Interactive Imitation Learning

Sham Kakade and Wen Sun

CS 6789: Foundations of Reinforcement Learning

Announcements

Final report: NeurIPS format

Maximum 9 pages for main tex (not including references and appendix)

Recap

Offline IL and Hybrid Setting:

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Ground truth reward $r(s, a) \in [0,1]$ is unknown;
assume expert is a near optimal policy $\pi^\star$

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Offline IL and Hybrid Setting:

Ground truth reward $r(s, a) \in [0,1]$ is unknown;
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We have a dataset $\mathscr{D} = (s_i^\star, a_i^\star)_{i=1}^M \sim d^{\pi^\star}$

Recap

Maximum Entropy IRL formulation:

$$\max_{\pi} \text{entropy}[\rho^{\pi}]$$

$$s.t, \mathbb{E}_{s,a\sim d^{\pi}}\phi(s,a) = \mathbb{E}_{s,a\sim d^{\pi^\star}}\phi(s,a)$$

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We can rewrite it in the max-min formulation:

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Algorithm: incremental update on cost function θ, exact update on policy π (soft VI):

$$\theta_{t+1} := \theta_t + \eta \nabla_{\theta} J(\theta_t, \pi_t), \quad \pi_{t+1} = \arg\min_{\pi} J(\theta_{t+1}, \pi)$$

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Interactive Imitation Learning Setting

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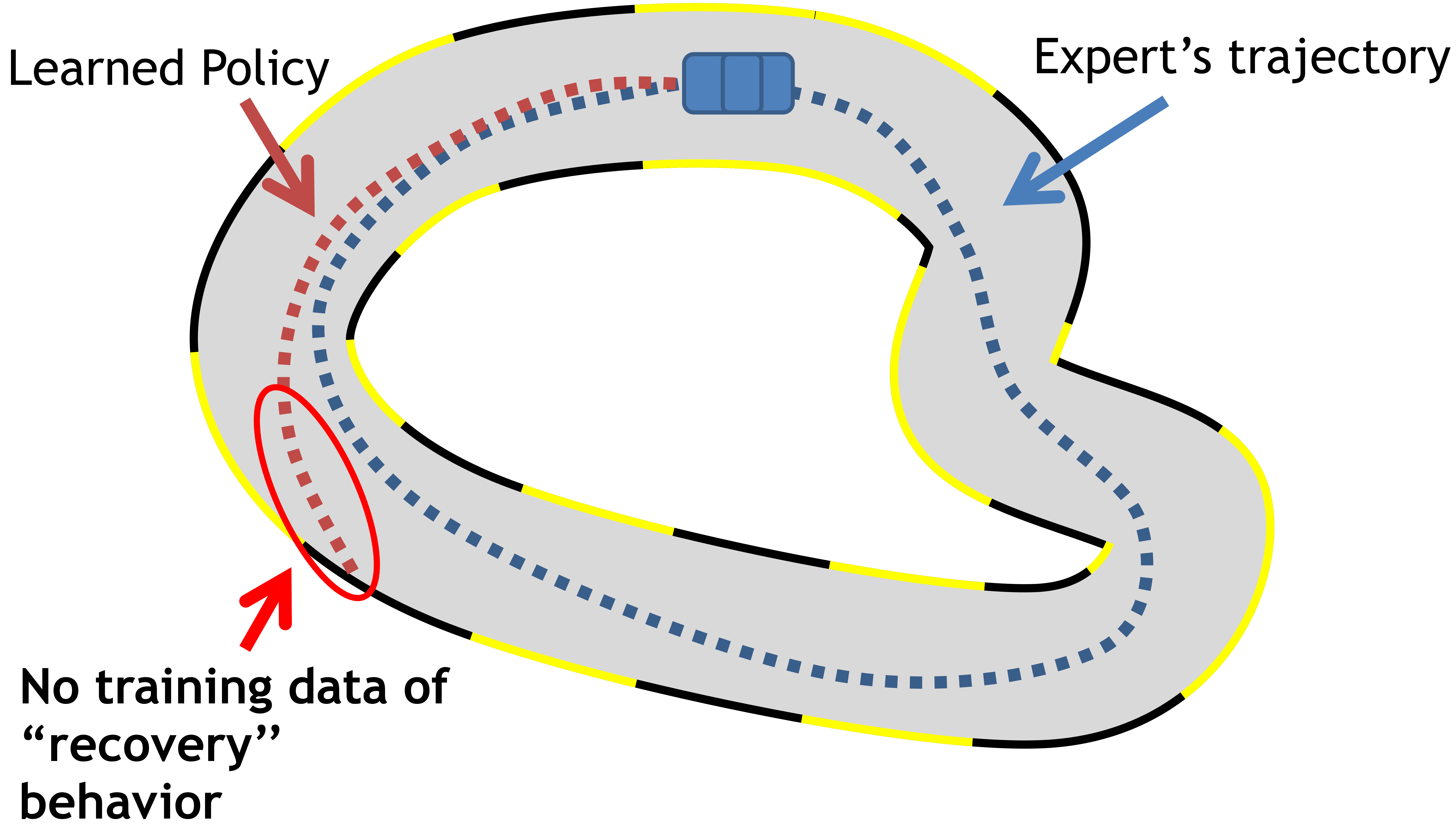
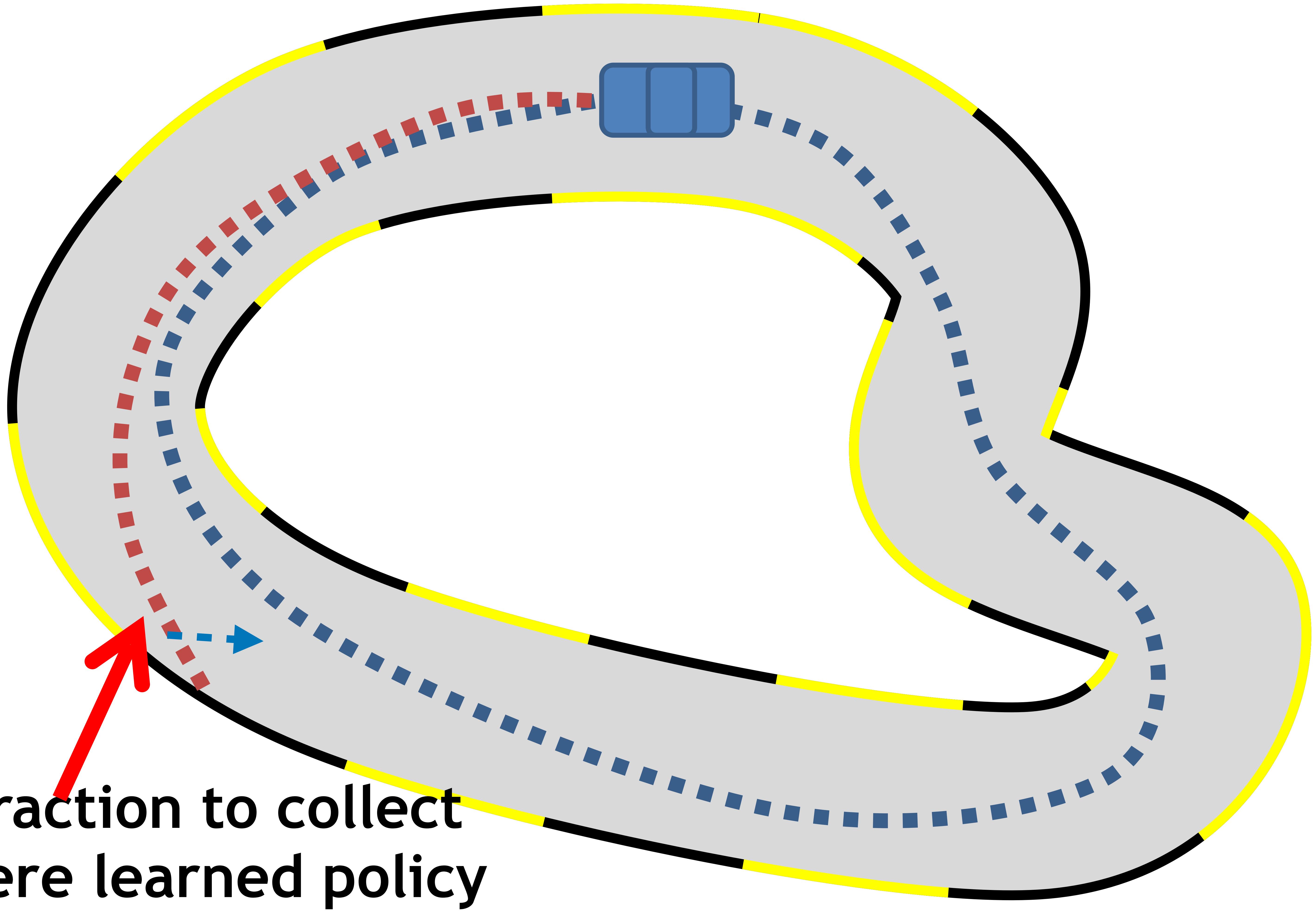
(Recall that previously we only had an offline dataset $\mathscr{D} = (s_i^\star, a_i^\star)_{i=1}^M \sim d^{\pi^\star}$)

Recall the Main Problem from Behavior Cloning:

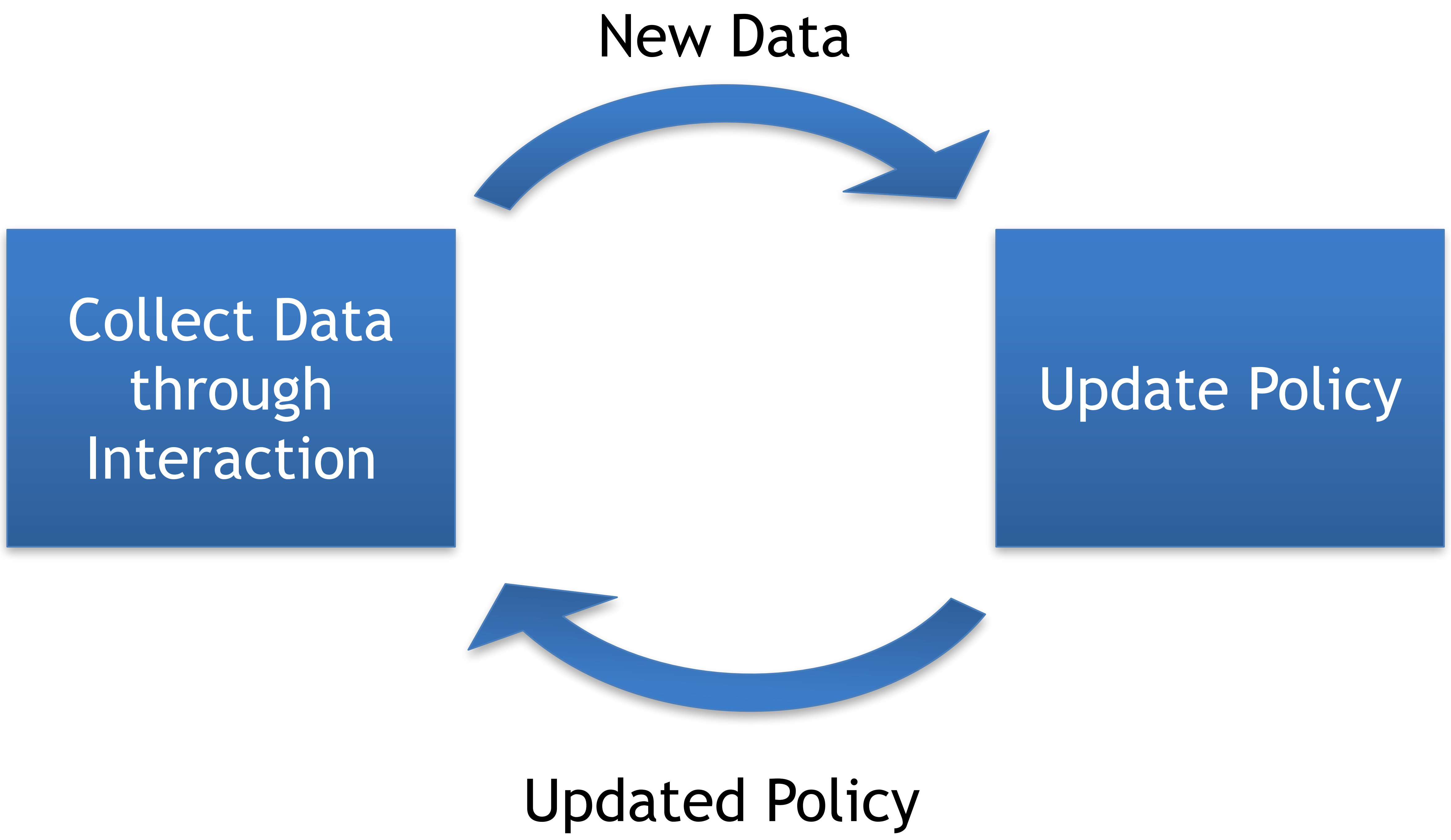


Intuitive solution: **Interaction**



Use interaction to collect data where learned policy goes

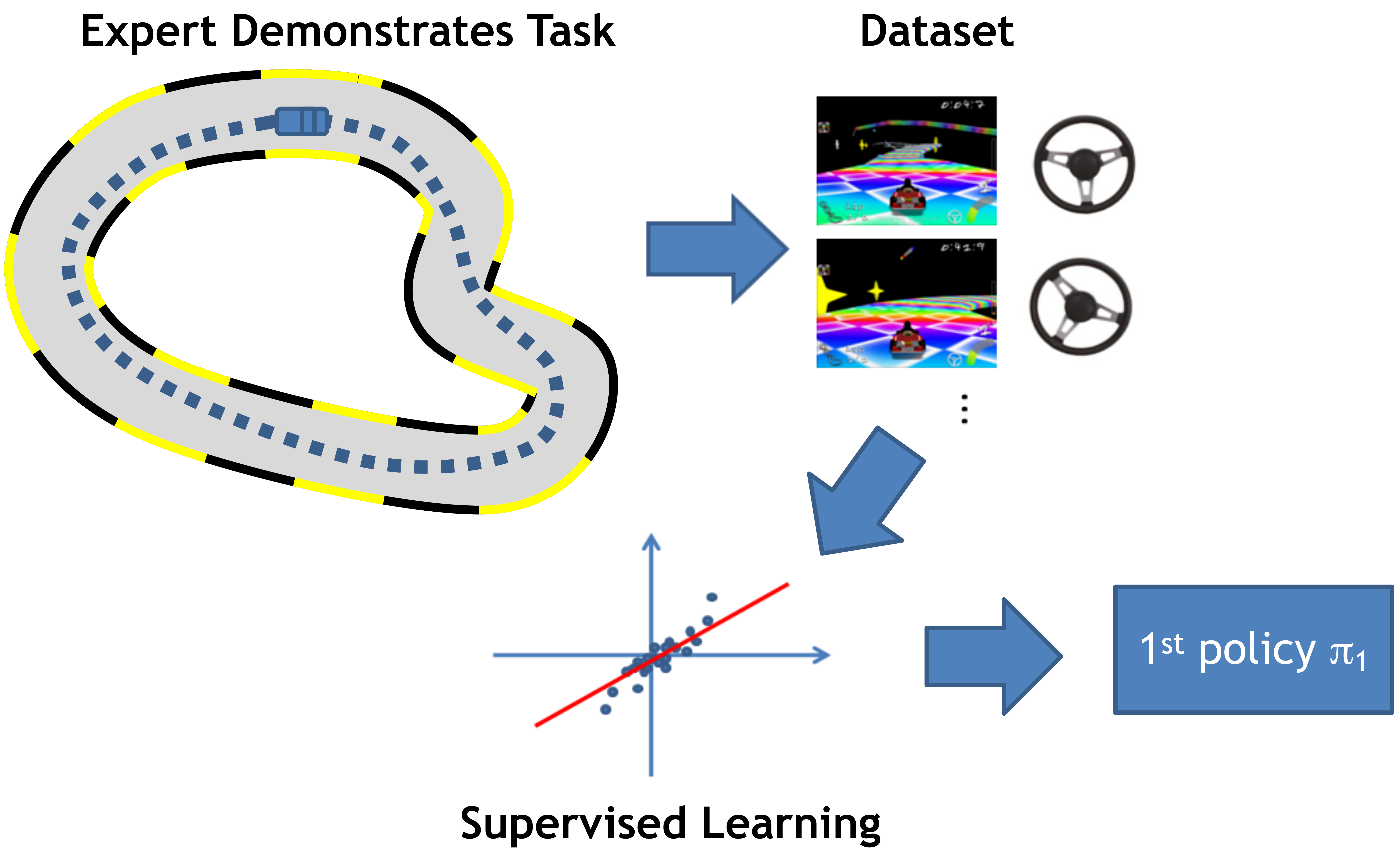
General Idea: Iterative Interactive Approach



All DAgger slides credit: Drew Bagnell, Stephane Ross, Arun Venktraman

DAgger: Dataset Aggregation [Ross11a]

0th iteration



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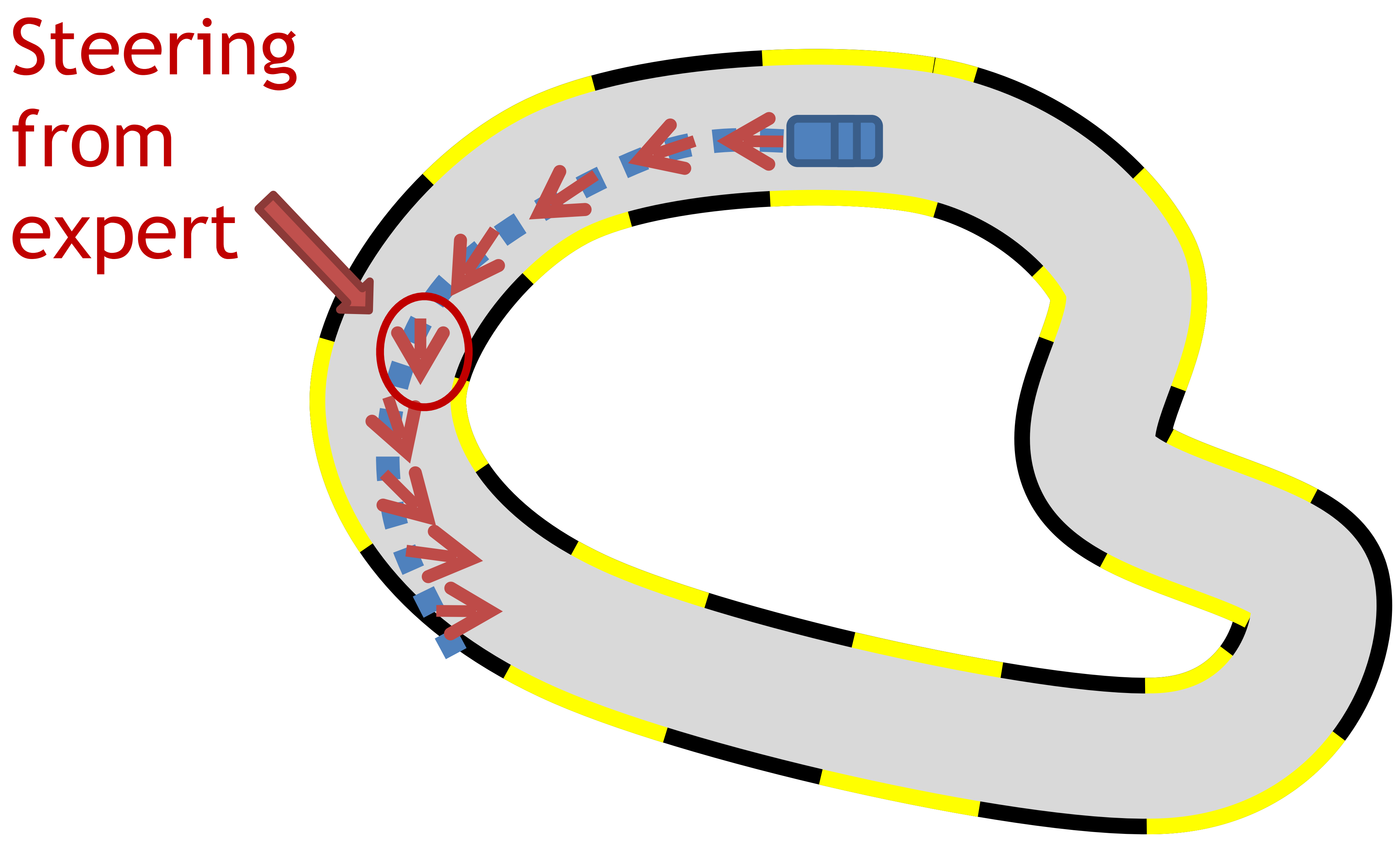
1st iteration

Execute π_1 and Query Expert



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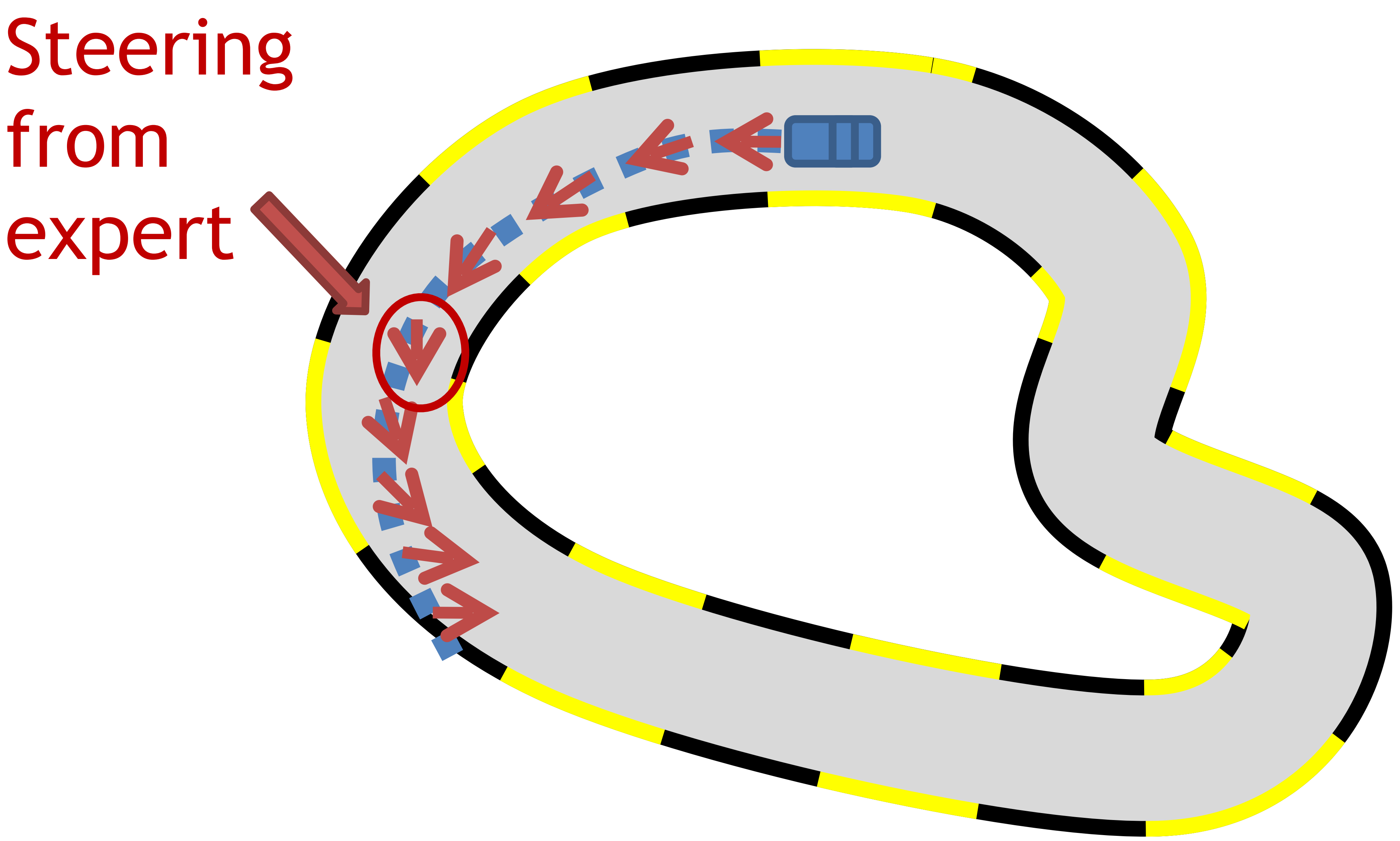
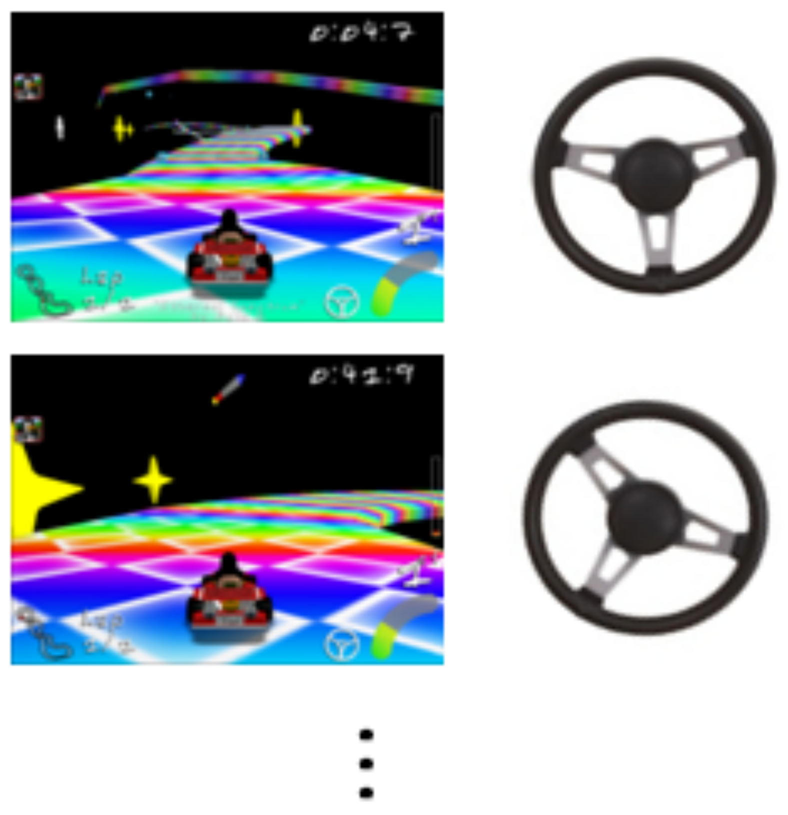


New Data



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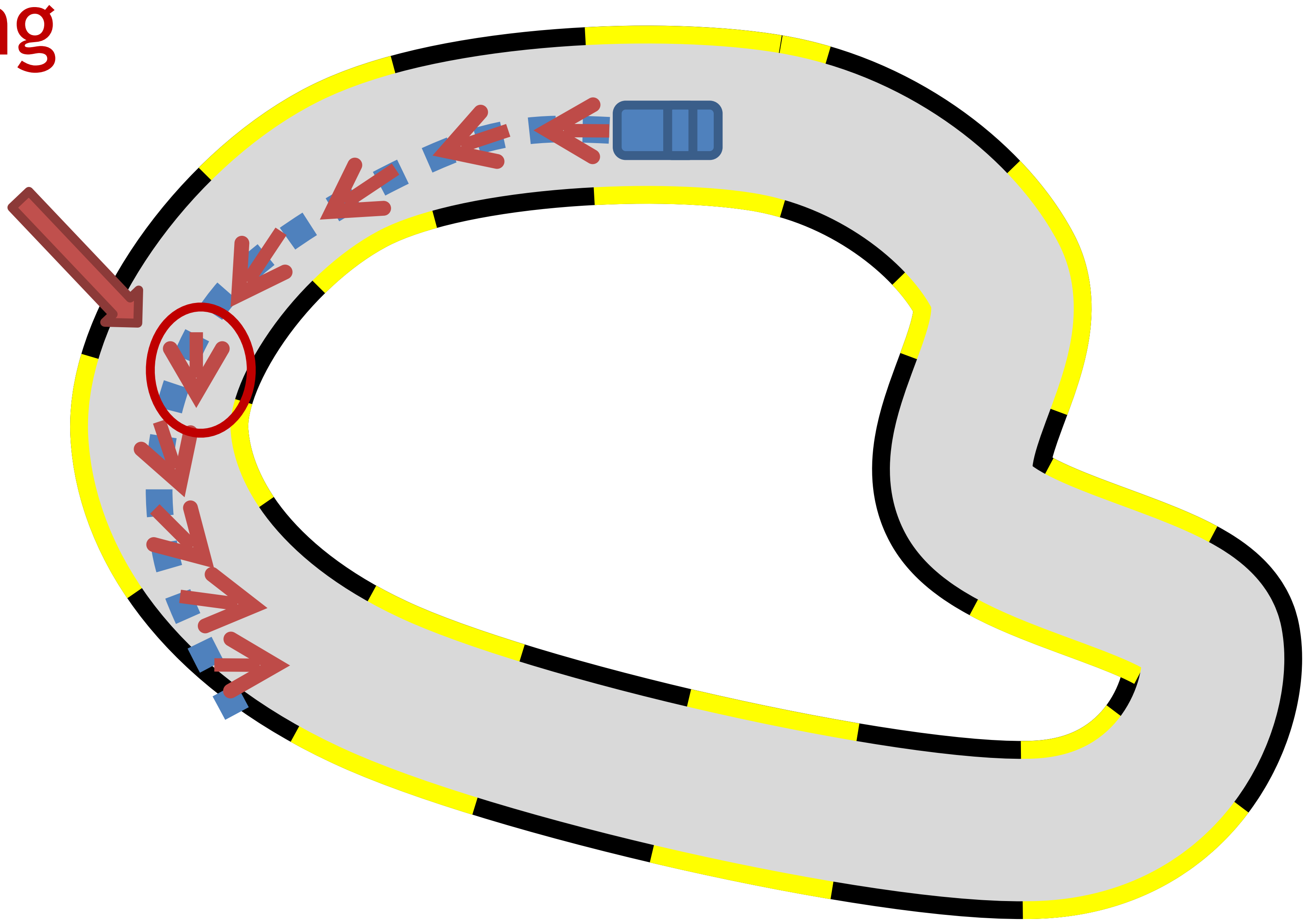
Steering
from
expert



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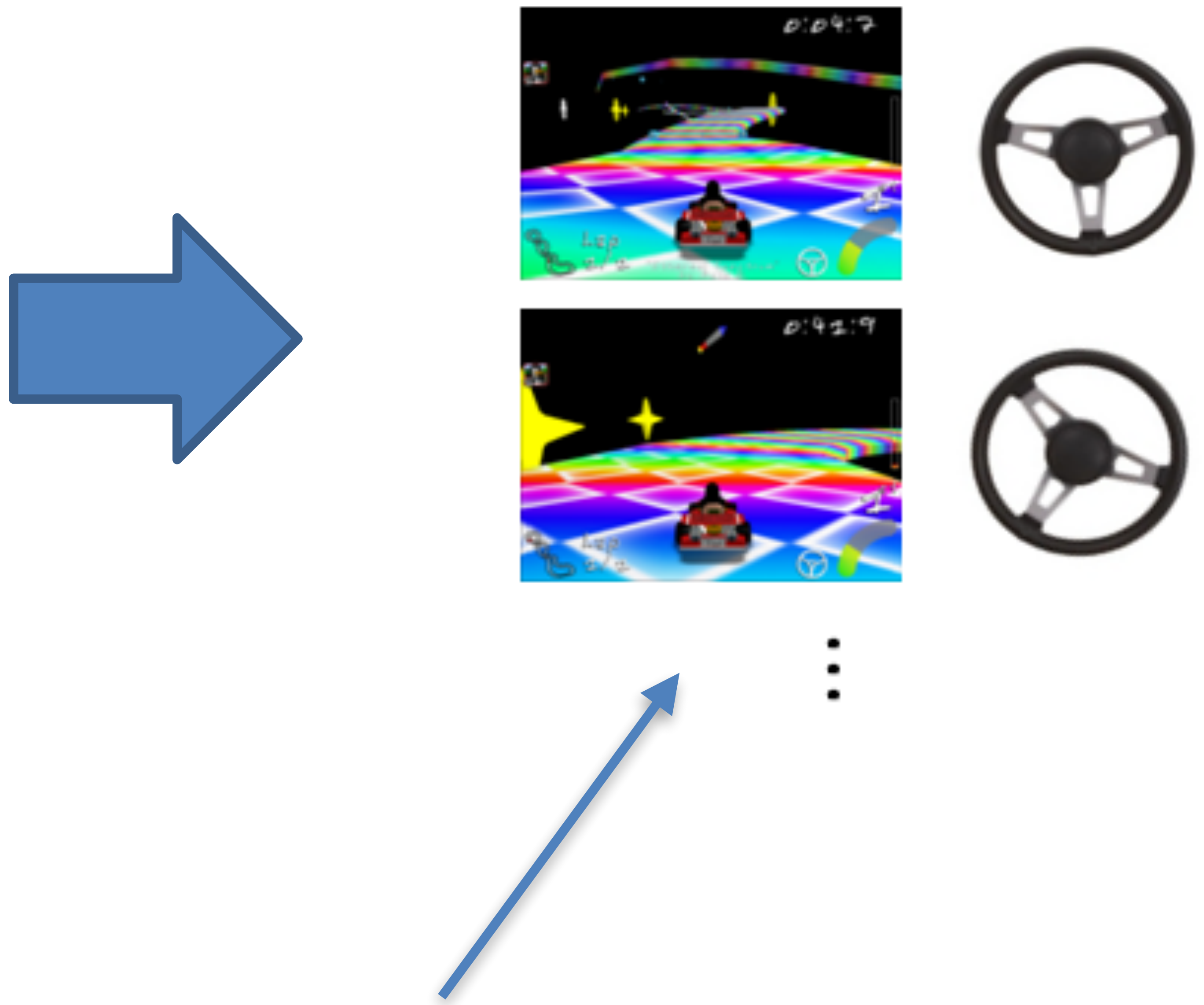

States from
the learned policy

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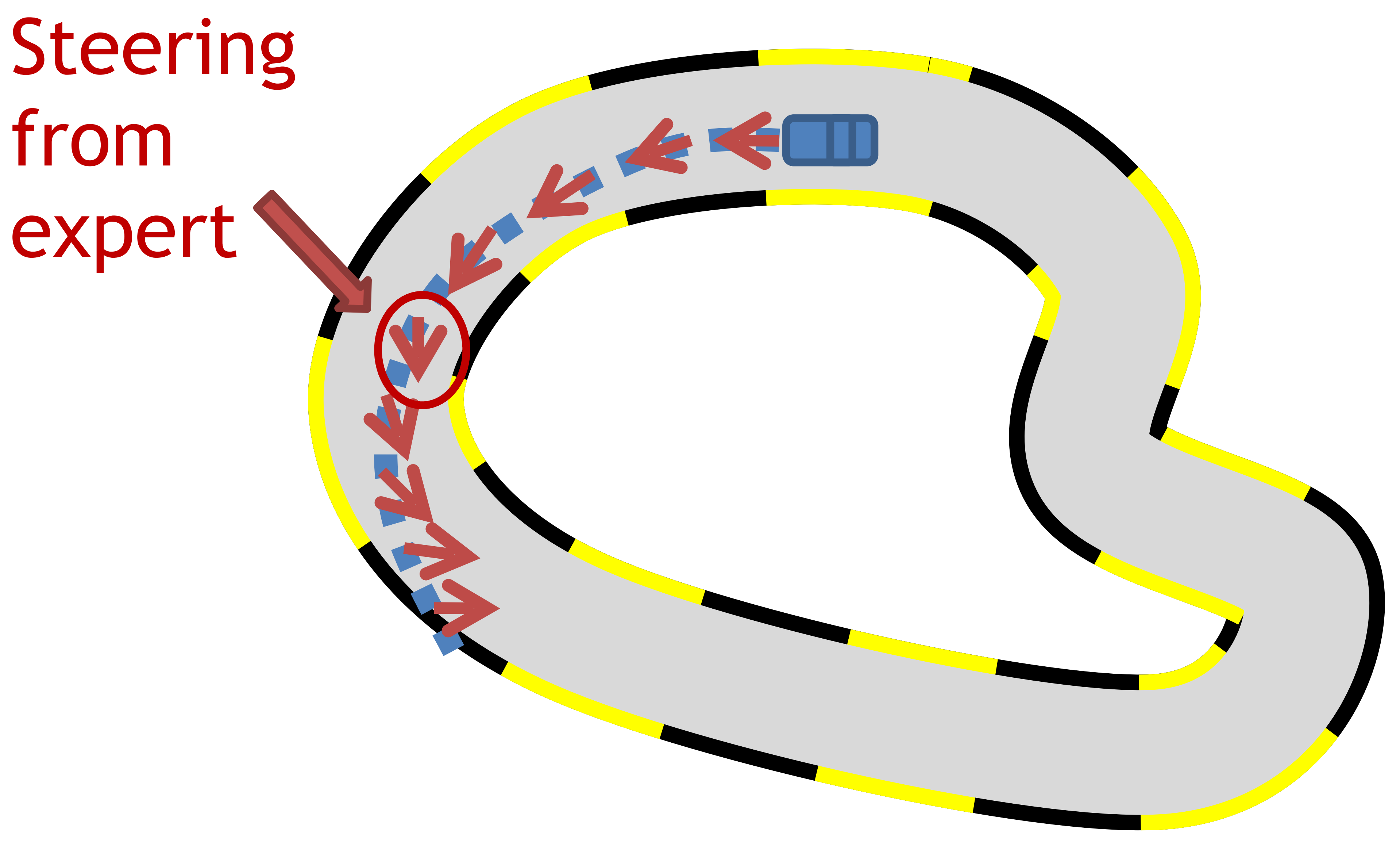

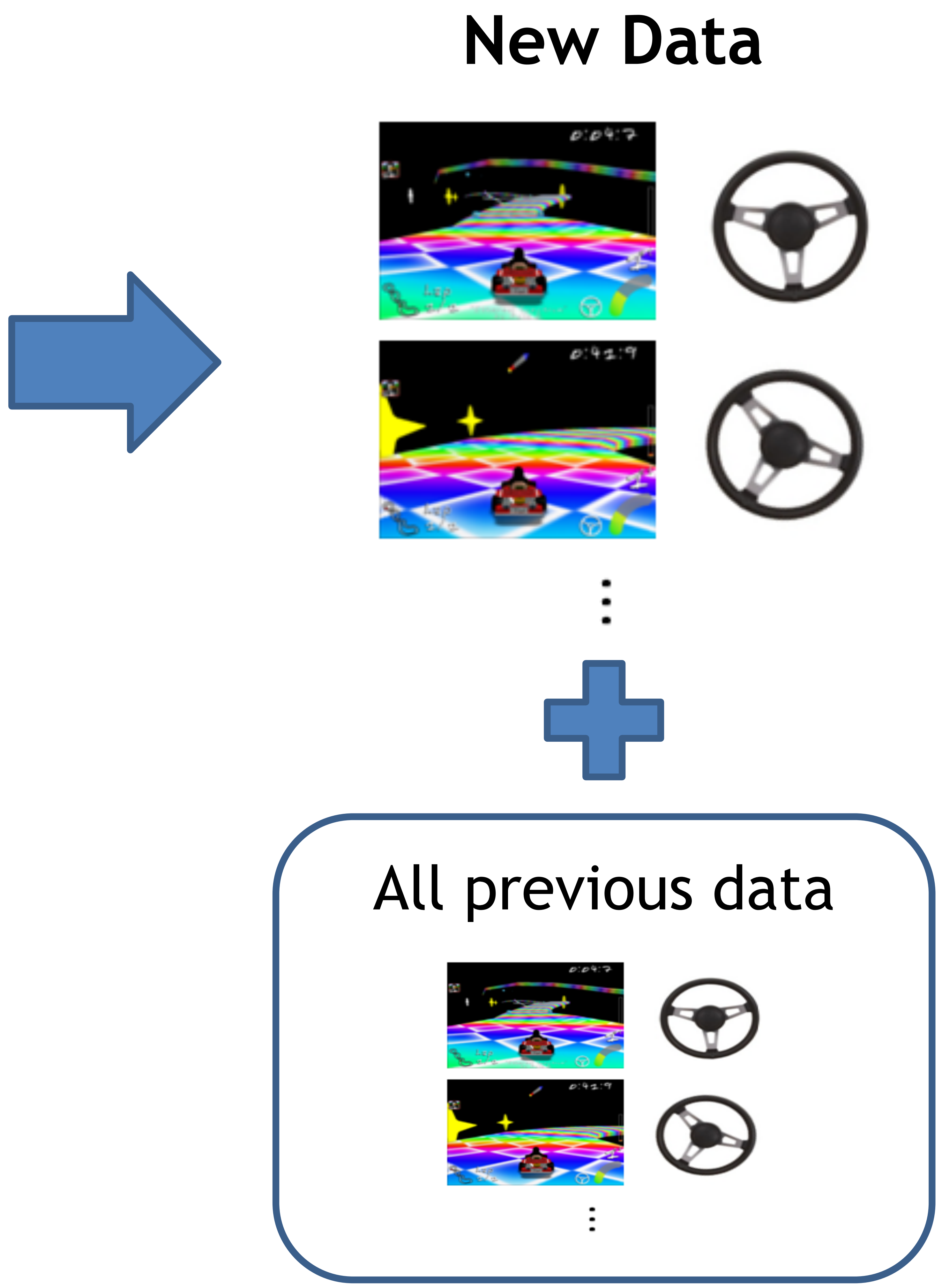

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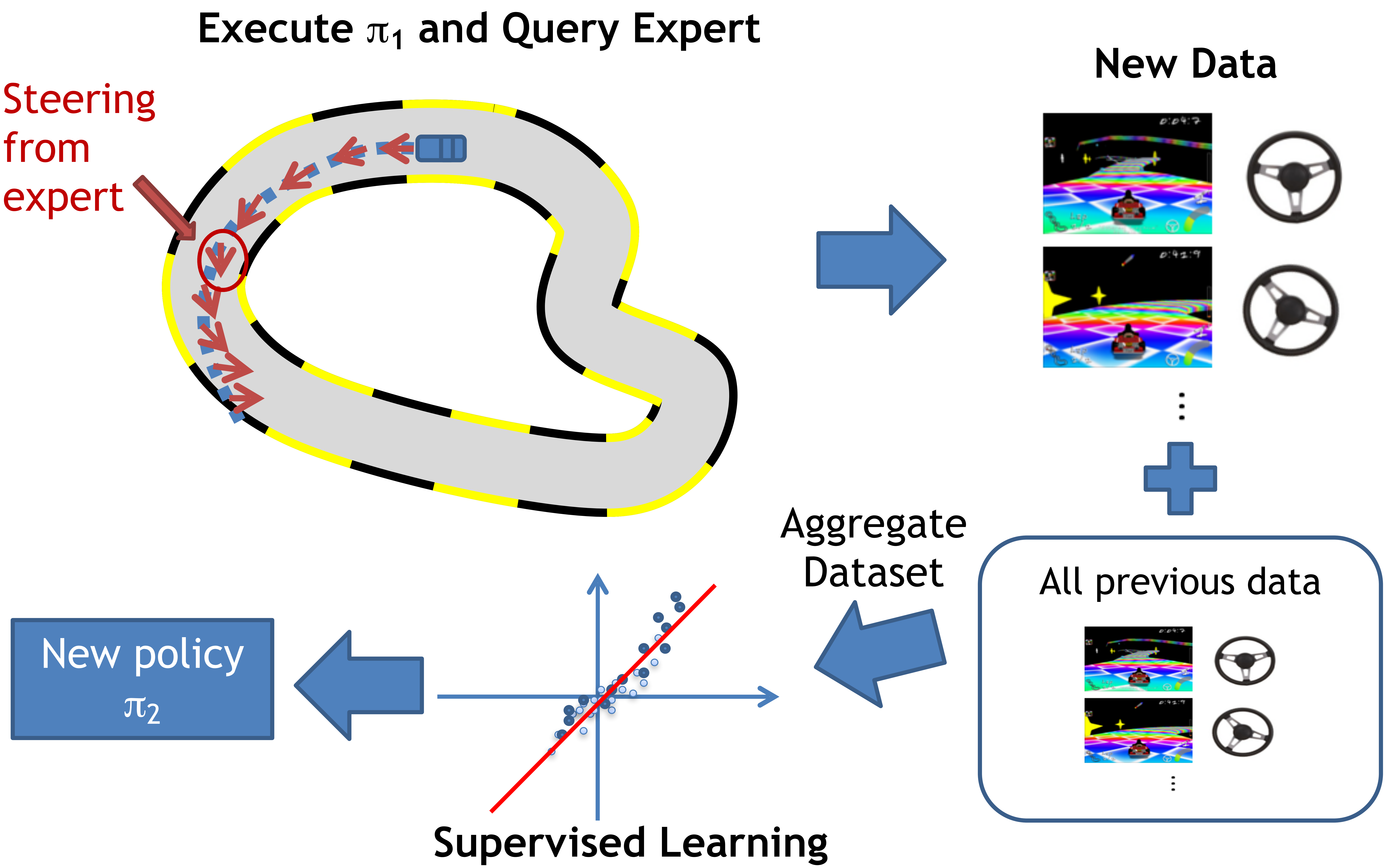
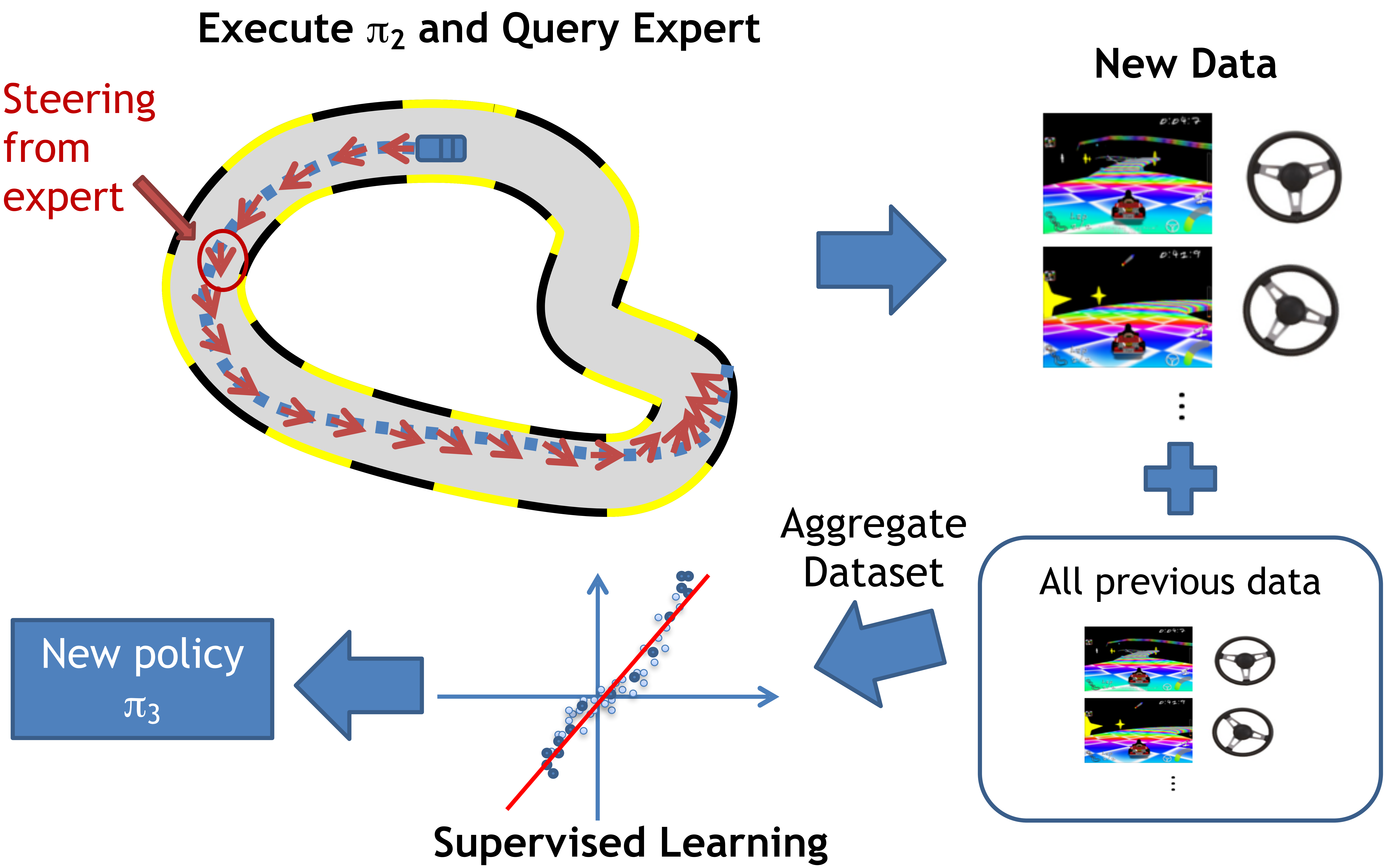


DAgger: Dataset Aggregation

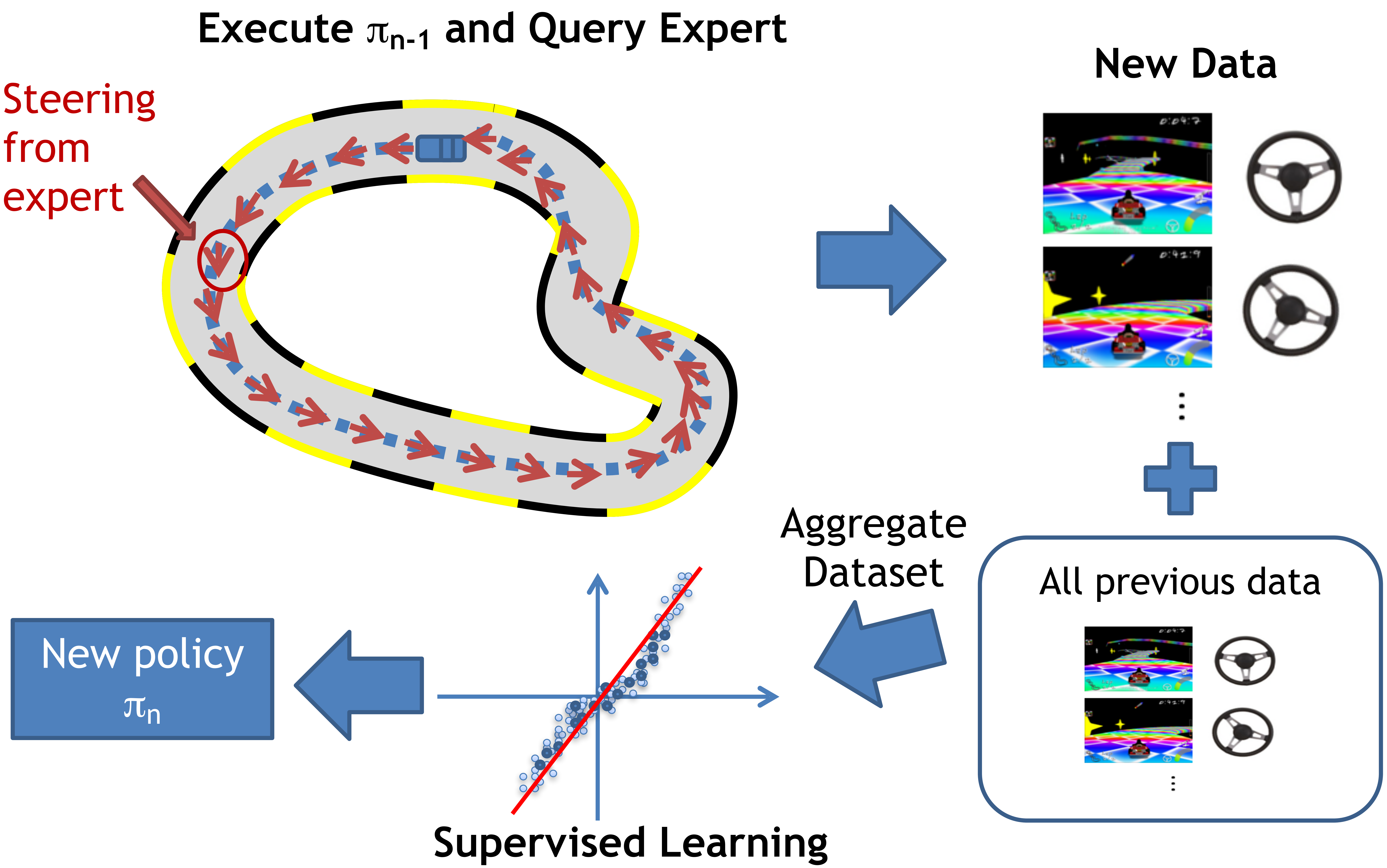
[Ross11a]

2nd iteration



DAgger: Dataset Aggregation [Ross11a]

n^{th} iteration



Success!

[Ross AISTATS 2011]

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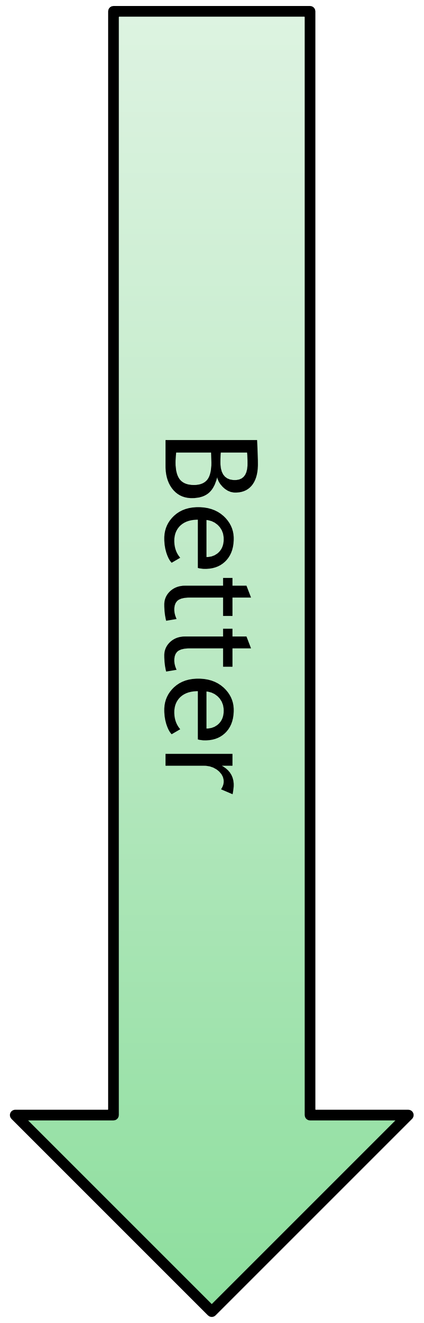
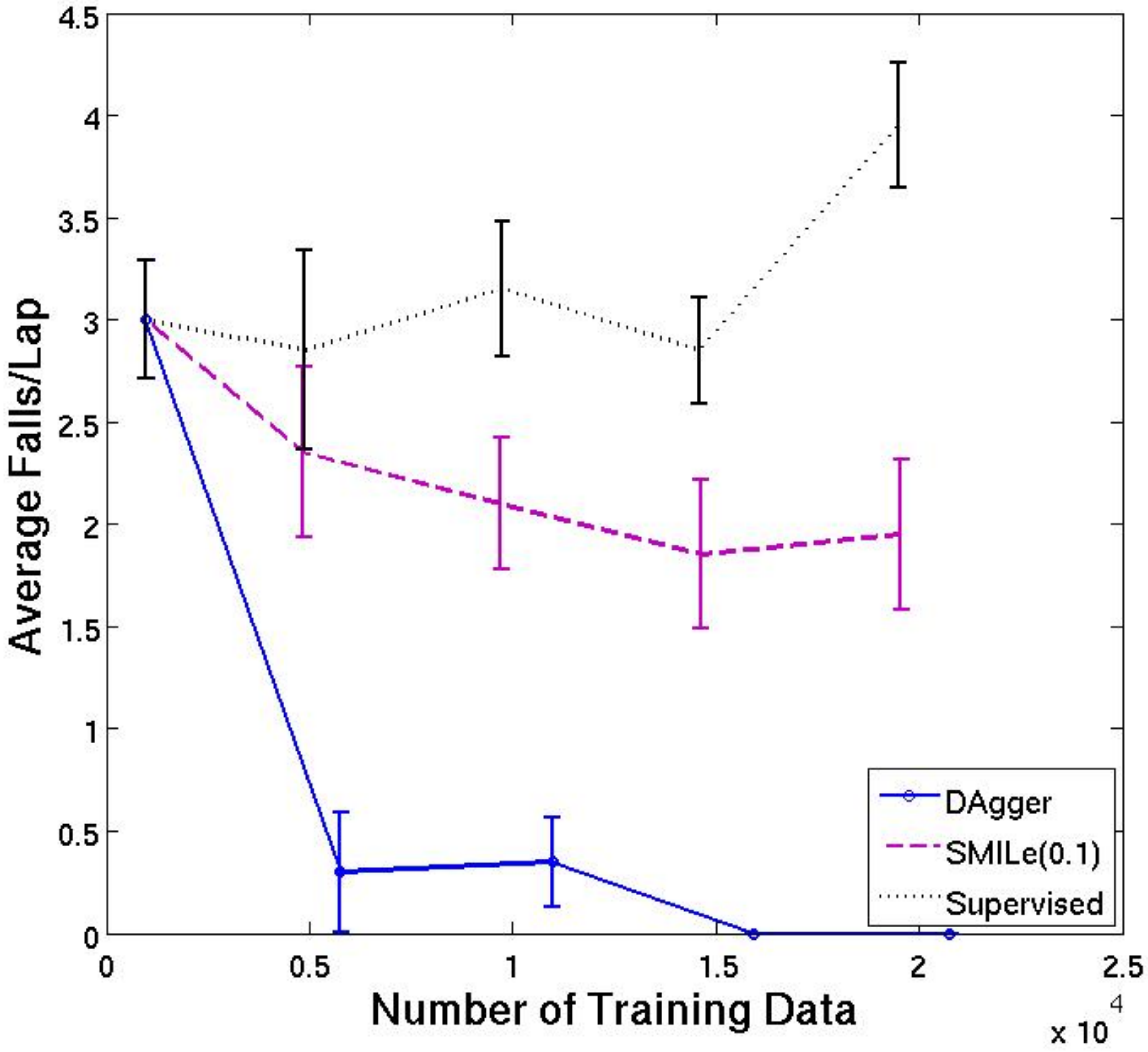
Average Falls/Lap





More fun than Video Games…

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Forms of the Interactive Experts

Interactive Expert is expensive, especially when the expert is human…

But expert does not have to be human…

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Example: high-speed off-road driving
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Fig. 4: The AutoRally car and the test track.

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Fig. 4: The AutoRally car and the test track.

Goal: learn a racing control policy that maps from data on cheap on-board sensors (raw-pixel imagine) to low-level control (steer and throttle)

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(a) raw image

→ Steering + throttle

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Their Setup:
At Training, we have expensive sensors for accurate state estimation
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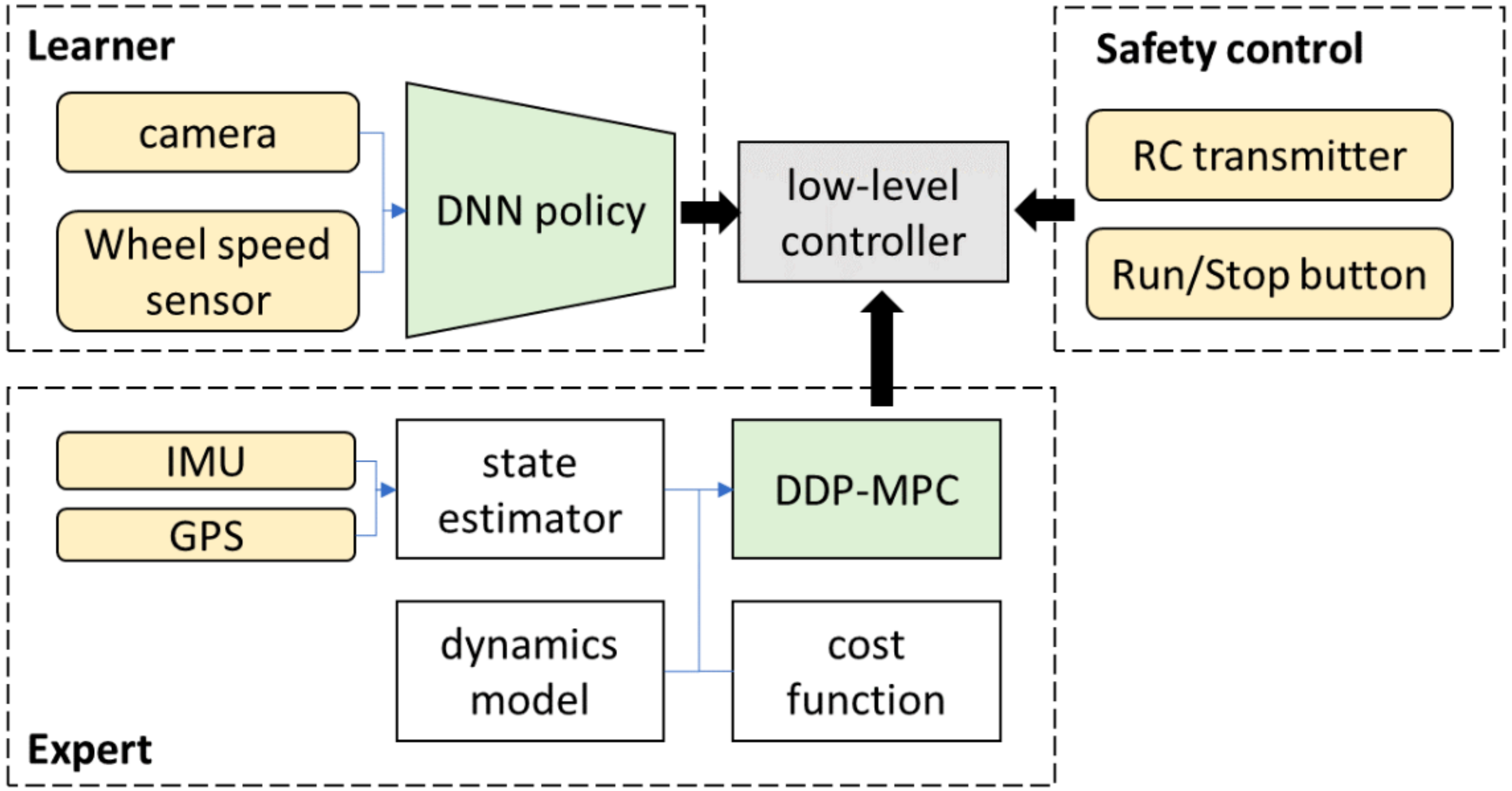


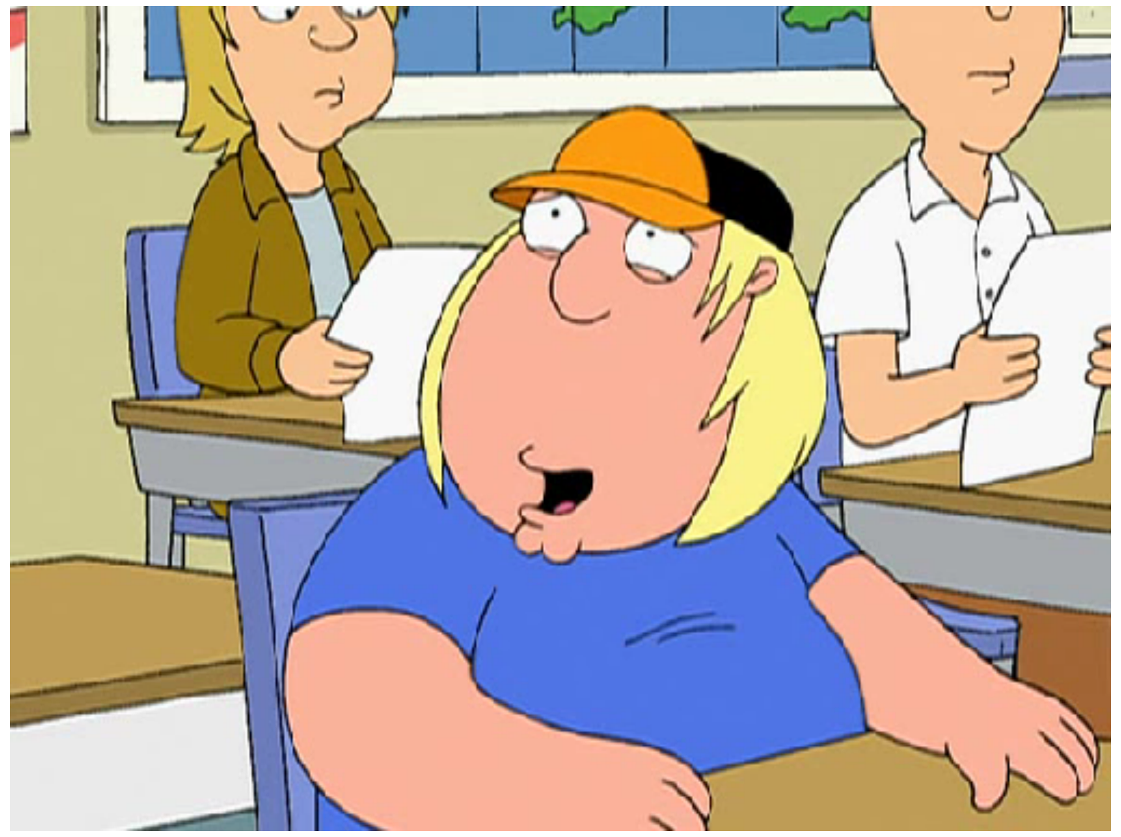
Analysis of DAgger

First let's do a quick introduction of online no-regret learning

[Vovk92,Warmuth94,Freund97,Zinkevich03,Kalai05,Hazan06,Kakade08]

Online Learning

Learner



convex Decision set $\mathcal{X}$

Adversary

...

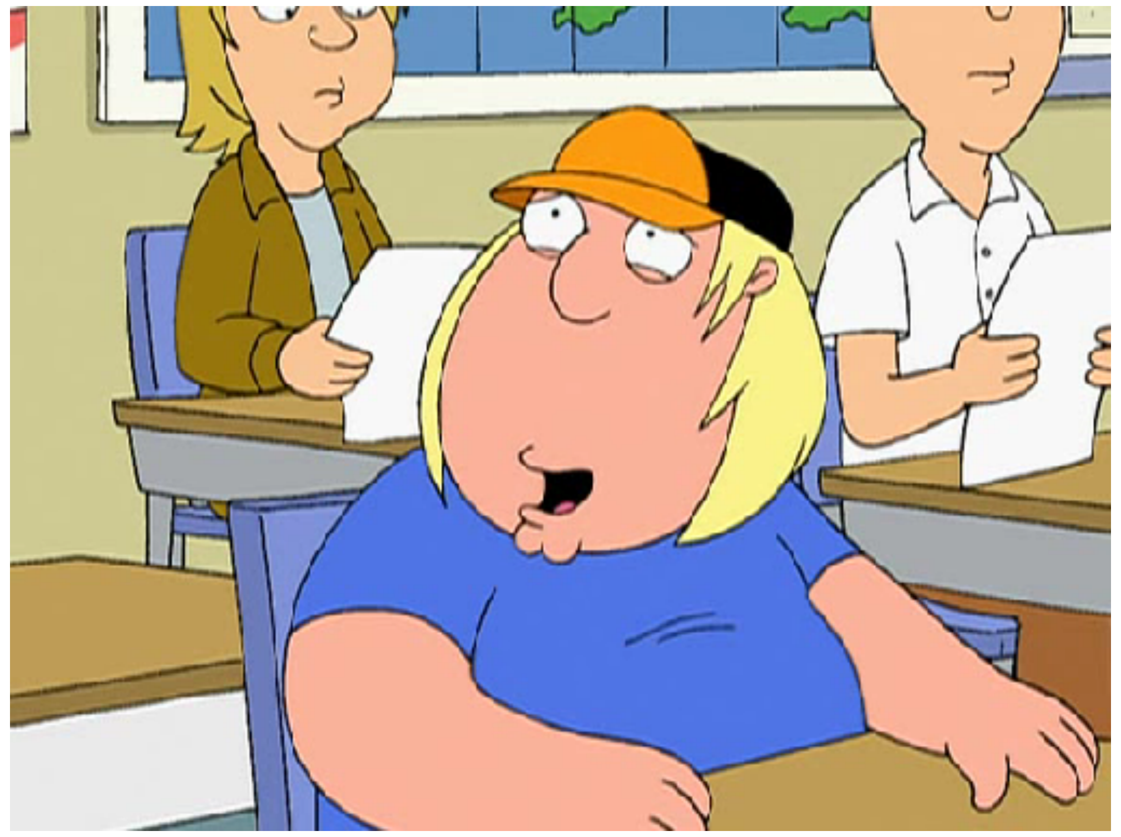
[Vovk92,Warmuth94,Freund97,Zinkevich03,Kalai05,Hazan06,Kakade08]

Online Learning

Learner

Learner picks a decision x_0

Adversary



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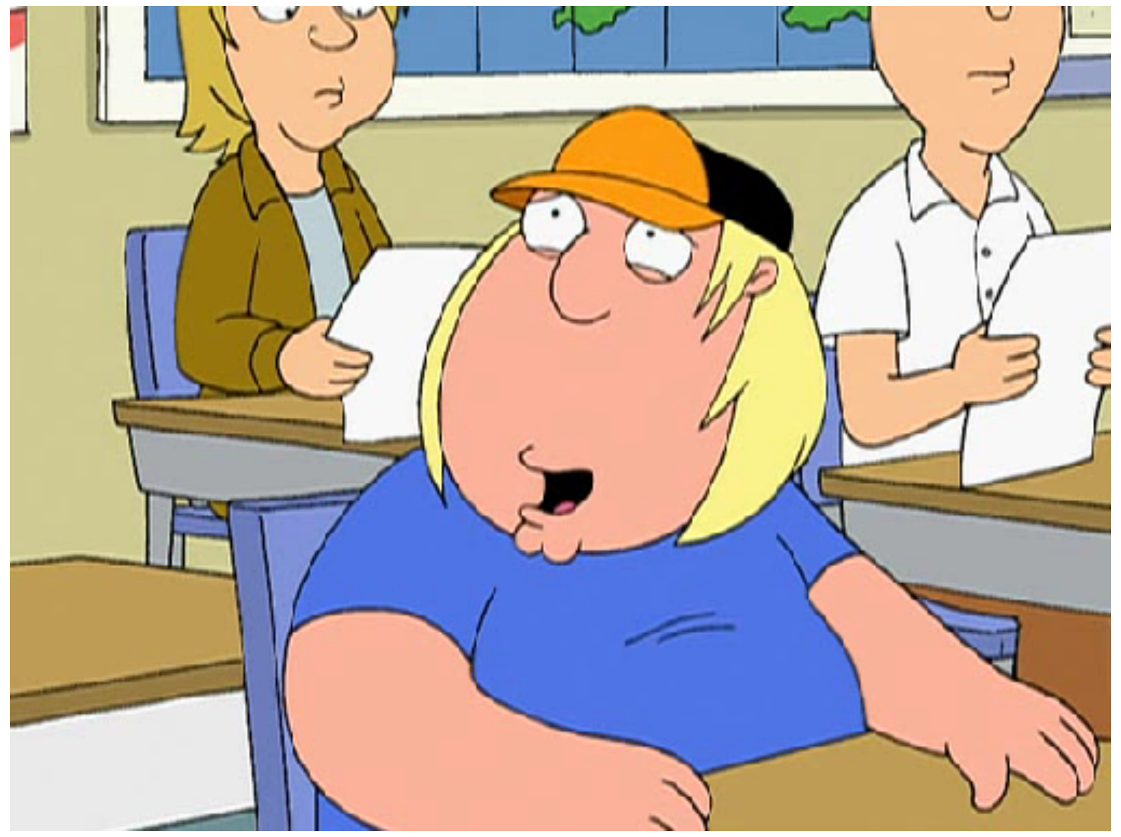

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Learner picks a decision x_0

Adversary picks a loss $\ell_0 : \mathscr{X} \to \mathbb{R}$

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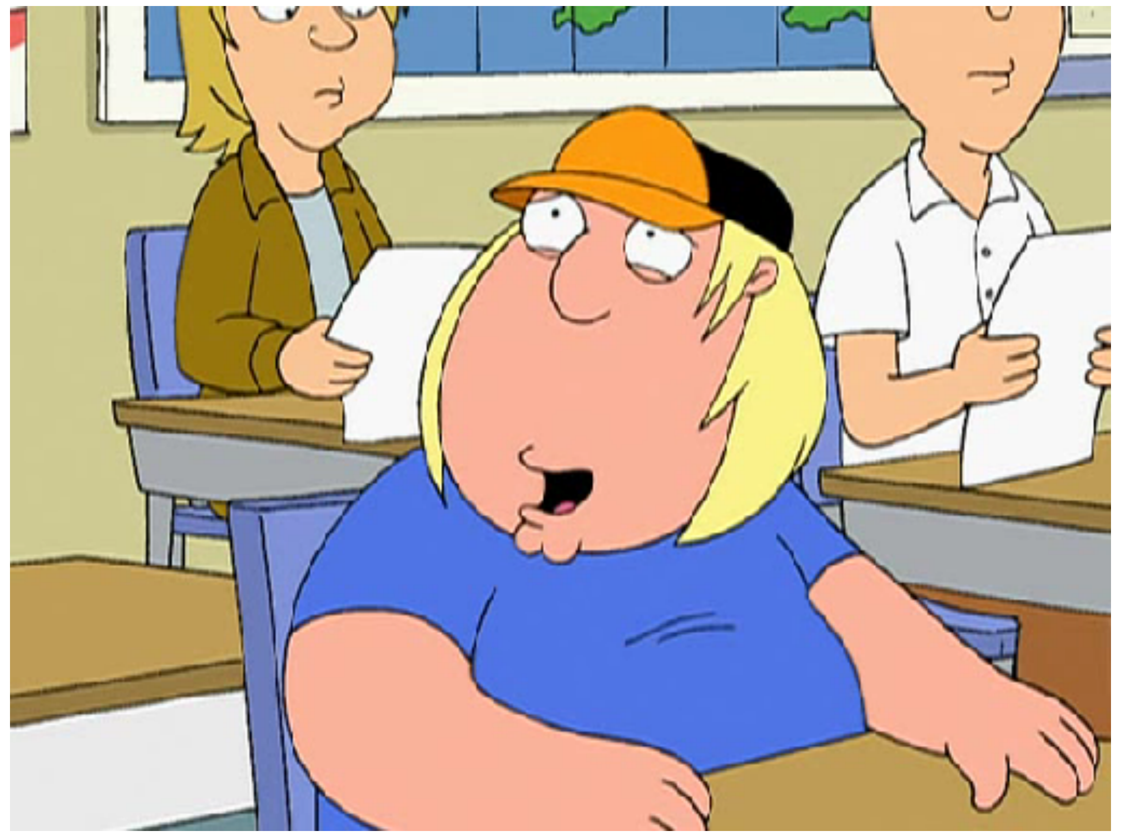
Learner



convex Decision set $\mathscr{X}$

Learner picks a decision x_0 →

← Adversary picks a loss $\ell_0 : \mathscr{X} \to \mathbb{R}$

Learner picks a new decision x_1 →

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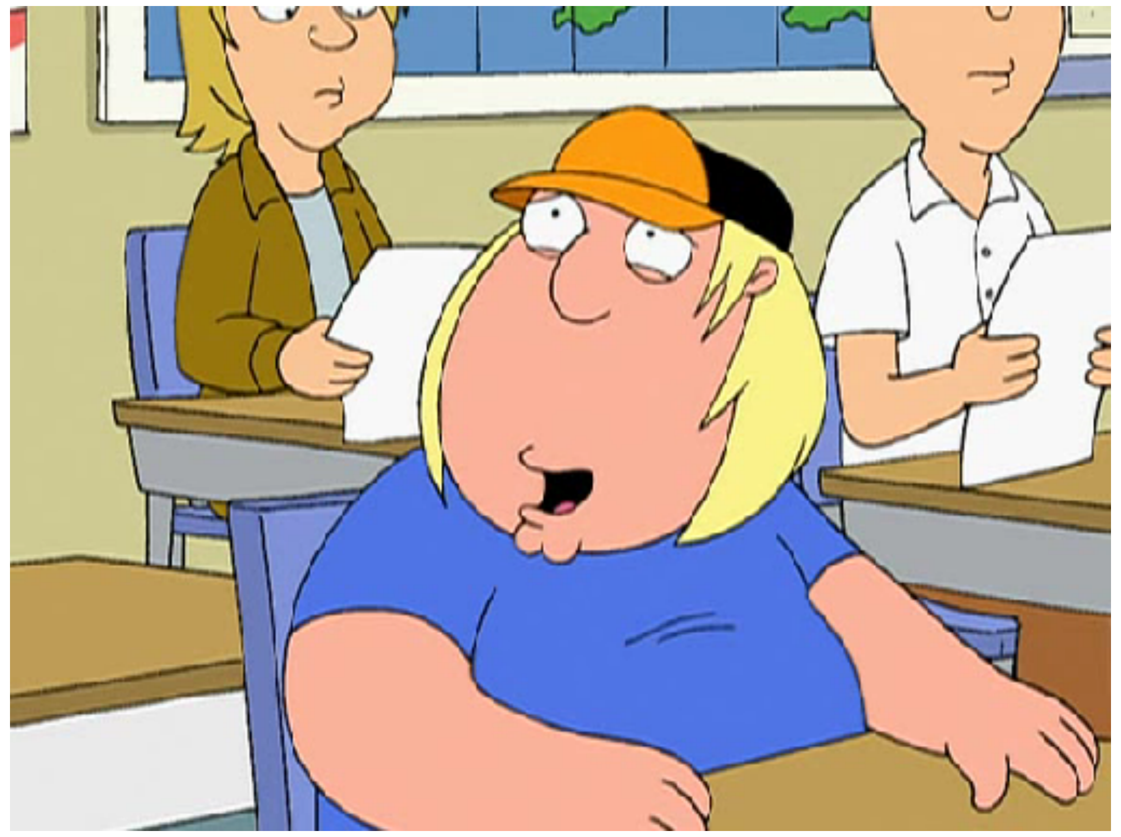

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Adversary picks a loss $\ell_0 : \mathscr{X} \to \mathbb{R}$ ←

Learner picks a new decision x_1 →

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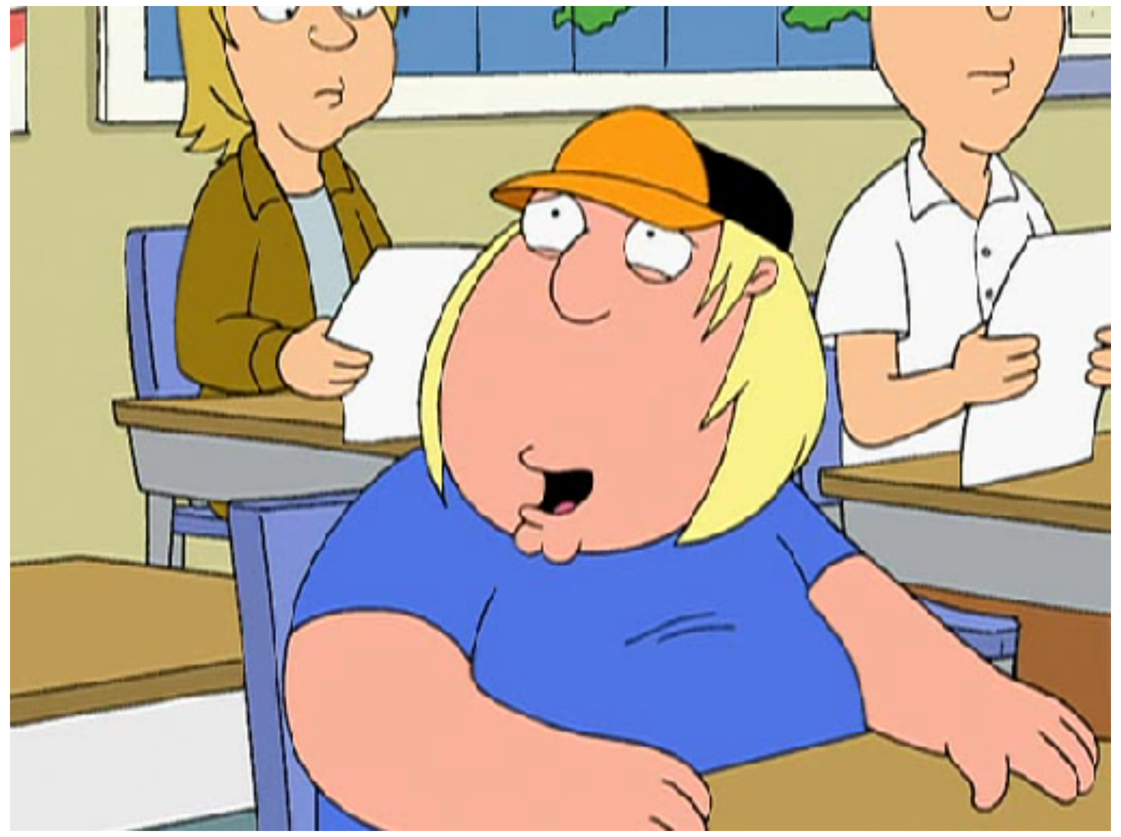

convex Decision set $\mathcal{X}$

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←

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→

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←

...

Adversary

$$\text{Regret} = \sum_{t=0}^{T-1} \ell_t(x_t) - \min_{x \in \mathcal{X}} \sum_{t=0}^{T-1} \ell_t(x)$$

A no-regret algorithm: Follow-the-Leader

At time step t, learner has seen $\ell_0, \ldots \ell_{t-1}$, which new decision she could pick?

$$\textbf{FTL: } x_t = \min_{x \in \mathcal{X}} \sum_{i=0}^{t-1} \ell_i(x)$$

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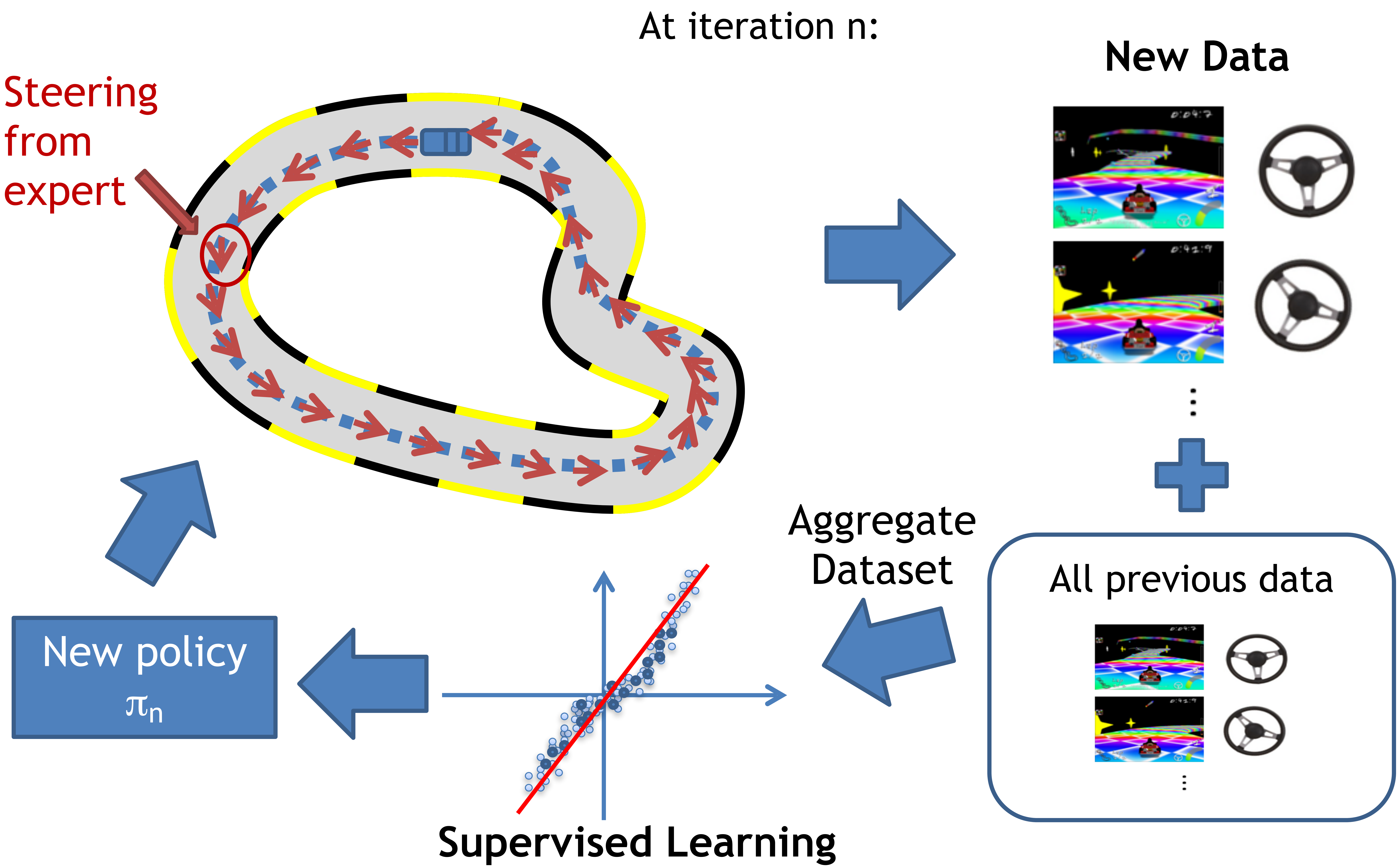
Theorem (FTL): if $\mathcal{X}$ is convex, and ℓ_t is strongly convex for all t, then for regret of FTL, we have:

$$\frac{1}{T}\left[\sum_{t=0}^{T-1} \ell_t(x_t) - \min_{x \in \mathcal{X}} \sum_{t=0}^{T-1} \ell_t(x)\right] = O\left(\frac{\log(T)}{T}\right)$$

DAgger Revisit



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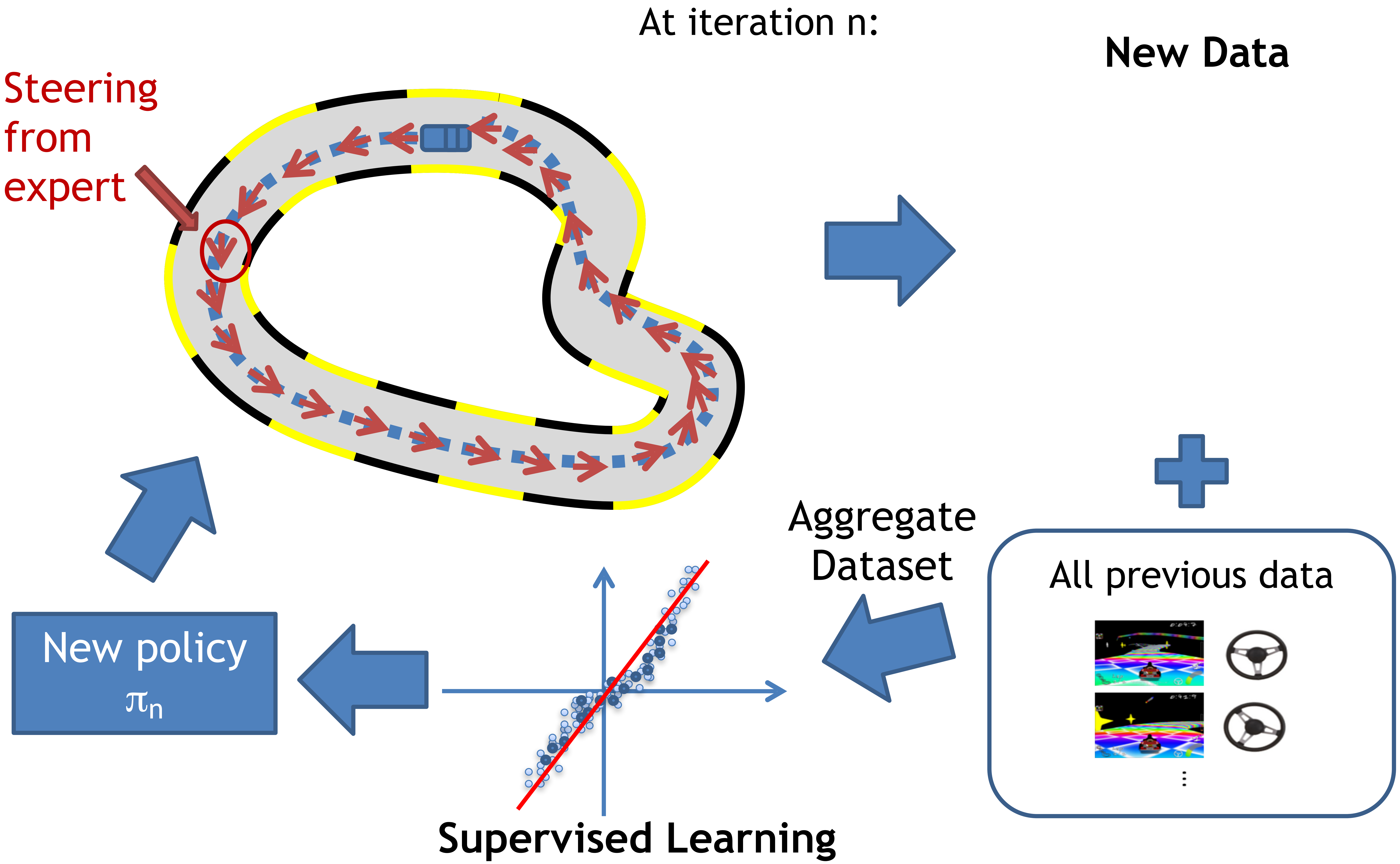

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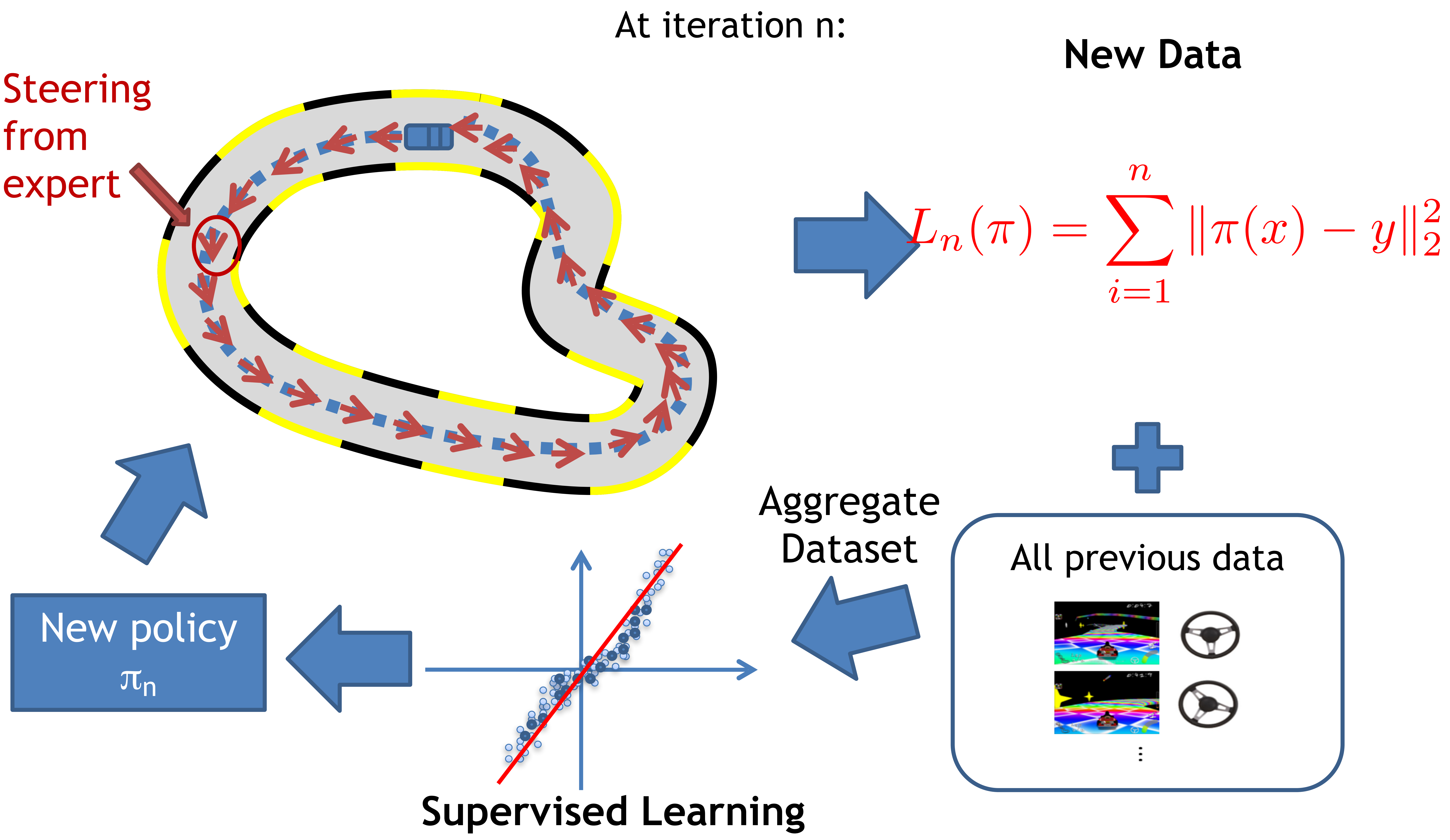

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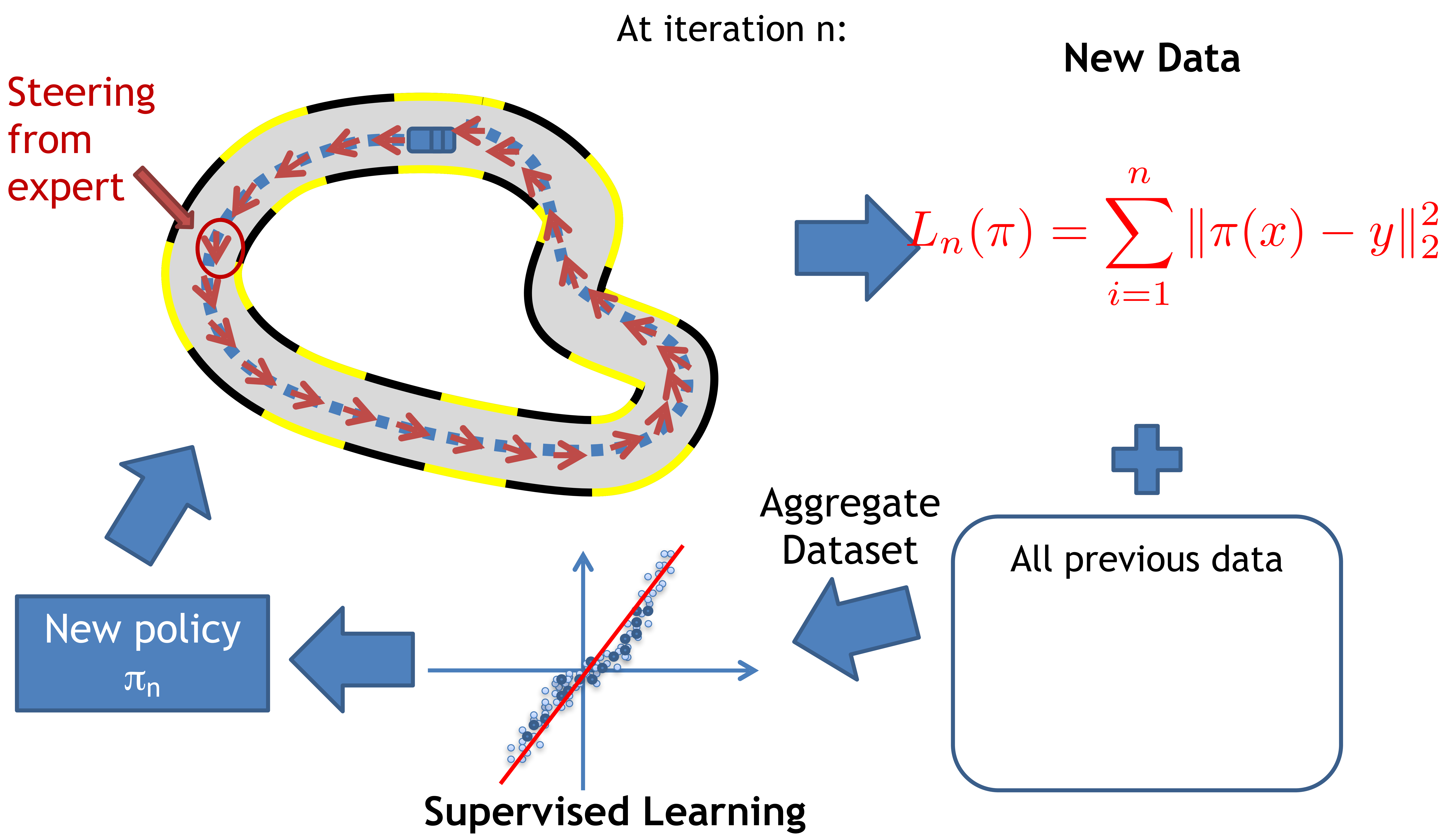

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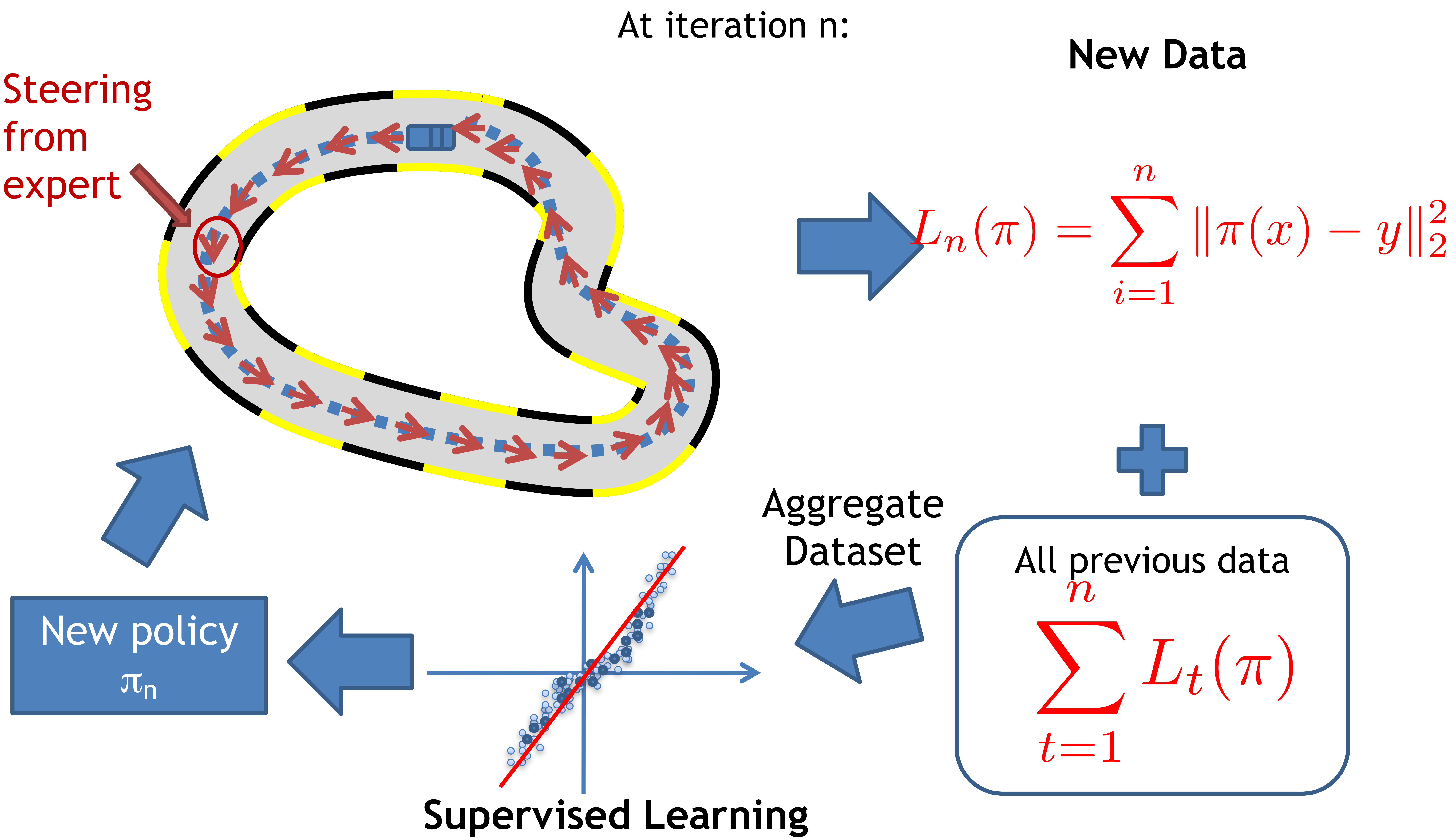
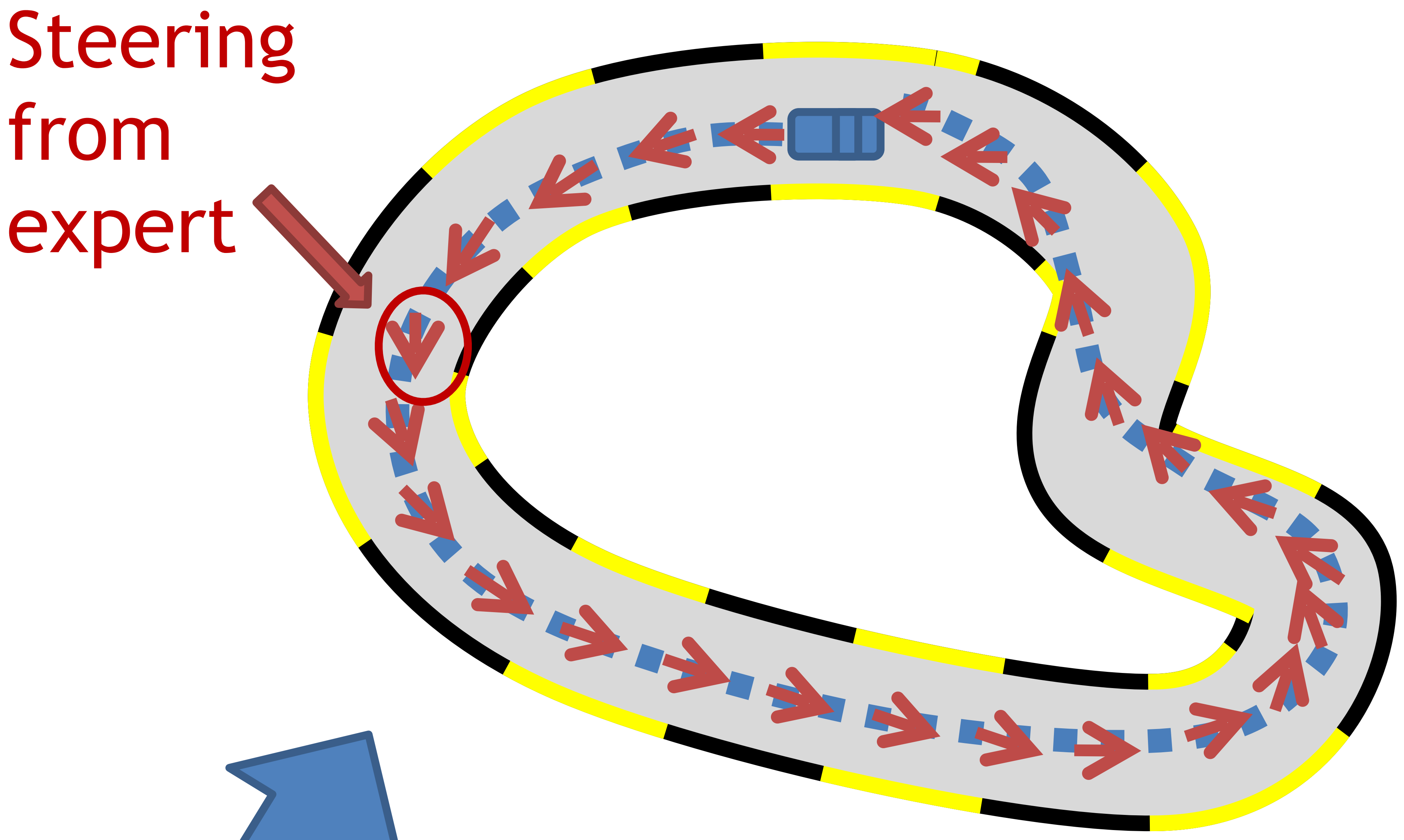


DAgger Revisit

At iteration n:



New Data

$$L_n(\pi) = \sum_{i=1}^{n} \|\pi(x) - y\|_2^2$$

All previous data

$$\sum_{t=1}^{n} L_t(\pi)$$

Aggregate Dataset

Supervised Learning

New policy π_n

DAgger Revisit

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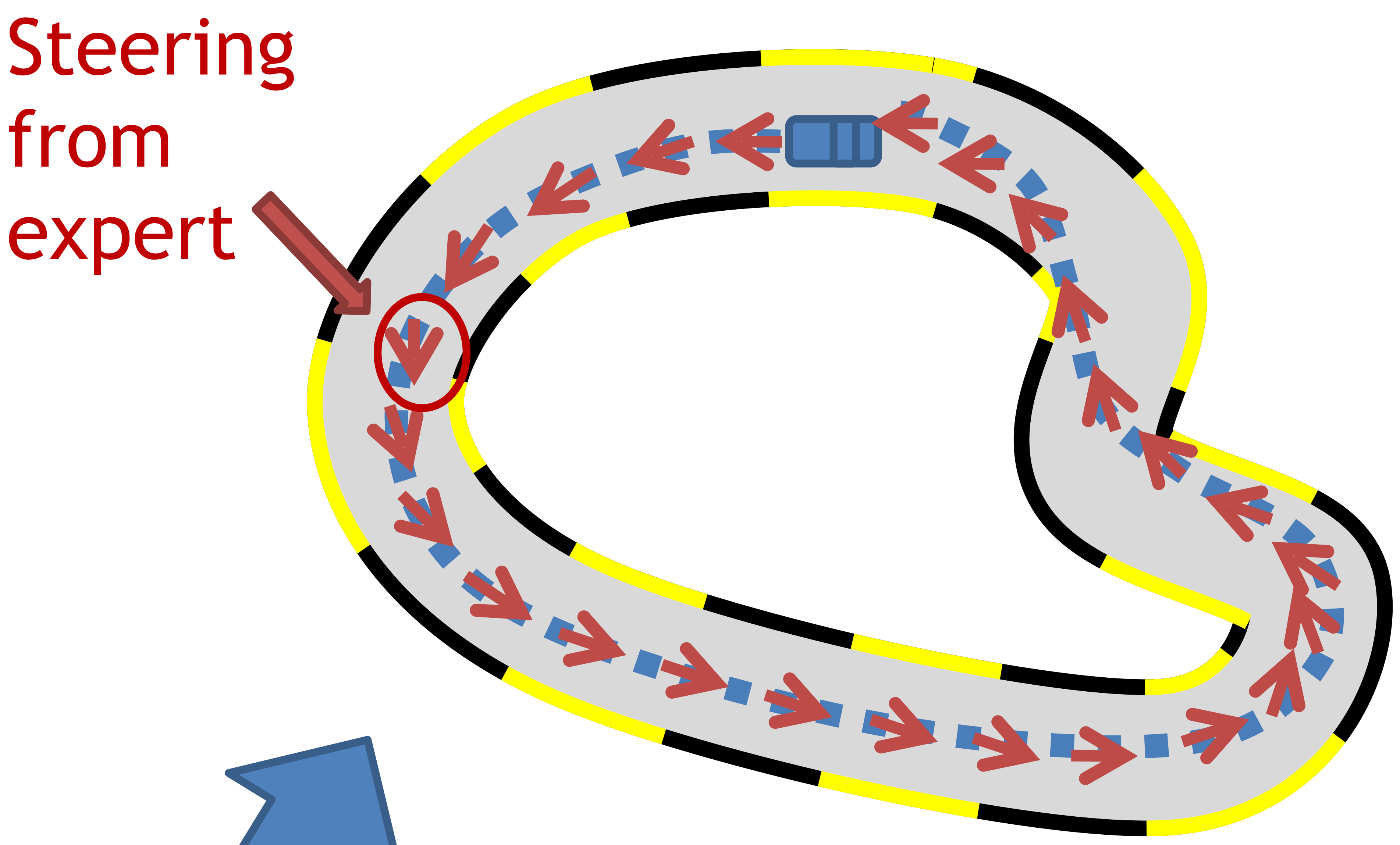

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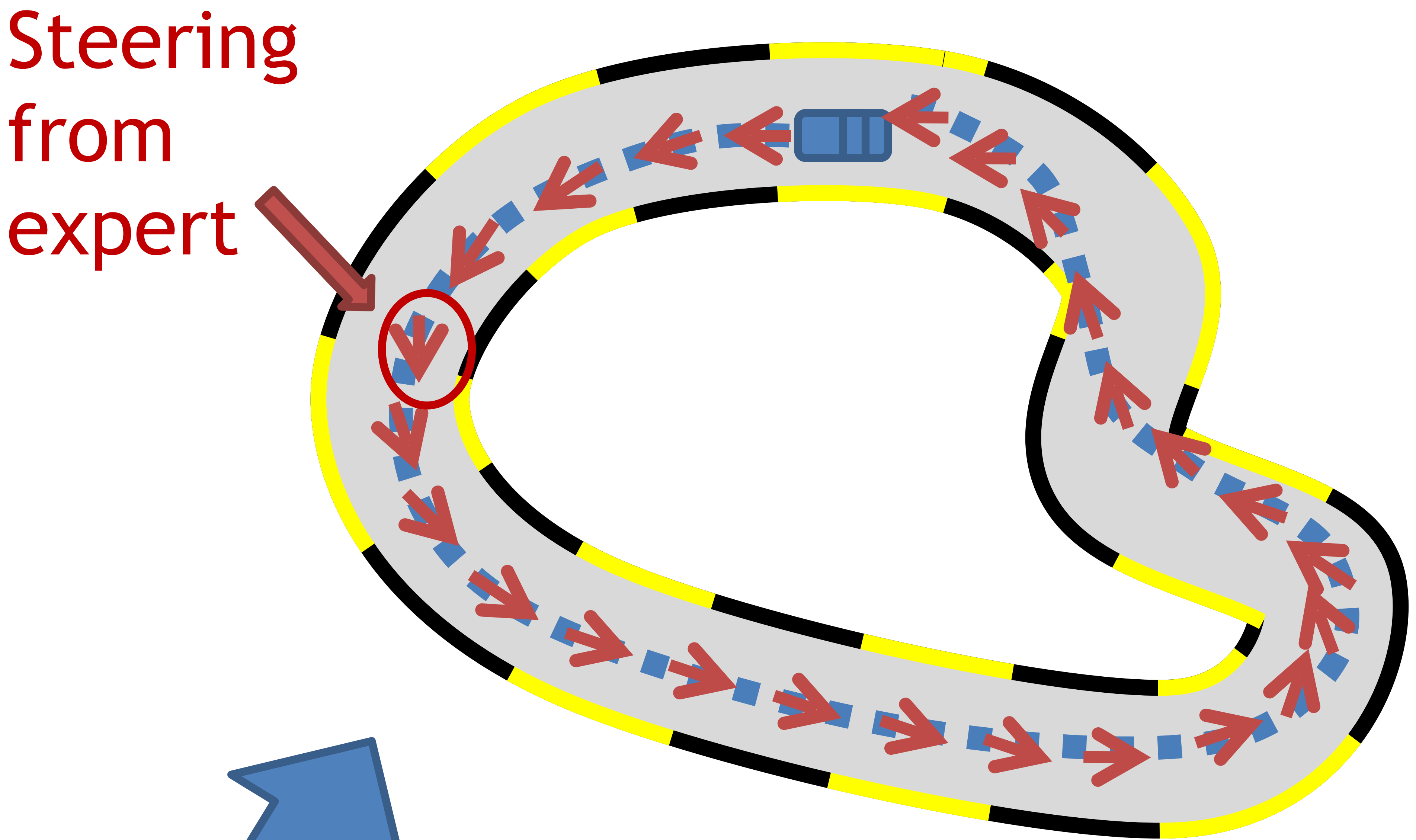

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Data Aggregation = Follow-the-Leader Online Learner

DAgger Analysis: A reduction to no-regret online learning

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Decision set $\Pi := \{\pi : S \mapsto A\}$ (restricted policy class, $\pi^\star$ may not be inside Π)

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(Here ℓ could be any convex surrogate loss for classification, .e.g, hinge loss)

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If the online learning procedure ensures no-regret, then

$$\frac{1}{T}\left[\sum_{t=0}^{T-1} \ell_t(\pi_t) - \min_{\pi\in\Pi} \sum_{t=0}^{T-1} \ell_t(\pi)\right] = o(T)/T$$

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Under the assumption that surrogate loss upper bounds zero-one loss:

$$\mathbb{E}_{s \sim d^{\pi_{\hat{t}}}} \left[\pi_{\hat{t}}(s) \neq \pi^\star(s) \right] \leq \mathbb{E}_{s \sim d^{\pi_{\hat{t}}}} \left[\ell(\pi_{\hat{t}}(s), \pi^\star(s)) \right] \leq \epsilon_{avg-reg} + \epsilon_\Pi$$

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