Planning in MDPs

Wen Sun

CS 6789: Foundations of Reinforcement Learning

Recap: Infinite Horizon MDPs

$$\mathcal{M} = \{S, A, P, r, \gamma\}$$

$$P : S \times A \mapsto \Delta(S), \quad r : S \times A \to [0,1], \quad \gamma \in [0,1)$$

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Value function $V^\pi(s) = \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r(s_h, a_h) \middle| s_0 = s, a_h \sim \pi(s_h), s_{h+1} \sim P(\cdot \mid s_h, a_h)\right]$

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Q function $Q^\pi(s, a) = \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r(s_h, a_h) \middle| (s_0, a_0) = (s, a), a_h \sim \pi(s_h), s_{h+1} \sim P(\cdot \mid s_h, a_h)\right]$

Recap: Bellman Optimality

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$$P : S \times A \mapsto \Delta(S), \quad r : S \times A \to [0,1], \quad \gamma \in [0,1)$$

Theorem 1: Bellman Optimality (Q-version)

$$Q^\star(s, a) = r(s, a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[\max_{a' \in A} Q^\star(s', a') \right]$$

Main Question for Today:

Given an MDP $\mathscr{M} = (S, A, P, r, \gamma)$, How to find $\pi^\star$ (stationary & deterministic)

Outline

1. Bellman optimality — property of $V^\star$

2. Optimal planning: Value Iteration

Bellman Optimality

Theorem 2:

For any $V : S \to \mathbb{R}$, if $V(s) = \max_a \left[r(s,a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} V(s') \right]$ for all s,

then $V(s) = V^\star(s), \forall s$

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$$|V(s) - V^\star(s)| = \left| \max_a (r(s,a) + \gamma \mathbb{E}_{s' \sim P(s,a)} V(s')) - \max_a (r(s,a) + \gamma \mathbb{E}_{s' \sim P(s,a)} V^\star(s')) \right|$$

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$$\leq \max_a \left| (r(s,a) + \gamma \mathbb{E}_{s' \sim P(s,a)} V(s')) - (r(s,a) + \gamma \mathbb{E}_{s' \sim P(s,a)} V^\star(s')) \right|$$

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$$\leq \max_a \gamma \mathbb{E}_{s' \sim P(s,a)} \left| V(s') - V^\star(s') \right|$$

$$\leq \max_a \gamma \mathbb{E}_{s' \sim P(s,a)} \left(\max_{a'} \gamma \mathbb{E}_{s'' \sim P(s',a')} \left| V(s'') - V^\star(s'') \right| \right)$$

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\begin{aligned}
|V(s) - V^\star(s)| &= \left| \max_a (r(s,a) + \gamma \mathbb{E}_{s' \sim P(s,a)} V(s')) - \max_a (r(s,a) + \gamma \mathbb{E}_{s' \sim P(s,a)} V^\star(s')) \right| \\
&\le \max_a \left| (r(s,a) + \gamma \mathbb{E}_{s' \sim P(s,a)} V(s')) - (r(s,a) + \gamma \mathbb{E}_{s' \sim P(s,a)} V^\star(s')) \right| \\
&\le \max_a \gamma \mathbb{E}_{s' \sim P(s,a)} \left| V(s') - V^\star(s') \right| \\
&\le \max_a \gamma \mathbb{E}_{s' \sim P(s,a)} \left(\max_{a'} \gamma \mathbb{E}_{s'' \sim P(s',a')} \left| V(s'') - V^\star(s'') \right| \right) \\
&\le \max_{a_1, a_2, \ldots a_{k-1}} \gamma^k \mathbb{E}_{s_k} |V(s_k) - V^\star(s_k)|
\end{aligned}
$$

Bellman Optimality for $Q^\star$

What about $Q^\star$?

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We should have:

For any $Q : S \times A \to \mathbb{R}$, if $Q(s, a) = r(s, a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} \max_{a'} Q(s', a')$

for all s, then $Q(s, a) = Q^\star(s, a), \forall s, a$

Outline

1. Bellman optimality — property of $V^\star$

2. Optimal planning: Value Iteration

Define Bellman Operator $\mathscr{T}$:

Given a function $f : S \times A \mapsto \mathbb{R}$,

$$\mathscr{T}f : S \times A \mapsto \mathbb{R},$$

$$\left(\mathscr{T}f\right)(s,a) := r(s,a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} \max_{a' \in A} f(s',a'), \forall s,a \in S \times A$$

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Q: what is $\mathcal{T} Q^\star$?

Value Iteration Algorithm:

1. Initialization: $Q^0 : \|Q^0\|_\infty \in (0, \frac{1}{1-\gamma})$

2. Iterate until convergence: $Q^{t+1} = \mathcal{T} Q^t$

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Via Bellman optimality theorem:

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$$x_0, x_{t+1} = \ell(x_t), t = 0, \ldots,$$

$$|x_t - x^\star| = |\ell(x_{t-1}) - \ell(x^\star)| \leq L |x_{t-1} - x^\star|$$

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If $L < 1$ (i.e., contraction), then it converges exponentially fast

Convergence of Value Iteration:

Lemma [contraction]: Given any Q, Q', we have:

$$\|\mathcal{T}Q - \mathcal{T}Q'\|_\infty \leq \gamma \|Q - Q'\|_\infty$$

Proof:

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Final Quality of the Policy:

$$\pi^t : \pi^t(s) = \arg\max_a Q^t(s, a)$$

Theorem: $V^{\pi^t}(s) \geq V^\star(s) - \frac{2\gamma^t}{1-\gamma}\|Q^0 - Q^\star\|_\infty \forall s \in S$

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$$\geq \gamma\mathbb{E}_{s'\sim P(s,\pi^t(s))}\left(V^{\pi^t}(s') - V^\star(s')\right) - 2\gamma^t\|Q^0 - Q^\star\|_\infty$$ ...Recursion

Summary for today

Planning algorithm (no learning so far):

VI: fixed point iteration $Q^{t+1} = \mathcal{T} Q^t$

1. Bellman operator is a contraction map

2. $||Q^t - Q^\star||_\infty$ being small implies V^{π^t} & $V^\star$ are close