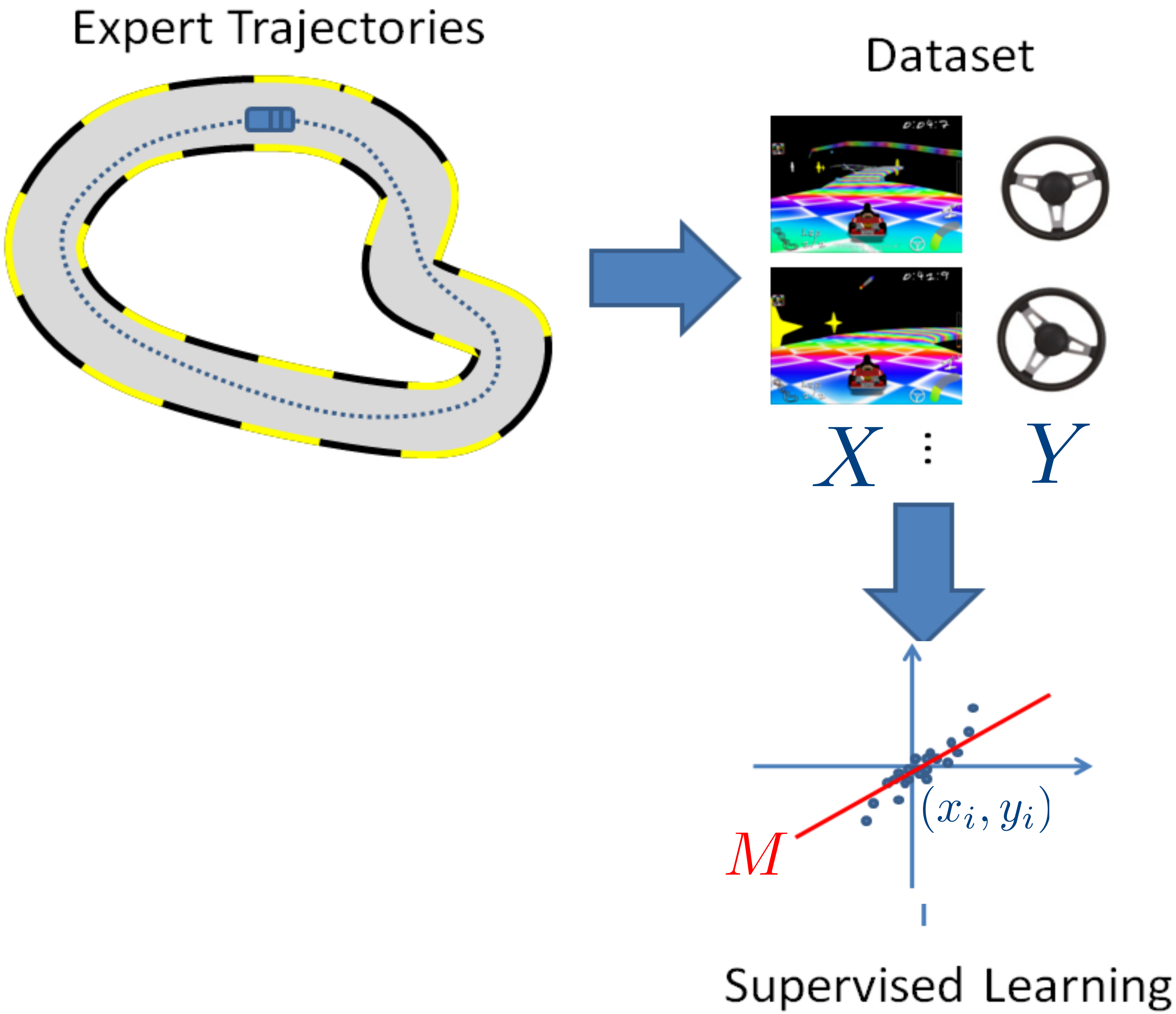

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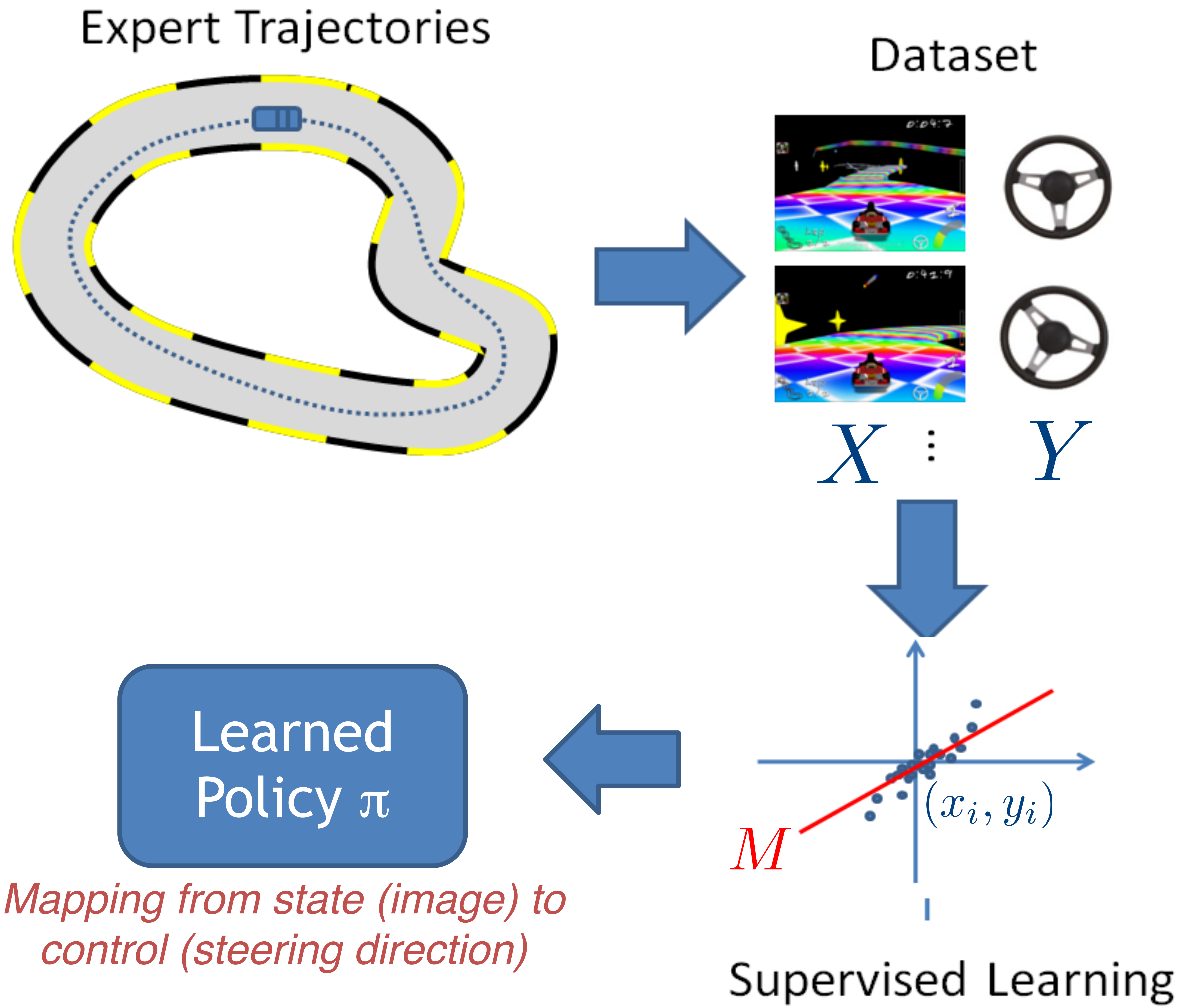


Ready!


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Let's formalize the offline IL Setting and the Behavior Cloning algorithm

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We have a dataset $\mathscr{D} = (s_i^\star, a_i^\star)_{i=1}^M \sim d^{\pi^\star}$

Goal: learn a policy from $\mathscr{D}$ that is as good as the expert $\pi^\star$

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(We can reduce it to other supervised learning oracles such as classification, regression)

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Analysis

Assumption: Π is discrete, and realizable, i.e., $\pi^\star \in \Pi$

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Theorem [MLE Guarantee] With probability at least $1 - \delta$, we have:

$$\mathbb{E}_{s \sim d^{\pi^\star}} \left\| \widehat{\pi}(\cdot \mid s) - \pi^\star(\cdot \mid s) \right\|_{tv} \leq \sqrt{\frac{\ln(|\Pi|/\delta)}{M}}$$

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This $1/\sqrt{M}$ rate should be expected:
no training and testing mismatch at this stage!

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Theorem [BC Sample Complexity] With probability at least $1-\delta$, BC returns a policy $\hat{\pi}$:

$$V^{\pi^\star} - V^{\hat{\pi}} \leq \frac{2}{(1-\gamma)^2} \underbrace{\sqrt{\frac{\ln(|\Pi|/\delta)}{M}}}_{\text{MLE error}}$$

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Note that $1/(1-\gamma)^2$ quadratic dependency on effective horizon

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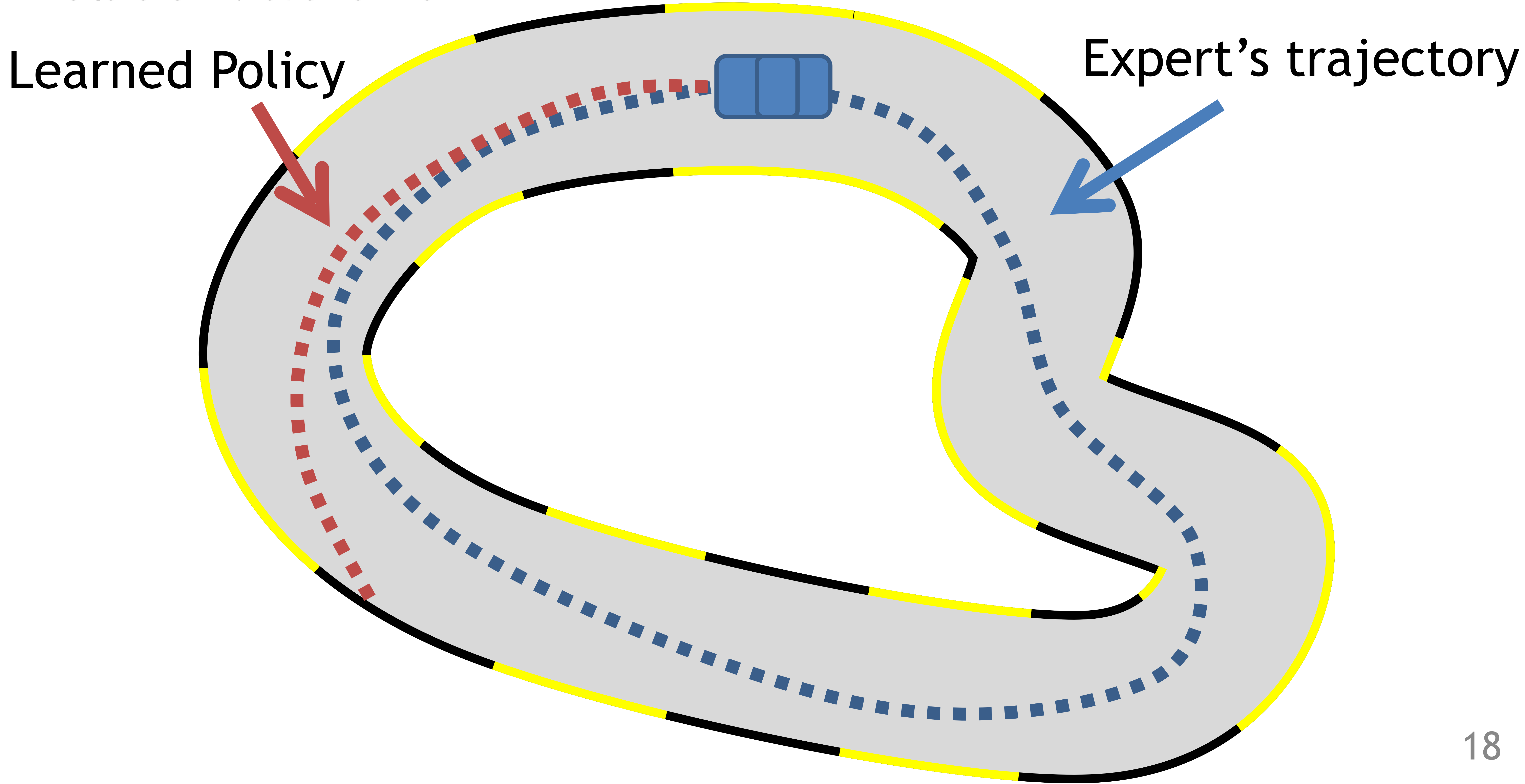
What could go wrong?

[Pomerleau89,Daume09]

- Predictions affect future inputs/ observations



Distribution Shift: Intuitive Explanation

Let's just focus on finite horizon (H) and deterministic policies here:

$$\mathbb{E}_{s \sim d_h^{\pi^\star}} \widehat{\pi}(s) \neq \pi^\star(s) \leq \epsilon, \forall h$$

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s_0

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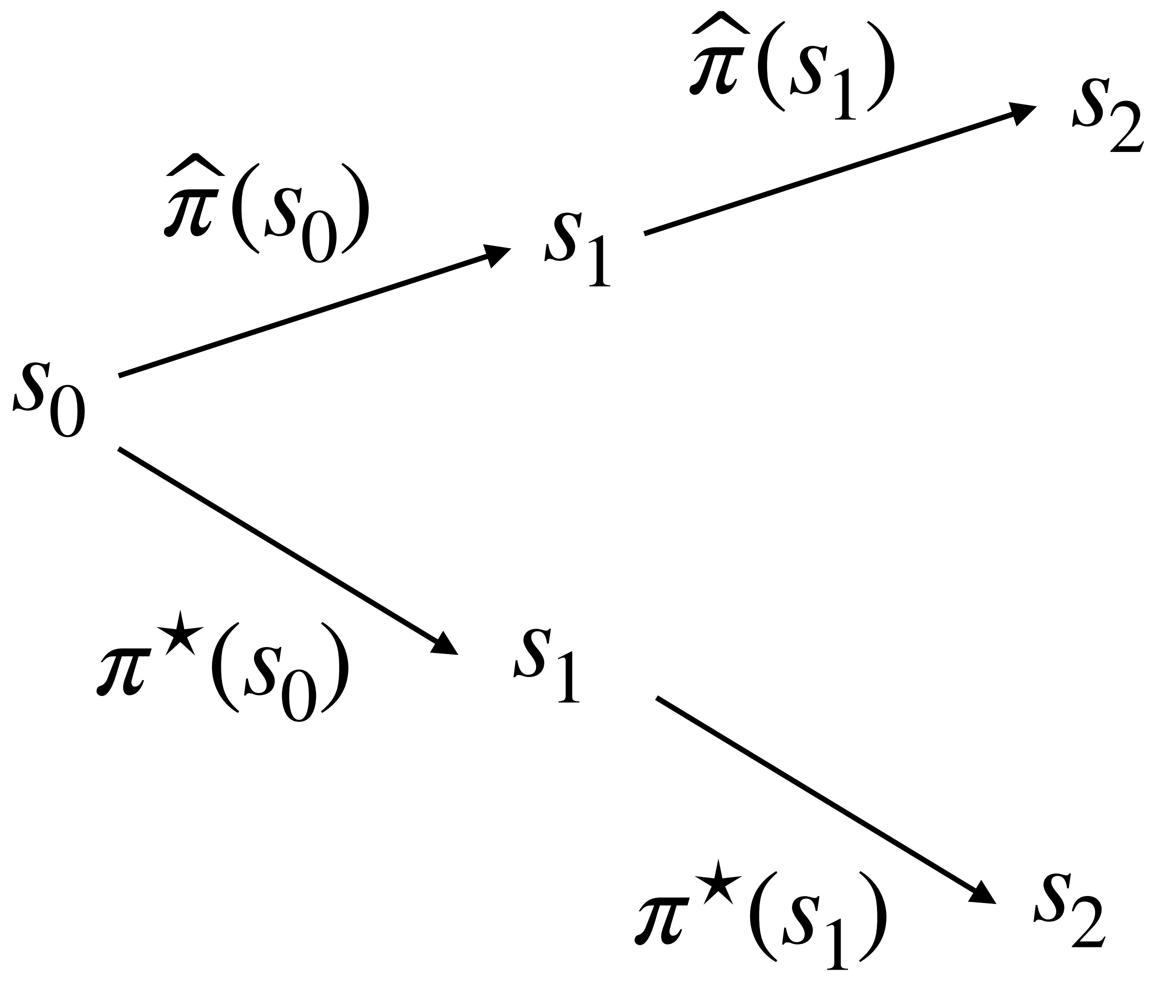

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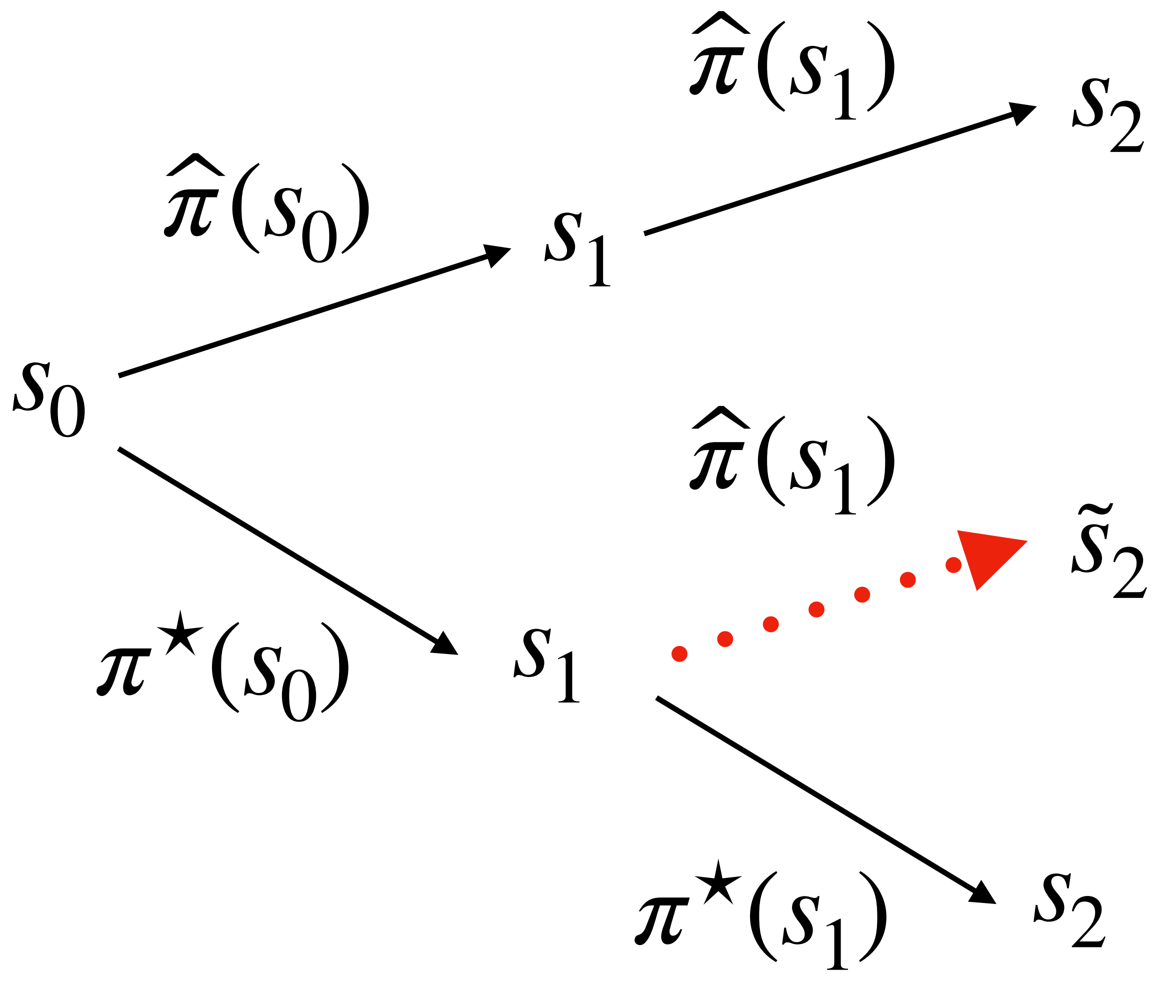

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"If the network is not presented with sufficient variability in its training exemplars to cover the conditions it is likely to encounter…[it] will perform poorly"

A potential Fix

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Let’s roll out our policy in the real world, and compare our trajectories to the expert’s trajectories, and then refine our learned model.

The Hybrid Imitation Learning Setting:

Recall BC: we only use offline expert data—no interaction with the environment

Hybrid setting: offline expert data + simulator (e.g., known transition P)

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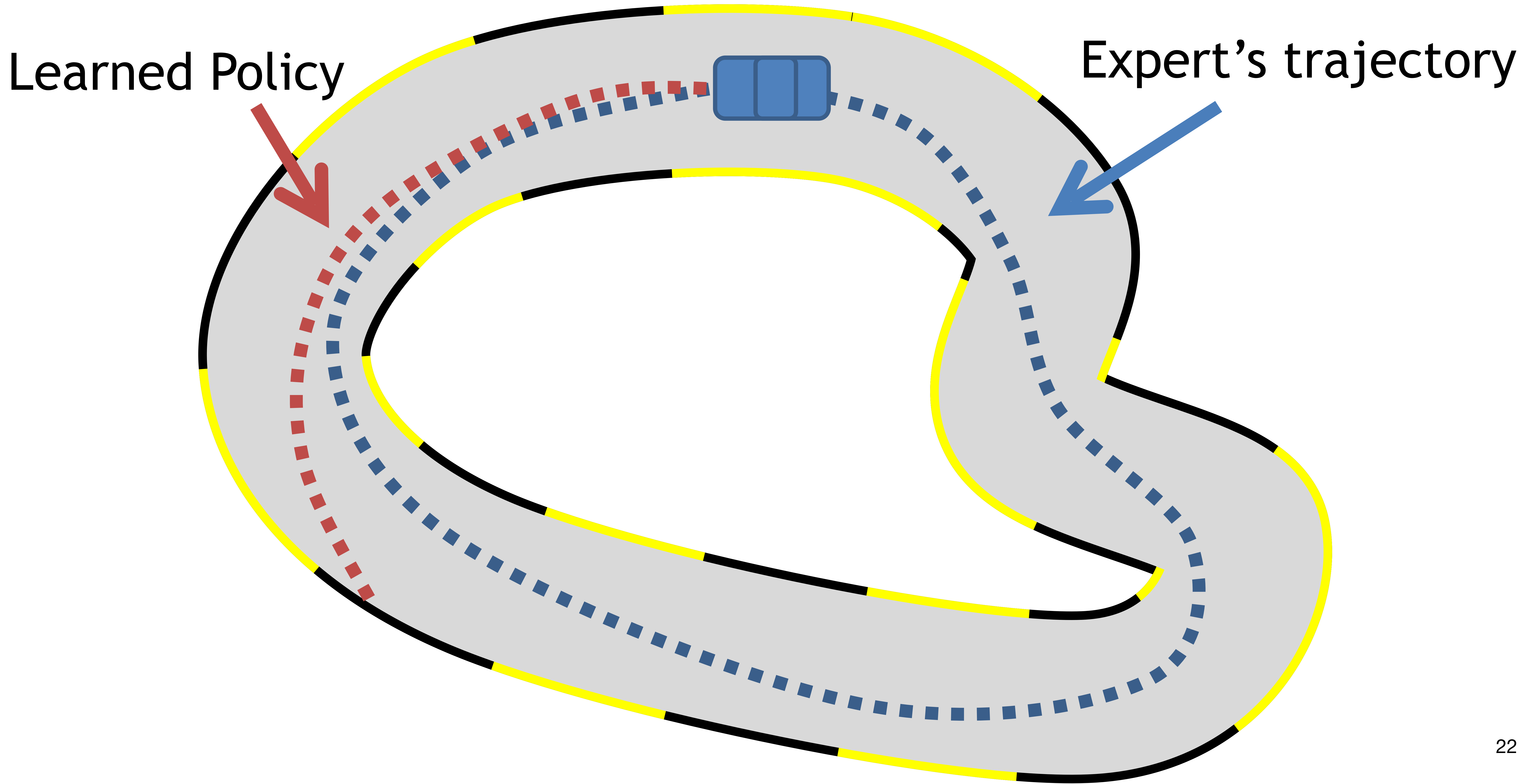

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Key Q: can we do better than offline IL Behavior Cloning (statistically at least—assuming infinite computation power)?

Key Idea: distribution matching

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Metric measures the divergence between two distributions

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$$\mathrm{IPM}_{\mathscr{F}}(p_1, p_2) = \max_{f \in \mathscr{F}} \left[\mathbb{E}_{x \sim p_1} f(x) - \mathbb{E}_{x \sim p_2} f(x) \right]$$

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$$\mathscr{F} = \{f : f \text{ is 1-Lipschitz}\} \Rightarrow \text{IPM}_{\mathscr{F}}(p_1, p_2) := \text{wasserstein dis}(p_1, p_2)$$

Algorithm: Distribution Matching TV distance

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$$f_{\pi,\pi'} = \arg\max_{f \in \mathscr{F}} \left[\mathbb{E}_{s,a\sim d^\pi} f(s,a) - \mathbb{E}_{s,a\sim d^{\pi'}} f(s,a) \right]$$

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Set refined discriminator class $\widetilde{\mathscr{F}} := \{f_{\pi,\pi'} : \pi \,\&\, \pi' \in \Pi, \pi \neq \pi'\}$

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Q: what is the size of $\widetilde{\mathcal{F}}$?

Set refined discriminator class $\widetilde{\mathcal{F}} := \{f_{\pi,\pi'} : \pi \,\&\, \pi' \in \Pi, \pi \neq \pi'\}$

Algorithm: Distribution Matching TV distance

Consider the Discriminator class: $\mathscr{F} = \{f : \|f\|_\infty \leq 1\}$ (IPM corresponds to TV distance)

Same assumption as we had in BC:
Π is discrete, and realizable, i.e., $\pi^\star \in \Pi$

Step 1: for each $\pi \in \Pi$, compute $d^\pi \in \Delta(S \times A)$ (recall P is known);
(This step is computationally inefficient)

Step 2: select useful discriminators: for all pair π & π', with $\pi \neq \pi'$

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$$\forall \pi \,\&\, \pi' \in \Pi, \; \|d^\pi - d^{\pi'}\|_{tv} = \max_{f \in \widetilde{\mathscr{F}}} \mathbb{E}_{s,a\sim d^\pi} f(s,a) - \mathbb{E}_{s,a\sim d^{\pi'}} f(s,a)$$

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$$\widehat{\pi} := \arg\min_{\pi \in \Pi} \left[\max_{f \in \widetilde{\mathscr{F}}} \left[\mathbb{E}_{s,a\sim d^\pi} f(s,a) - \frac{1}{M} \sum_{i=1}^M f(s_i^\star, a_i^\star) \right] \right]$$

Theorem: Distribution Matching TV distance

Theorem [Dis-match w/ TV dist] With probability at least $1 - \delta$, our algorithm finds a policy $\widehat{\pi}$, s.t.,

$$V^{\pi^\star} - V^{\widehat{\pi}} \leq \mathcal{O}\left(\frac{1}{1-\gamma} \sqrt{\frac{\ln(|\Pi|/\delta)}{M}} \right)$$

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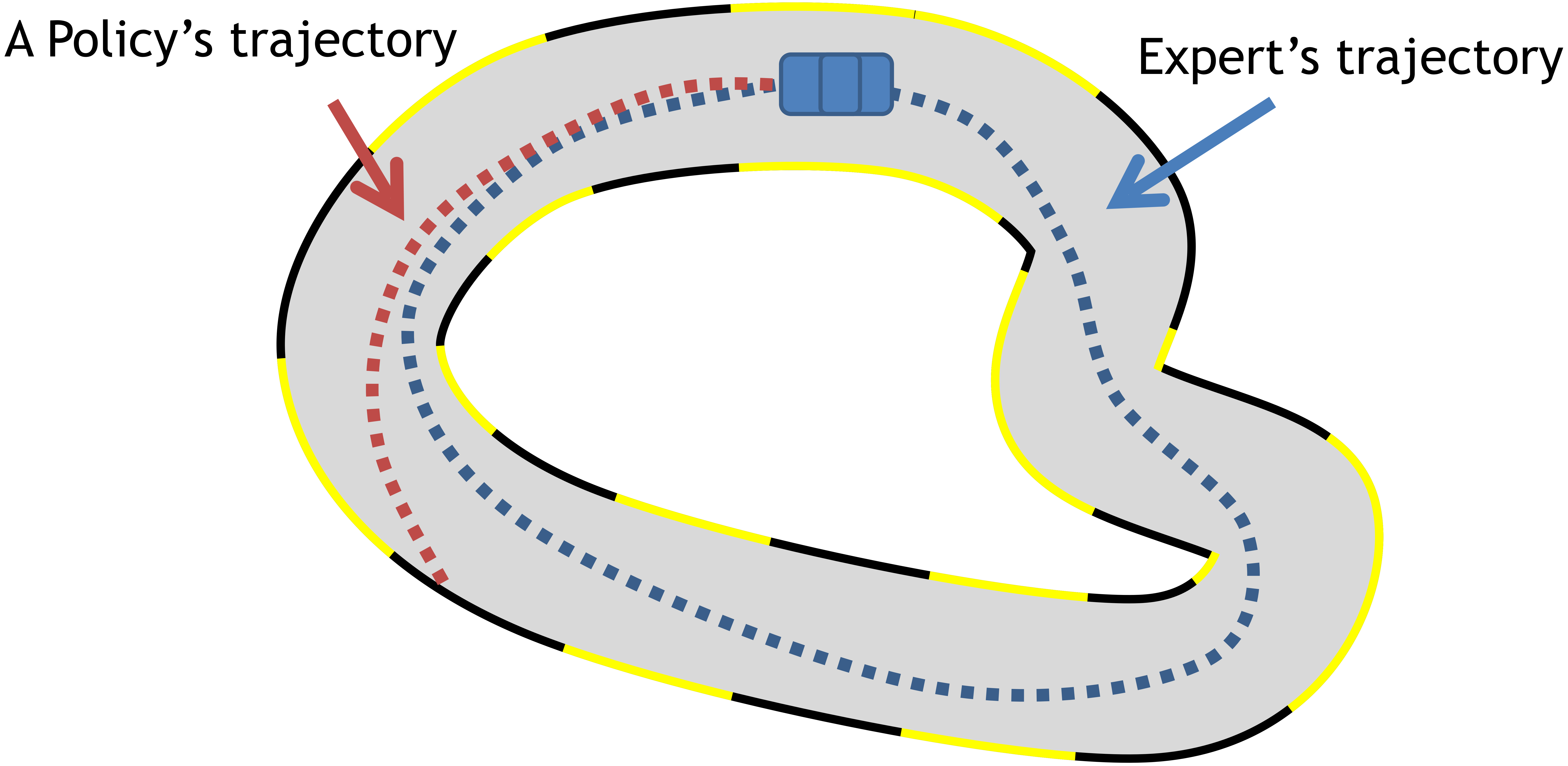
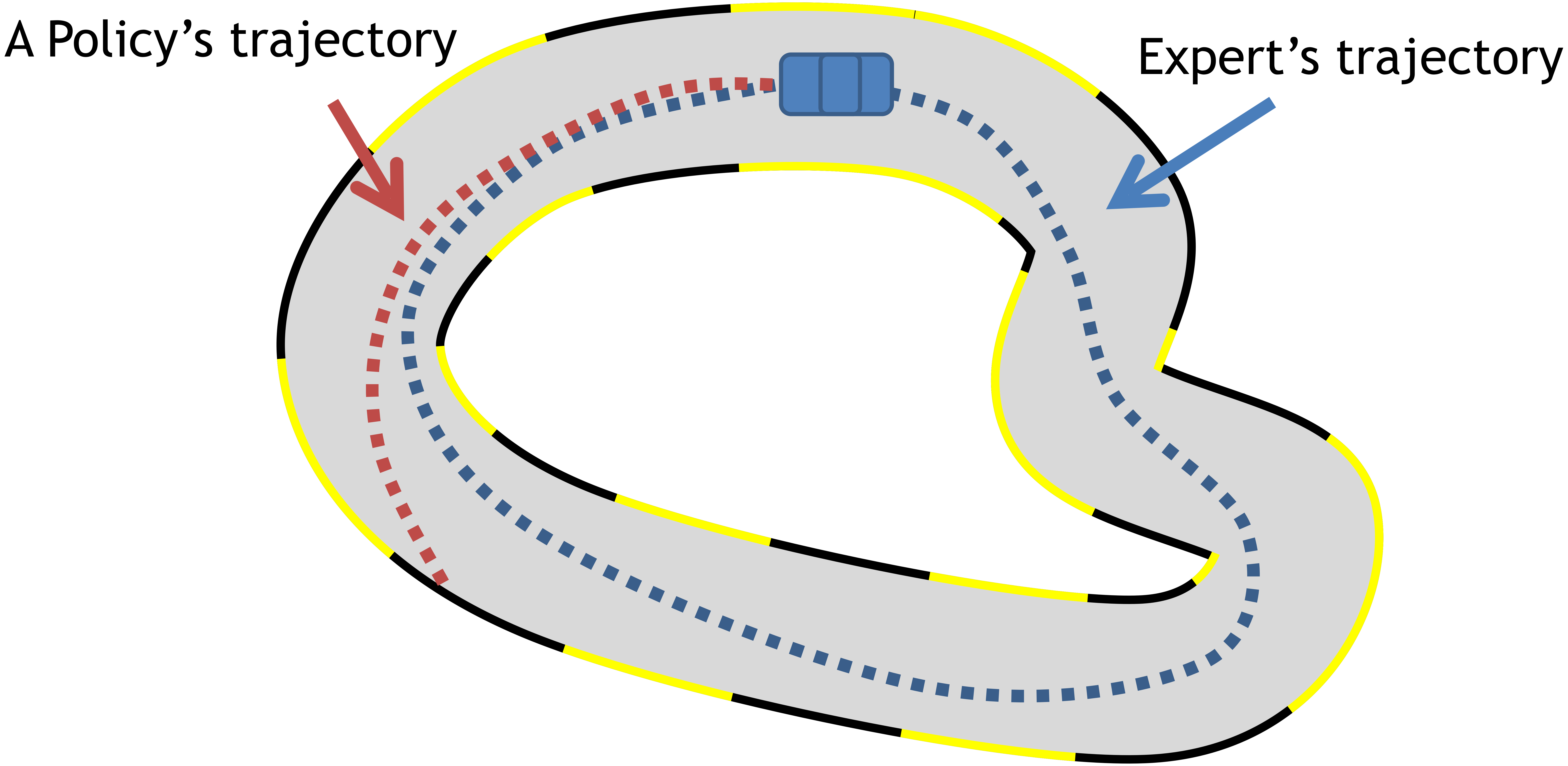
Theorem [Offline BC] With probability at least $1-\delta$, BC returns a policy $\hat{\pi}$:

$$V^{\pi^\star} - V^{\hat{\pi}} = \mathcal{O}\left(\frac{1}{(1-\gamma)^2}\sqrt{\frac{\ln(|\Pi|/\delta)}{M}}\right)$$


A Policy's trajectory
Expert's trajectory

d^{π} : generator that generators state-action pairs



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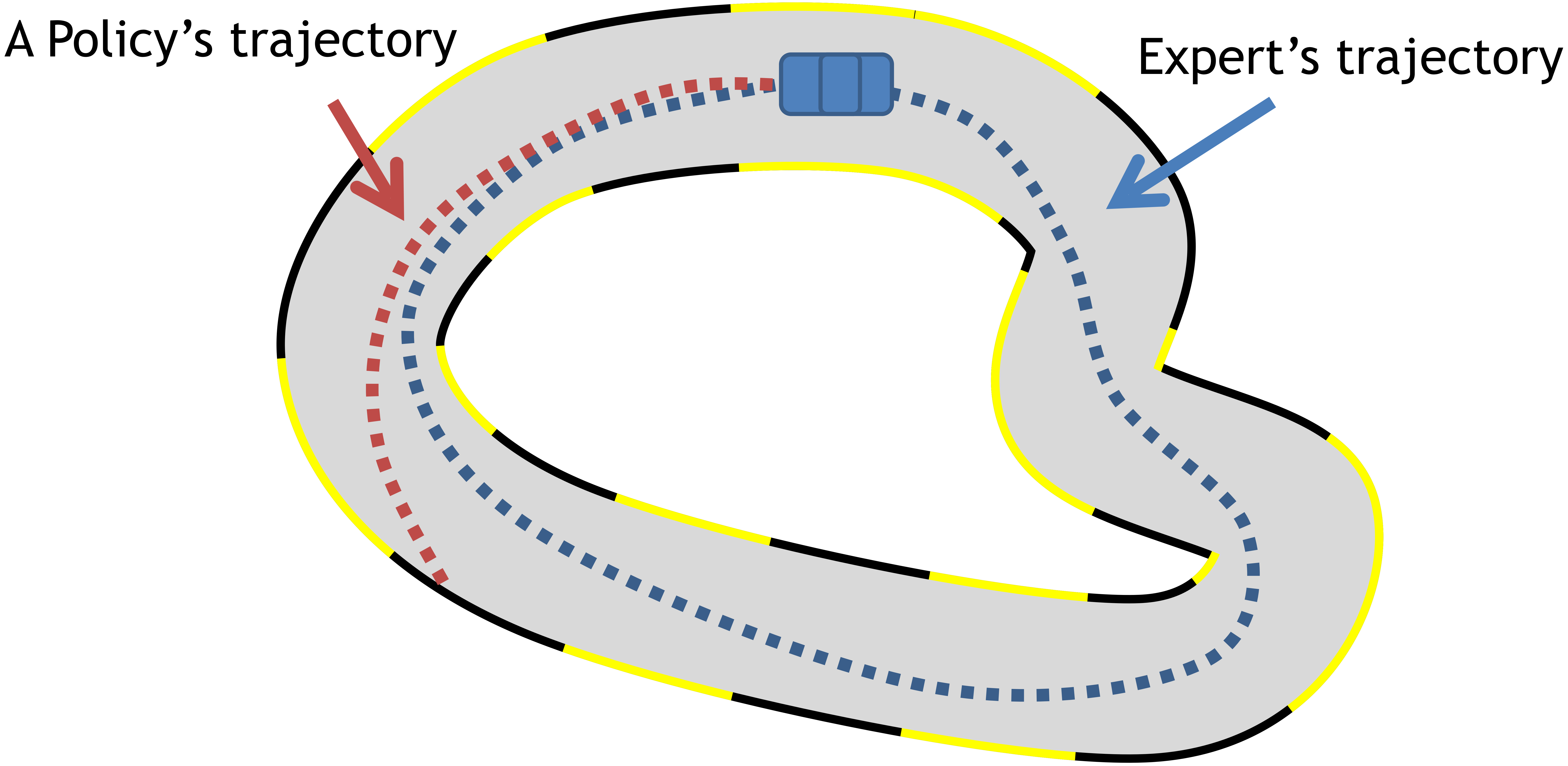

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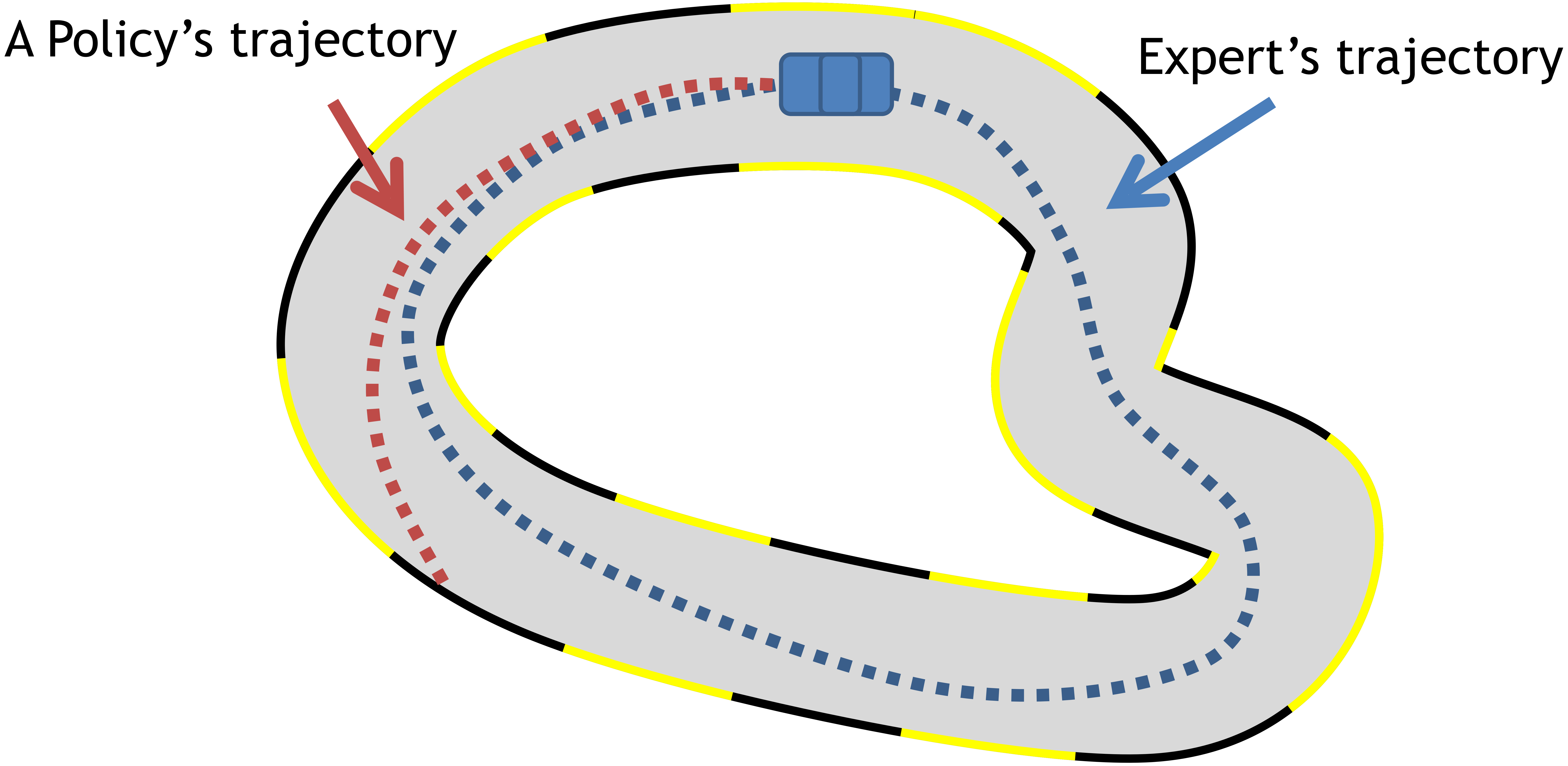
$\widetilde{\mathscr{F}}$: discriminators which distinguish red and blue



Next lecture we will talk about a computationally efficient algorithm in the hybrid setting

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1. Offline RL: only use offline expert data

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Statistically, Distribution-matching has linear dependency on horizon, but the algorithm is computationally inefficient

Take home message:

There is a provable statistical benefit from the hybrid setting!

Ps: the distribution matching algorithm is very new (it was discovered when I was writing the book chapter…)