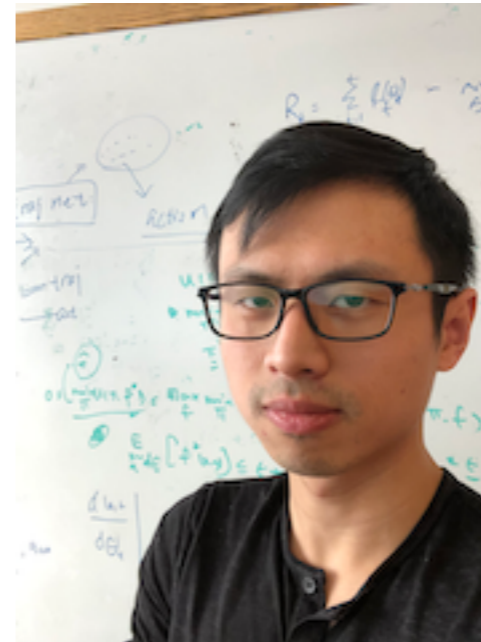


# Pessimistic Model-based Offline Reinforcement Learning under Partial Coverage

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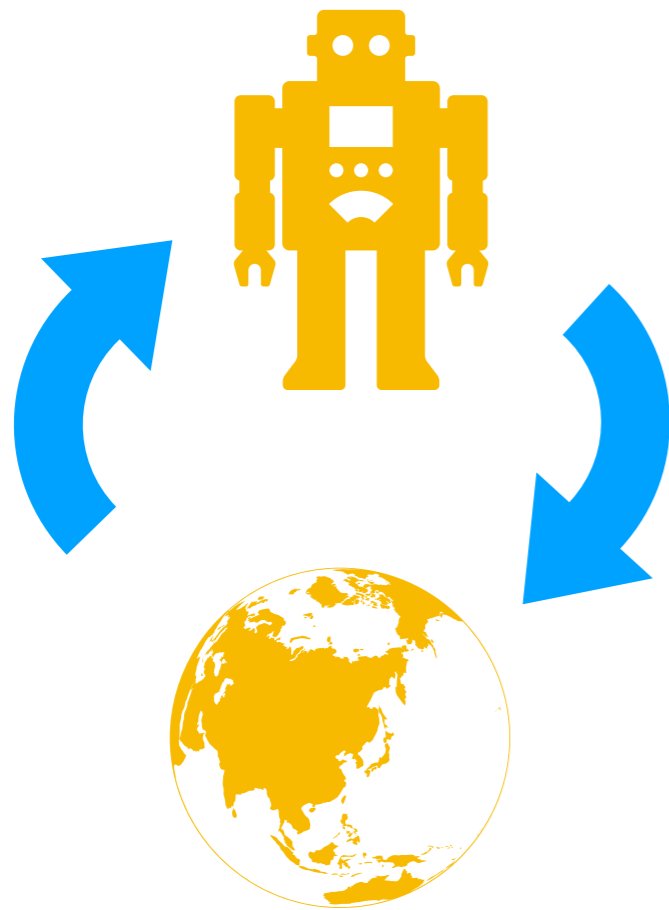


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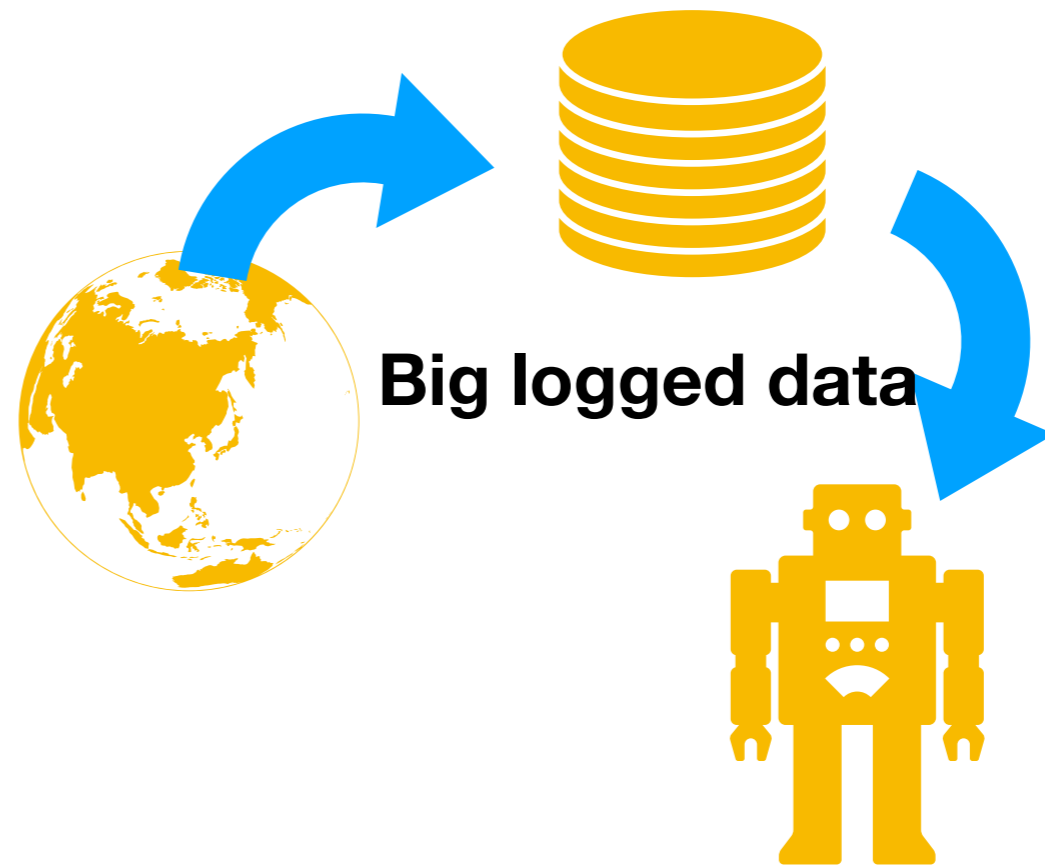
- **1: Overview**
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# Offline RL

## Online RL



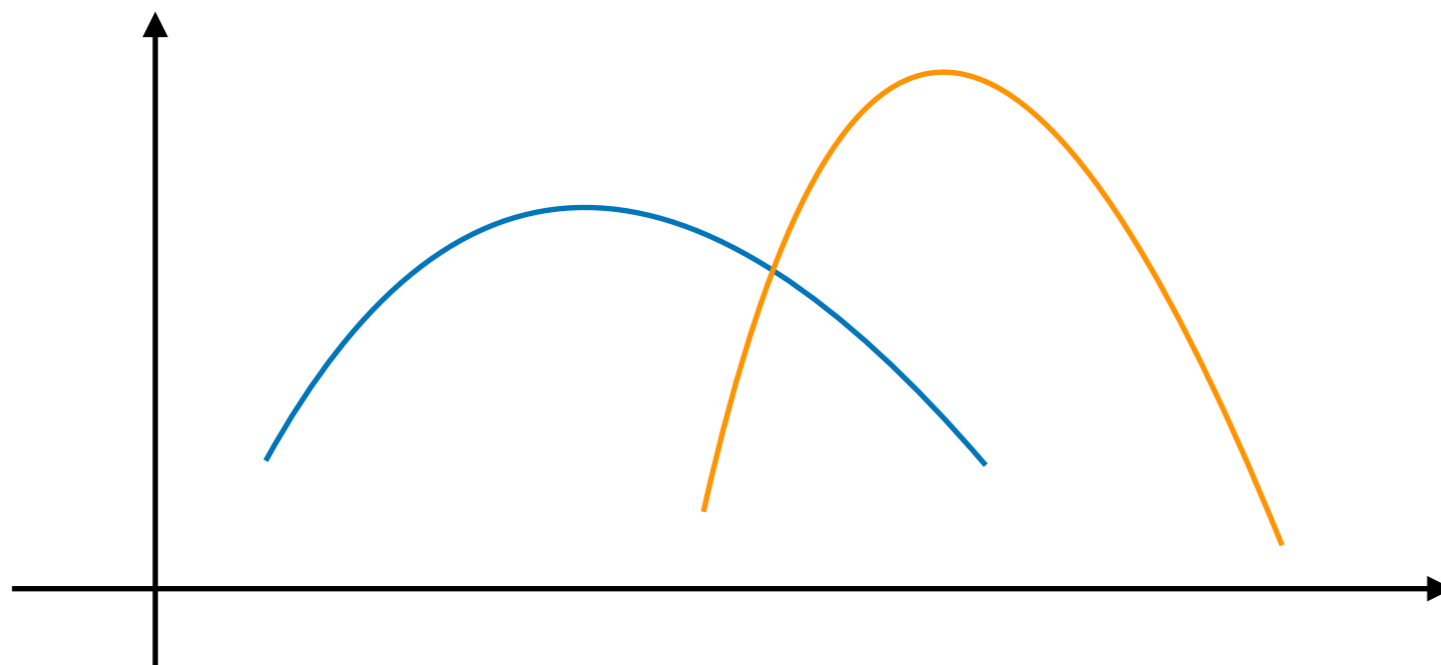
## Offline RL



- We only have access to logged data.
- We want to learn high-quality policies from the logged data.

# Question

- Unfortunately, the offline data is often not exploratory.
- Q. Can we still learn good policies when the offline data is not fully exploratory? (with **realizability** of the model)



Offline data:  $\rho(s, a)$ .

Distribution induced by a policy  $\pi$ ,  $d^\pi(s, a)$

# Global vs. Partial Coverage

- Most of offline RL works assume **global coverage**. Under  $\max_{s,a} \frac{d^\pi(s,a)}{\rho(s,a)} < \infty \quad \forall \pi$ ,

they show the learned policy  $\hat{\pi}$  can compete with the global optimal policy [MS, 2008].

\*  $V^{\pi(P^*)} - V^{\hat{\pi}} = \text{Small}$ .  
(  $\pi(P^*)$  is the optimal policy.  $V^\pi$  is the policy value of  $\pi$ .)

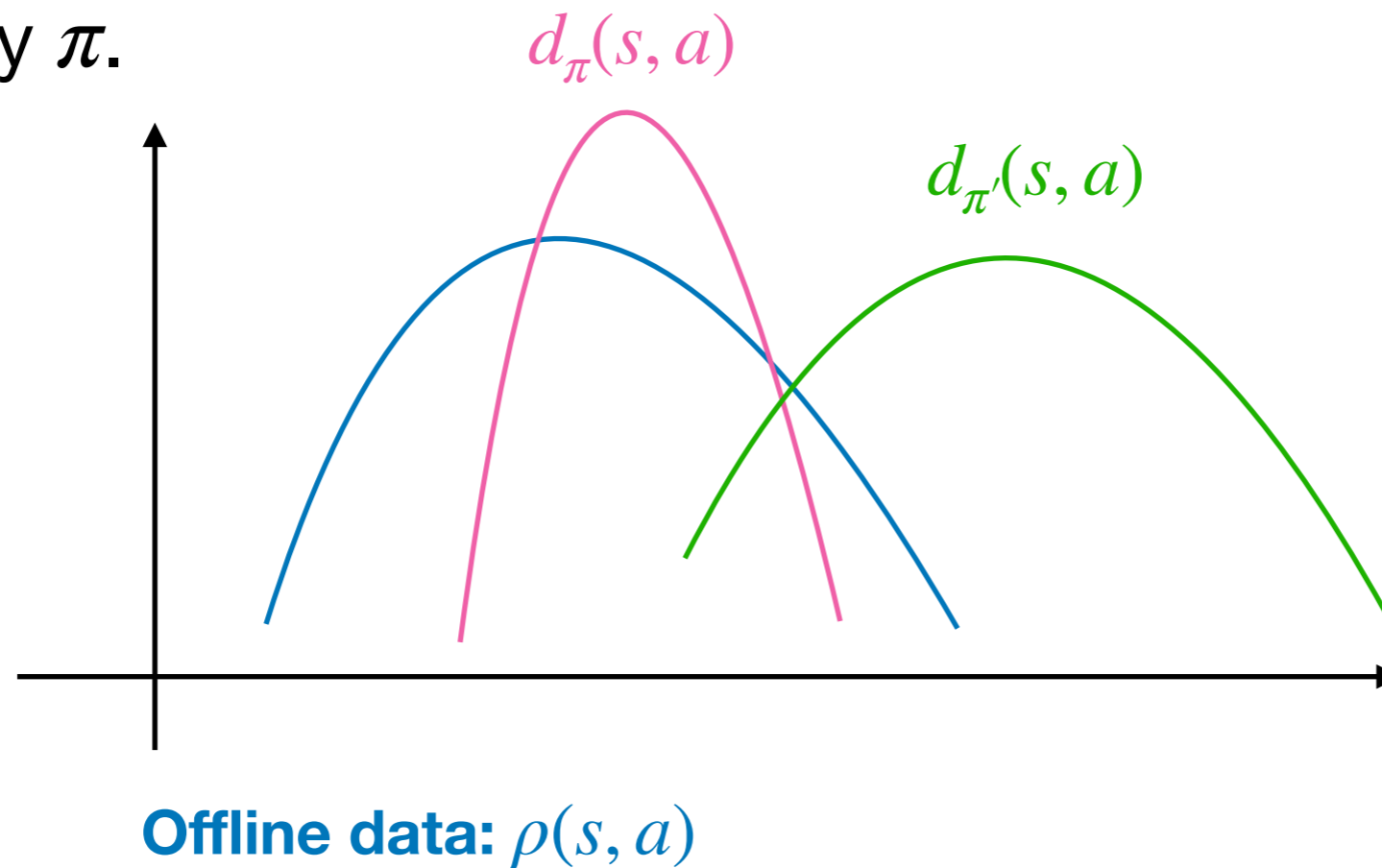
- In this work, we want to show results under **partial coverage**. We want to show the output policy can compete with any policies

$$\pi \text{ s.t. } \max_{s,a} \frac{d^\pi(s,a)}{\rho(s,a)} < \infty.$$

\*  $V^\pi - V^{\hat{\pi}} = \text{Small}$  for any  $\pi$  covered by offline data.

# Global vs. Partial Coverage

- Global coverage is not satisfied in the following. ( $\pi'$  is not covered by offline data)
- But, under partial coverage, we can still compete with a policy  $\pi$ .



# What We Know So Far.

- There are many works under global coverage [MS, 2008].
- In **particular (linear)** models, there exists a model-based algorithm under partial coverage [CUSKS21]. **But not for any models!**
- Several papers under partial coverage in the model-free setting [RZMIR21, JYW21, ZCZS21, X CJMA21, ZWB21], which assume **completeness** as well as realizability.

# What We Show

- We propose a model-based offline RL algorithm CPPO. We show the PAC guarantee under partial coverage assuming the realizability of the model.
- This works for **any** MDPs 😊.
- When we have more structures, the density-ratio based partial coverage concept is refined.
  - Examples: **linear mixture MDPs**, KNRs, **low-rank MDPs** (models with unknown features), **factored MDPs**.



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# Notation

- MDP:  $\langle \mathcal{S}, \mathcal{A}, r, P, \gamma, d_0 \rangle$ . Discount factor  $\gamma \in [0, 1)$ ,  $\mathcal{S}$ : State space,  $\mathcal{A}$ : Action space.

**Transition Dynamics**

$$P : \mathcal{S} \times \mathcal{A} \rightarrow \Delta(\mathcal{S})$$

**Reward function**

$$r : \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$$

**Initial distribution**

$$d_0 \in \Delta(\mathcal{S})$$

- We have an offline dataset:  $\mathcal{D} = \{s^{(i)}, a^{(i)}, s'^{(i)}\}_{i=1}^n$  following  $(s, a) \sim \rho, s' \sim P^\star(s, a)$ . ( $P^\star$  is the true unknown transition density)

- $d^\pi = (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t d_t^\pi$  is a state action discounted occupancy

distribution under  $\pi$  and  $P^\star$ .

- $V_P^\pi$  is an expected cumulative reward of  $\pi$  under P:

$$\mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r_h \mid s_0 \sim d_0, a_0 \sim \pi(s_0), s_1 \sim P(s_0, a_0), \dots\right].$$

# Function Classes We Use

- We need two function classes:
  - Model class  $\mathbf{M}$  (  $\subset \{ \mathcal{S} \times \mathcal{A} \rightarrow \Delta(\mathcal{S}) \}$  ) to learn the true transition  $P^*$  .
  - Policy class  $\mathbf{\Pi}$  (  $\subset \{ \mathcal{S} \rightarrow \Delta(\mathcal{A}) \}$  ). Throughout this presentation, this is the unrestricted policy class.

# Model-based RL

**Step 1: MLE.**  $\hat{P}_{\text{MLE}} = \operatorname{argmax}_{P \in \mathcal{M}} \sum_{i=1}^n \log P(s^{(i)} | s^{(i)}, a^{(i)})$ .

**Step 2: Policy Optimization.**  $\hat{\pi} = \operatorname{argmax}_{\pi \in \Pi} V_{\hat{P}_{\text{MLE}}}^{\pi}$ .

- Under global coverage 🥲 ( $\max_{s,a} \frac{d^{\pi}(s,a)}{\rho(s,a)} \leq C, \forall \pi$ ), the output can compete with the global optimal policy  $\pi(P^{\star})$  with  $1 - \delta$ :  
 $V_{P^{\star}}^{\pi(P^{\star})} - V_{\hat{P}_{\text{MLE}}}^{\hat{\pi}} = O((1 - \gamma)^{-2} \sqrt{C \ln(|\mathcal{M}|/\delta)/n})$ .

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# Algorithm

## CPPO: Constrained Pessimistic Policy Optimization

$$\text{Step 1: MLE. } \hat{P}_{\text{MLE}} = \operatorname{argmax}_{P \in M} \sum_{i=1}^n \log P(s^{(i)} | s^{(i)}, a^{(i)}).$$

Step 2: Solve constrained Optimization.

$$\hat{\pi} = \operatorname{argmax}_{\pi \in \Pi} \min_{P \in M_{\mathcal{D}}} V_P^{\pi} \text{ where}$$
$$M_{\mathcal{D}} = \left\{ P \mid P \in M, \frac{1}{n} \sum_{i=1}^n \|\hat{P}_{\text{MLE}}(\cdot | s^{(i)}, a^{(i)}) - P(\cdot | s^{(i)}, a^{(i)})\|_1^2 \leq \xi \right\}.$$

\*  $\xi$  is a hyperparameter

- Search for the least favorable model in terms of  $V_P^{\pi}$  that is feasible w.r.t the constraint.
- Why? Pessimistic principle (being conservative on uncovered regions) is employed.

# Model-based Concentrability Coefficient

[Definition] Model-based concentrability coefficient:

$$C_{\pi}^{\dagger} = \sup_{P' \in \mathcal{M}} \frac{\mathbb{E}_{(s,a) \sim d^{\pi}} [\|P'(\cdot | s, a) - P(\cdot | s, a)\|_1^2]}{\mathbb{E}_{(s,a) \sim \rho} [\|(P'(\cdot | s, a) - P(\cdot | s, a))\|_1^2]} .$$

- Smaller than the density ratio:  $C_{\pi}^{\dagger} \leq \max_{s,a} d^{\pi}(s, a) / \rho(s, a)$ .
- Adaptive to model classes. If the model class is small,  $C_{\pi}^{\dagger}$  is small either.

# Guarantee of CPPPO

[PAC Bound for CPPPO] Suppose  $P^\star \in \mathcal{M}$ . (by choosing  $\xi$  properly)

With probability  $1 - \delta$ ,

$$\forall \pi^\star; V_{P^\star}^{\pi^\star} - V_{P^\star}^{\hat{\pi}} = O\left((1 - \gamma)^{-2} \sqrt{C_{\pi^\star}^\dagger \ln(|\mathcal{M}|/\delta)/n}\right).$$

- The output can **simultaneously** compete with any comparator policies satisfying partial coverage  $C_{\pi^\star}^\dagger < \infty$ .
- Even if  $\pi^\star$  is the optimal policy  $\pi(P^\star)$ ,  $C_{\pi(P^\star)}^\dagger < \infty$  is still weaker than the global coverage  $(\max_{s,a} d^\pi(s, a)/\rho(s, a) < \infty, \forall \pi)$ .
- When  $|\mathcal{M}|$  is infinite, we can still use localized Rademacher complexities.



# Derivation

- Define  $\hat{V}^\pi = \min_{P \in M_D} V_P^\pi$ . then,  $\hat{\pi} = \operatorname{argmax}_\pi \hat{V}^\pi$ .
- We can show  $P^\star \in M_D$  in high probability.
- We have  $\hat{V}^\pi \leq V_{P^\star}^\pi, \forall \pi \in \Pi$  (**Pessimism**).
- $$V_{P^\star}^{\pi^\star} - V_{P^\star}^{\hat{\pi}} = V_{P^\star}^{\pi^\star} - \hat{V}^{\pi^\star} + \hat{V}^{\pi^\star} - V_{P^\star}^{\hat{\pi}} \leq V_{P^\star}^{\pi^\star} - \hat{V}^{\pi^\star} + \hat{V}^{\hat{\pi}} - V_{P^\star}^{\hat{\pi}} \leq V_{P^\star}^{\pi^\star} - \hat{V}^{\pi^\star}.$$

**Definition of  $\hat{\pi}$ .** **Pessimism.**
- Finally, use performance difference lemma. Done 🍌

# Model free vs. Model-based

- The error in CPPO does not include  $|\Pi|$ . As a result, the policy class  $\Pi$  can be **unrestricted**. More strongly, we can compete with **any history dependent policies**.
- [XCJMA21] shows the PAC guarantee under partial coverage, realizability and **Bellman completeness** of Q-function class for any policy in  $\Pi$ , i.e. ,  $\mathcal{T}^\pi Q \subset Q$ .
  - \*  $\mathcal{T}^\pi$  is the **Bellman operator** for a policy  $\pi$ .
- Thus,  **$\Pi$  needs to be generally restricted** .
- It cannot compete with history dependent policies.

# Comparison to Existing Pessimistic Algorithms

- CPPPO use the MLE guarantee:

$$\mathbb{E}_{(s,a) \sim \rho} [\|\hat{P}_{\text{MLE}}(\cdot | s, a) - P^*(\cdot | s, a)\|_1^2] \lesssim \sqrt{\ln |M| / \delta} / n .$$

- For **linear** models, [CUSKS21, JYW21] (existing offline RL papers using negative bonus terms) use

$$\text{Distance}(\hat{P}(\cdot | s, a), P^*(\cdot | s, a))^2 \lesssim \text{Poly}(1/n, \ln(1/\delta), \dots), \forall (s, a) .$$

- **Average error (over offline data) guarantees** are weaker than **pointwise error guarantees** 😞
- But **average error guarantees** are enough for the pessimism and obtained for any **nonlinear** models 😊

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# Next Questions

- $C_{\pi^*}^\dagger$  is very abstract. Can we replace it with more interpretable quantities? (and tighter than the density ratio.)
- To see them, we analyze on four models:
  - Linear mixture MDPs (including linear MDPs),
  - KNRs (generalization of LQRs),
  - Low-rank MDPs (with unknown features),
  - Factored MDPs.

# 1: Linear MDPs

Definition: Linear MDPs [YW20]

The true  $P^\star$  is  $\mu^\top(s')M^\star\phi(s, a)$  ( Unknown  $M^\star \in \mathbb{R}^{d_1 \times d_2}$  ) given feature vectors  $\phi(s, a) : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}^{d_2}$ ,  $\mu(s) : \mathcal{S} \rightarrow \mathbb{R}^{d_1}$ .



$|\mathcal{S}| \times |\mathcal{S}| |\mathcal{A}|$

=



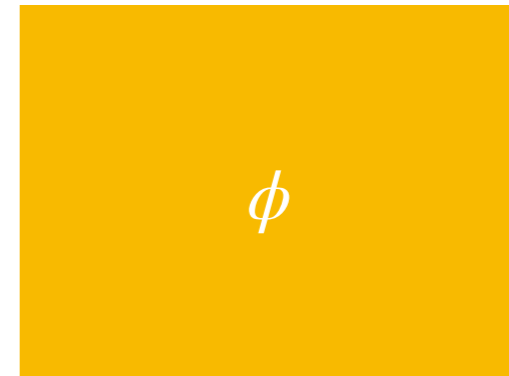
$|\mathcal{S}| \times d_1$

×



$d_1 \times d_2$

×



$d_2 \times |\mathcal{S}| |\mathcal{A}|$

# 1: Linear MDPs

Definition: Linear MDPs [YW20]

The true  $P^*$  is  $\mu^\top(s')M^*\phi(s, a)$  ( Unknown  $M^* \in \mathbb{R}^{d_1 \times d_2}$  ) given feature vectors  $\phi(s, a) : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}^{d_2}$ ,  $\mu(s) : \mathcal{S} \rightarrow \mathbb{R}^{d_1}$ .

[Concentrability Coefficient for Linear MDPs]

$$\bar{C}_{\pi^*} = \sup_{x \in \mathbb{R}^d} \frac{x^\top \mathbb{E}_{(s,a) \sim d^{\pi^*}} [\phi(s, a) \phi(s, a)^\top] x}{x^\top \mathbb{E}_{(s,a) \sim \rho} [\phi(s, a) \phi(s, a)^\top] x}.$$

- Smaller than the density ratio, i.e.,  $\bar{C}_{\pi^*} \leq \max_{s,a} d^{\pi^*}(s, a) / \rho(s, a)$ .
- If  $\bar{C}_{\pi^*}$  is small, this implies the offline data sufficiently covers the subspace that the comparator policy  $\pi^*$  visits measured by  $\phi(s, a)$ .
- In tabular MDPs,  $\bar{C}_{\pi^*} = \max_{s,a} d^{\pi^*}(s, a) / \rho(s, a)$ .

# 1: Linear MDPs

[PAC Bound for CPPPO] Suppose  $P^\star \in \mathcal{M}$ . With probability  $1 - \delta$ ,

$$\forall \pi^\star; V_{P^\star}^{\pi^\star} - V_{P^\star}^{\hat{\pi}} = \tilde{O}\left((1 - \gamma)^{-2} \sqrt{\bar{C}_{\pi^\star} d^2 \ln(1/\delta)/n}\right).$$

- Partial coverage is refined as  $\bar{C}_{\pi^\star} < \infty$ .
- $\mathbb{E}_{(s,a) \sim \rho}[\phi(s,a)\phi(s,a)^\top]$  can be **singular**. (Previous works assume the non-singularity.)



# 1: Linear Mixture MDPs

Definition: Linear Mixture MDPs [AJSWY20, MJTS 20]

The true  $P^*$  is  $\theta^{*\top} \psi(s, a, s')$  given a feature vector  $\psi(s, a, s') : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}^d$ .

- Linear MDPs belong to linear mixture MDPs.
- Define pseudo feature vectors:  $\psi_V(s, a) = \int \psi(s, a, s') V(s') d(s')$

[Concentrability Coefficient for Linear Mixture MDPs]

$$\bar{C}_{\pi^*, \text{mix}} = \sup_{P \in Z_{P^*}} \sup_{x \in \mathbb{R}^d} \frac{x^\top \mathbb{E}_{(s,a) \sim d^{\pi^*}} [\psi_{V_P^{\pi^*}}(s, a) \psi_{V_P^{\pi^*}}(s, a)^\top] x}{x^\top \mathbb{E}_{(s,a) \sim \rho} [\psi_{V_P^{\pi^*}}(s, a) \psi_{V_P^{\pi^*}}(s, a)^\top] x},$$

where  $Z_{P^*} = \{P : \mathbb{E}_{(s,a) \sim \rho} [TV(P(\cdot | s, a), P^*(\cdot | s, a))^2] \leq \xi\}$ .

- $\bar{C}_{\pi^*, \text{mix}}$  is defined for varying feature vectors  $\psi_{V_P^{\pi^*}}$ .
- In linear MDPs,  $\bar{C}_{\pi^*, \text{mix}}$  reduces to  $\bar{C}_{\pi^*}$ .

# 1: Linear Mixture MDPs

[PAC Bound for CPPPO] Suppose  $P^\star \in M$ . With probability  $1 - \delta$ ,  
$$\forall \pi^\star; V_{P^\star}^{\pi^\star} - V_{P^\star}^{\hat{\pi}} = \tilde{O}\left((1 - \gamma)^{-2} \sqrt{\bar{C}_{\pi^\star, \text{mix}} d^2 \ln(1/\delta)/n}\right).$$

- Partial coverage concept is refined as  $\bar{C}_{\pi^\star, \text{mix}} < \infty$ .

## 2 KNRs

Definition: Kernelized nonlinear regulators. The true  $P^\star$  is a Gaussian distribution  $\mathcal{N}(W^\star \phi(s, a), I)$  ( $W^\star \in \mathbb{R}^{d_S \times d}$ ) given a feature vector  $\phi : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}^d$ . ( $d_S$  is a dimension of  $\mathcal{S}$ )

- Include LQRs.
- Include RKHS models (GPs) .

[Concentrability Coefficient for KNRs]

$$\bar{C}_{\pi^\star} = \sup_{x \in \mathbb{R}^d} \frac{x^\top \mathbb{E}_{(s,a) \sim d^{\pi^\star}} [\phi(s, a) \phi(s, a)^\top] x}{x^\top \mathbb{E}_{(s,a) \sim \rho} [\phi(s, a) \phi(s, a)^\top] x} .$$

- This is exactly the same as the one in linear MDPs.

## 2 KNRs

[PAC Bound for CPPO]

Let  $\Sigma_\rho = \mathbb{E}_{(s,a) \sim \rho}[\phi(s,a)\phi(s,a)^\top]$ . Suppose  $P^\star \in M$ . With  $1 - \delta$ ,

$$\forall \pi^\star; V_{P^\star}^{\pi^\star} - V_{P^\star}^{\hat{\pi}} = \tilde{O} \left( (1 - \gamma)^{-2} \text{rank}(\Sigma_\rho)^3 \sqrt{d_S \bar{C}_{\pi^\star} \ln(1/\delta)/n} \right).$$

- Partial coverage concept is refined as  $\bar{C}_{\pi^\star} < \infty$ .
- $\Sigma_\rho$  can be **singular!!** The error depends on  $\text{rank}[\Sigma_\rho]$  but not  $d$ .
- $d$  can be infinite. Formally, extended to the infinite-dimensional setting,  $P^\star = \mathcal{N}(g^\star(s,a), I)$  where  $g^\star$  is an element of RKHS.

# 3: Low-rank MDPs

Definition: Low-rank MDPs [JKALS17, AKKS20]. The true  $P^\star$  is  $\mu^\star(s')^\top \phi^\star(s, a)$ . Both  $\mu^\star(\cdot)$ ,  $\phi^\star(\cdot)$  are unknown features. ( $\mu^\star : \mathcal{S} \rightarrow \mathbb{R}^d$ ,  $\phi^\star : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}^d$ )

$$\begin{array}{ccc} \begin{array}{|c|} \hline P^\star \\ \hline \end{array} & = & \begin{array}{|c|} \hline \mu^\star \\ \hline \end{array} \times \begin{array}{|c|} \hline \phi^\star \\ \hline \end{array} \\ \begin{array}{|c|} \hline |\mathcal{S}| \times |\mathcal{S}| \times |\mathcal{A}| \\ \hline \end{array} & & \begin{array}{|c|} \hline |\mathcal{S}| \times d \\ \hline \end{array} \times \begin{array}{|c|} \hline d \times |\mathcal{S}| \times |\mathcal{A}| \\ \hline \end{array} \end{array}$$

- **Features are unknown** 😞. We set the function classes  $\mu^\star \in \Psi$ ,  $\phi^\star \in \Phi$ .
- Low-rank MDPs include latent variable models, block MDPs and linear MDPs.

# 3: Low-rank MDPs

Definition: Low-rank MDPs [JKALS17, AKKS20]. The true  $P^\star$  is  $\mu^\star(s')^\top \phi^\star(s, a)$ . Both  $\mu^\star(\cdot)$ ,  $\phi^\star(\cdot)$  are unknown features. ( $\mu^\star : \mathcal{S} \rightarrow \mathbb{R}^d$ ,  $\phi^\star : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}^d$ )

[Concentrability Coefficient for Low-rank MDPs]

$$\bar{C}_{\pi^\star, \phi^\star} = \sup_{x \in \mathbb{R}^d} \frac{x^\top \mathbb{E}_{(s,a) \sim d^{\pi^\star}} [\phi^\star(s, a) \phi^\star(s, a)^\top] x}{x^\top \mathbb{E}_{(s,a) \sim \rho} [\phi^\star(s, a) \phi^\star(s, a)^\top] x}$$

- Looks similar to the one in linear MDPs and KNRs 🤔  $C_{\pi^\star, \phi^\star}^\dagger$  depends on the only **true feature**  $\phi^\star$  but not on other features.

# 3: Low-rank MDPs

[PAC Bound for CPPPO] Let  $\Sigma_{\rho, \phi^*} = \mathbb{E}_{(s,a) \sim \rho}[\phi^*(s,a)\phi^*(s,a)^\top]$ .

Suppose  $P^* \in M$  with  $1 - \delta$ ,

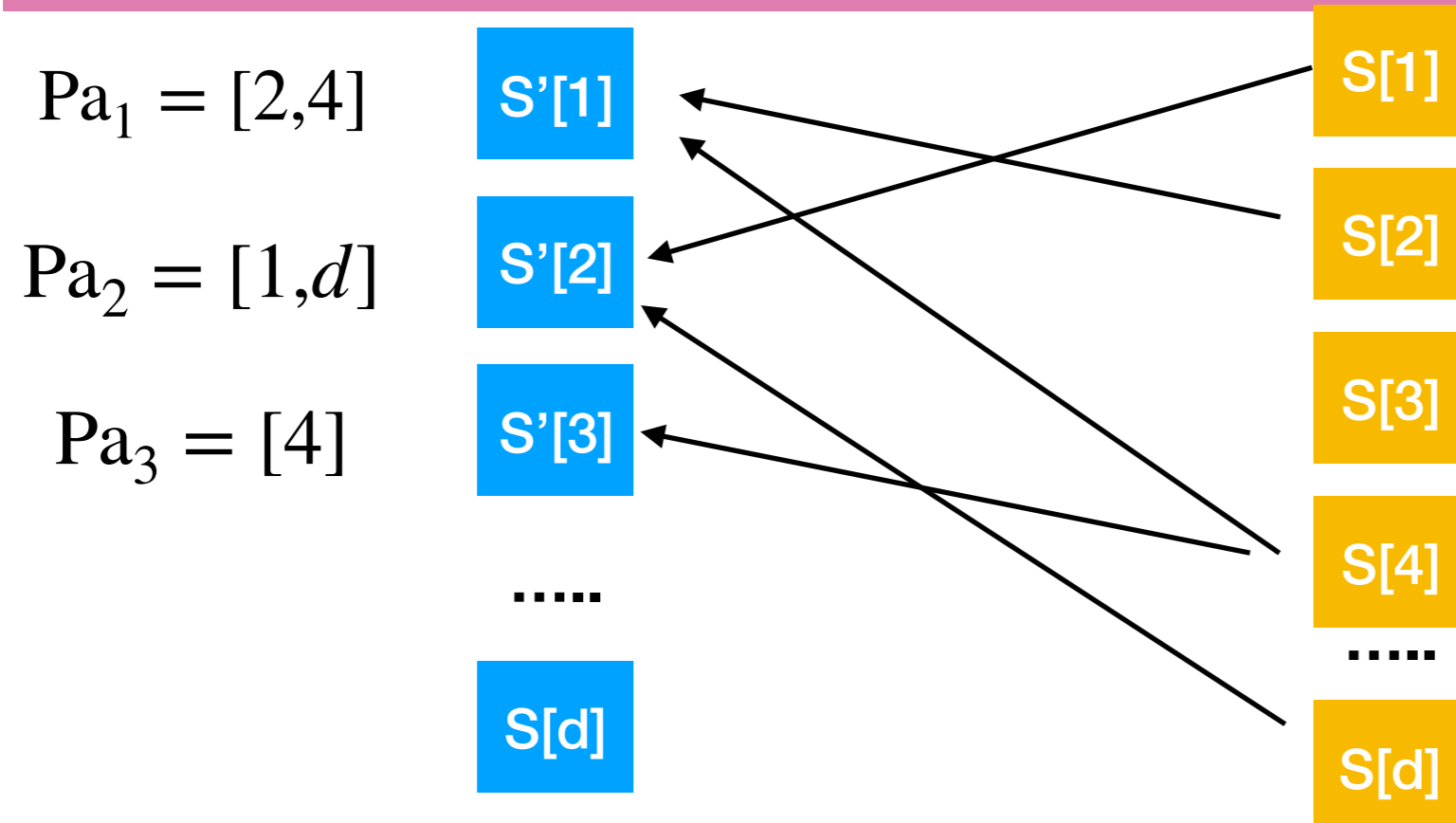
$$\forall \pi^*; V_{P^*}^{\pi^*} - V_{P^*}^{\hat{\pi}} = \tilde{O}((1 - \gamma)^{-2} \sqrt{\bar{C}_{\pi^*, \phi^*} \text{rank}(\Sigma_{\rho, \phi^*}) \ln(|M|/\delta)/n}).$$

- Partial coverage concept is refined as  $\bar{C}_{\pi^*, \phi^*} < \infty$ .
- Error depends on  $\text{rank}(\Sigma_{\rho, \phi^*})$  instead of  $d$ .
- Previous related work on sparse linear MDPs ([HDLSW20]) assumes the non-singularity of  $\Sigma_{\rho, \phi}$  for any  $\phi \in \Phi$ .

# 4. Factored MDPs

Definition: Factored (tabular) MDPs.  $\mathcal{S} = \mathcal{O}^d$

$$P^*(s' | s, a) = \prod_{i=1}^d P^*(s'[i] | s[\text{Pa}_i], a). \text{ Denote } \mathcal{S}_i = \mathcal{O}^{|\text{Pa}_i|}.$$



- Factored MDPs are governed by  $O(\sum_i |\mathcal{O}^{|\text{Pa}_i|})$  parameters.
- Non-factored MDPs are governed by  $O(|\mathcal{O}|^d)$  parameters.
- When  $|\text{Pa}_i| \ll d$ , the difference is huge.
- Our goal is leveraging this factored structure.



# 4. Factored MDPs

Definition: Factored (tabular) MDPs.  $\mathcal{S} = \mathcal{O}^d$

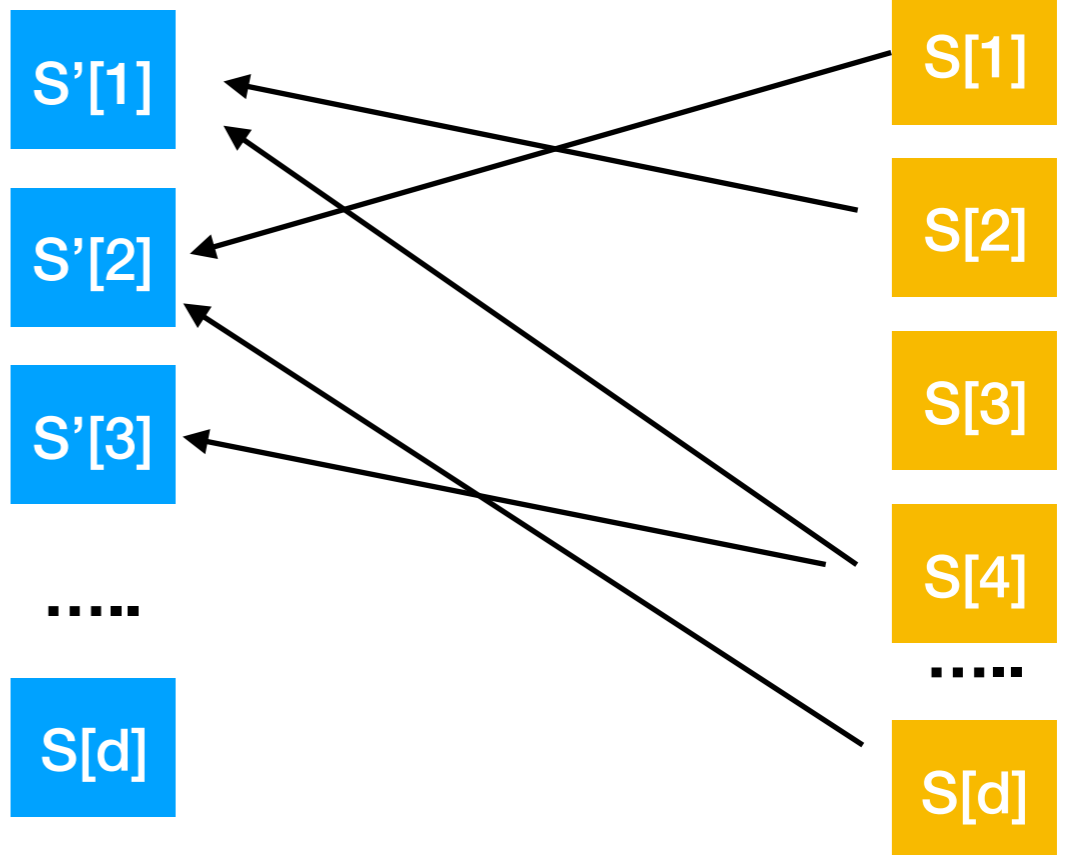
$$P^*(s' | s, a) = \prod_{i=1}^d P^*(s'[i] | s[\text{Pa}_i], a). \quad \text{Denote } \mathcal{S}_i = \mathcal{O}^{|\text{Pa}_i|}.$$

- Introduce  $\bar{C}_{\pi^*, \infty}^{[j]} = \max_{s_j \in \mathcal{S}_j, a \in \mathcal{A}} \frac{d^\pi(s_j, a)}{\rho(s_j, a)}$ ,  $\nu(s_j, a) = \sum_{s: \in \mathcal{S}, s[\text{Pa}_j]=s_j} \nu(s, a)$
- $\bar{C}_{\pi^*, \infty}^{[j]}$  is the marginal density ratio over each component.

$$\text{Pa}_1 = [2, 4]$$

$$\text{Pa}_2 = [1, d]$$

$$\text{Pa}_3 = [4]$$



## Example

$$\bar{C}_{\pi^*, \infty}^{[1]} = \max_{s_1 \in \{s[2], s[4]\}, a} \frac{d^\pi(s_1, a)}{\rho(s_1, a)}.$$

# 4. Factored MDPs

[Concentrability Coefficient for Factored MDPs]

$$\bar{C}_{\pi^*, \infty} = \max_{j \in [1, \dots, d]} \bar{C}_{\pi^*, \infty}^j$$

- $\bar{C}_{\pi^*, \infty}^{[j]}$  is smaller than the global density ratio  $\max_{s \in \mathcal{S}, a \in \mathcal{A}} \frac{d^\pi(s, a)}{\rho(s, a)}$  for any  $j \in [1, \dots, d]$ .
- Thus,  $\bar{C}_{\pi^*, \infty}$  is smaller than the global density ratio  $\max_{s \in \mathcal{S}, a \in \mathcal{A}} \frac{d^\pi(s, a)}{\rho(s, a)}$ .

# 4. Factored MDPs

[PAC Bound for CPPPO] Suppose  $P^\star \in M$ . With probability  $1 - \delta$ ,

$$\forall \pi^\star; V_{P^\star}^{\pi^\star} - V_{P^\star}^{\hat{\pi}} = \tilde{O}\left((1 - \gamma)^{-2} \sqrt{d\bar{C}_{\pi^\star, \infty} \sum_i |\mathcal{O}|^{Pa_i} \ln(1/\delta)/n}\right).$$

- Partial coverage concept is refined as  $\bar{C}_{\pi^\star, \infty} < \infty$ .
- This formally demonstrates the benefit of the factored structure in terms of the coverage condition.

# Disclaimer

- We claim CPPO works for **any** MDPs. What does it mean?
- **Any MDPs where the MLE has valid statistical guarantees.**
- CPPO does not work on (different) linear MDPs [JYWJ20] and linear Bellman complete MDPs 🙄.
- But, by taking a model-based perspective on them and modifying CPPO, we can still ensure the PAC guarantee under partial coverage.

# Conclusion

- CPPPO has the PAC guarantee under partial coverage assuming the realizability of the model. This works for **any** MDPs.
- Partial coverage concept is tailored to each model:
  - KNRs, linear mixture MDPs: relative condition numbers.
  - Low-rank MDPs: relative condition numbers defined on the **true unknown features**.
  - Factored MDPs: density ratios considering **the factored structures**.

# Future Directions

- Computationally efficient algorithm which has PAC guarantee under partial coverage.
- Lower bound results.
- Bayesian algorithms.

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