PC-PG: Policy Cover Directed Exploration for Provable Policy Gradient Learning

Joint work with Alekh Agarwal, Mikael Henaff, and Sham Kakade

Microsoft Research

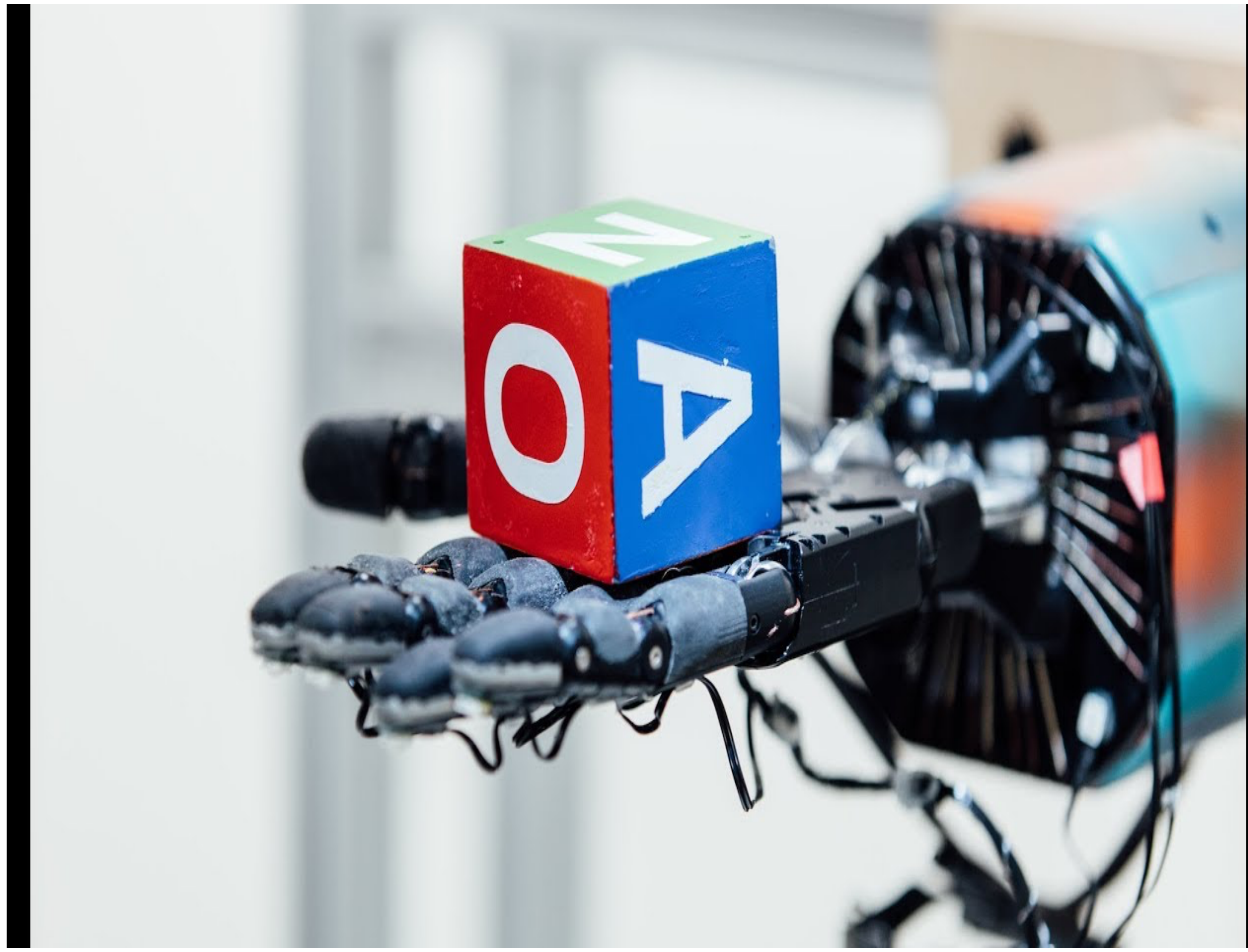
Facebook AI Research

W
UNIVERSITY of WASHINGTON

Policy Optimization

[AlphaZero, Silver et.al, 17]

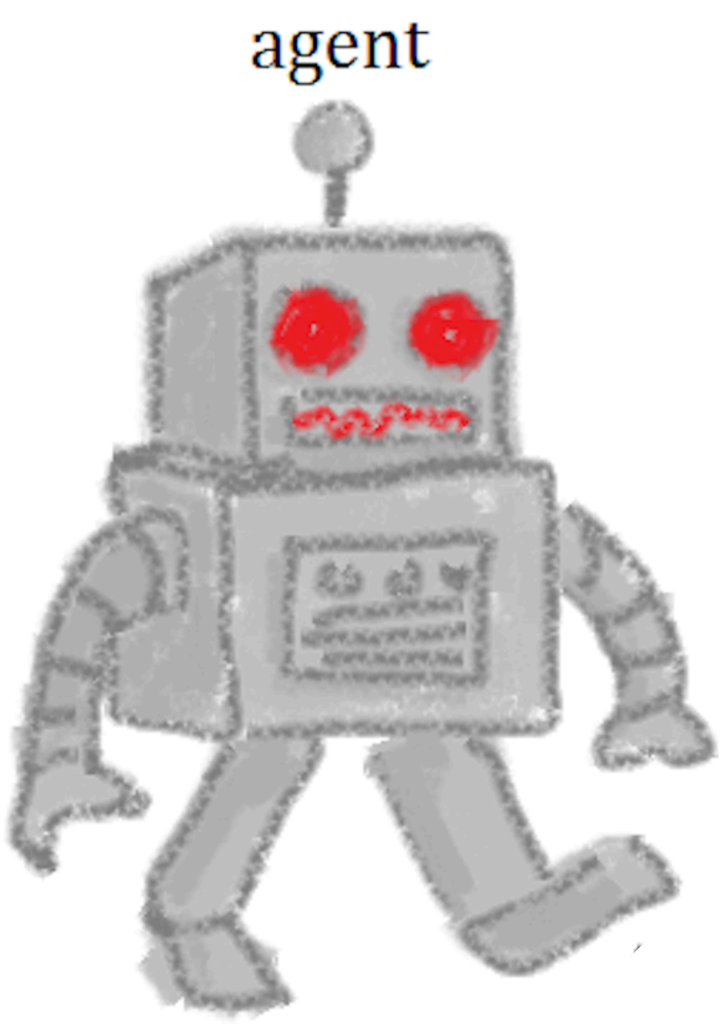
[OpenAI Five, 18]



[OpenAI,19]

Can we design Provably Correct Policy Gradient algorithms?

Infinite Horizon Discounted MDPs



Policy: state to action

$$\pi(s) \rightarrow a$$

Reward & Next State

$$r(s, a), s' \sim P(\cdot \mid s, a)$$

Objective: $\max_{\pi} \mathbb{E}_{\pi,P} \left[r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots \right]$

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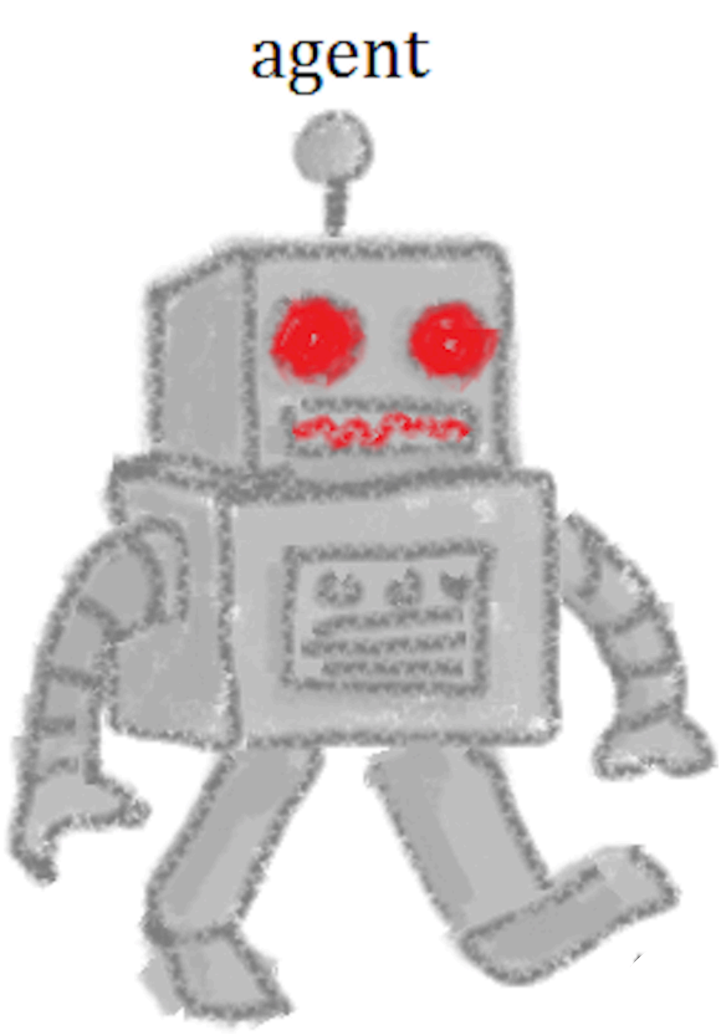

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Assume $s_0 \sim \mu_0$ and we can only reset from μ_0

Policy Gradient Methods

e.g., Reinforce, Natural Policy Gradient, TRPO, PPO:

(Williams 92, Kakade 02, Schulman et al 15, 17)

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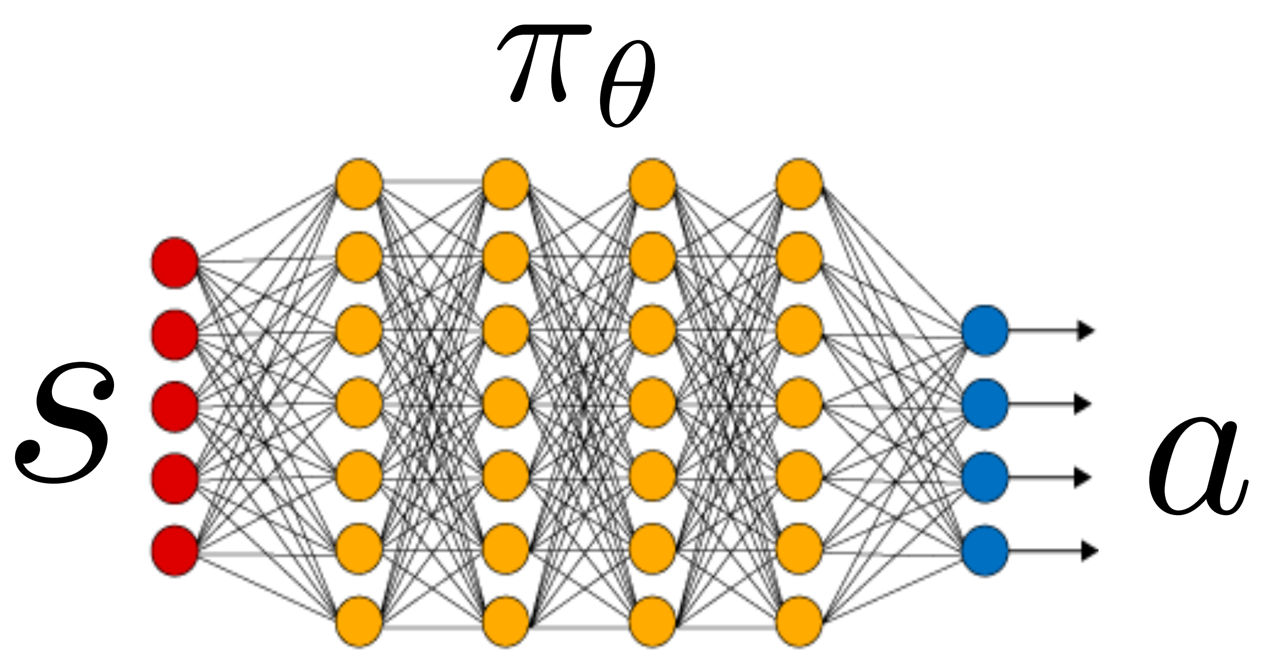

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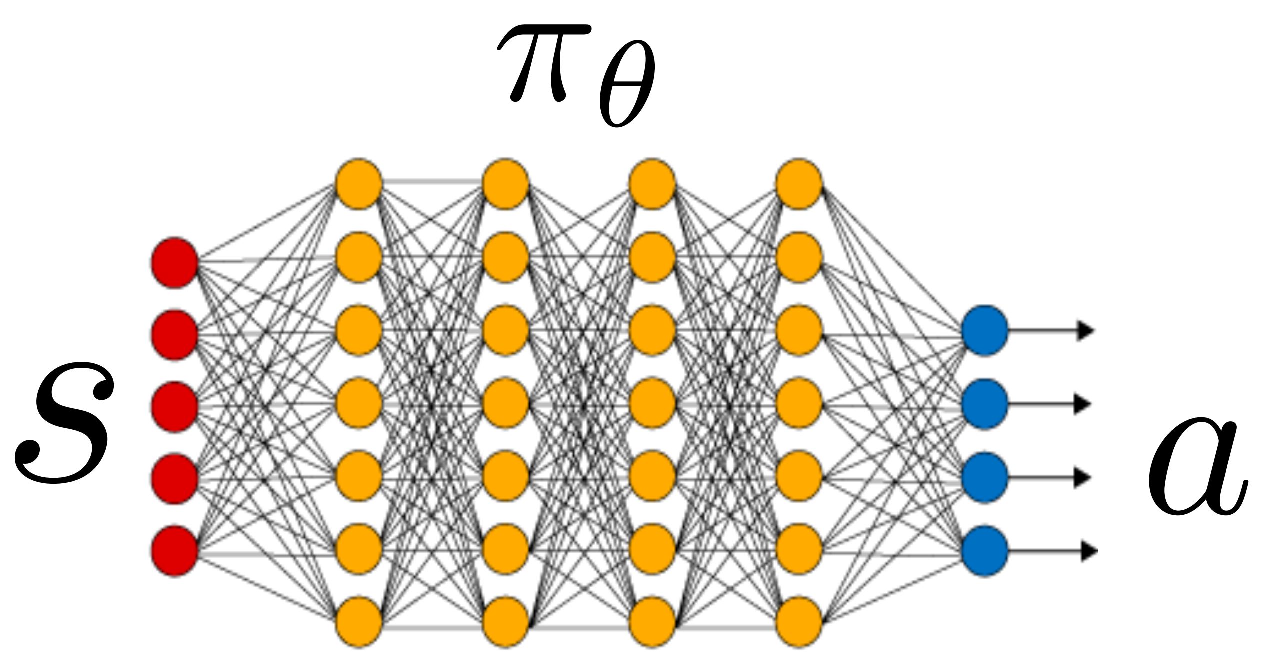

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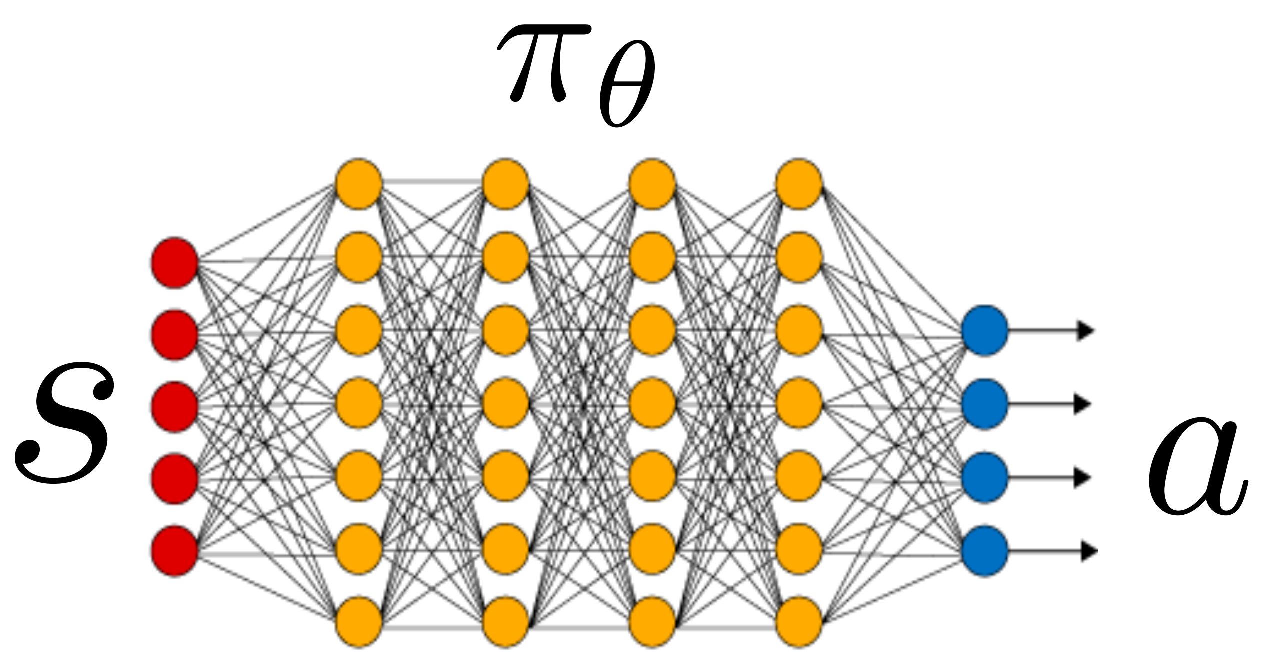

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$$\theta = \theta + \eta \nabla_\theta J(\pi_\theta)$$

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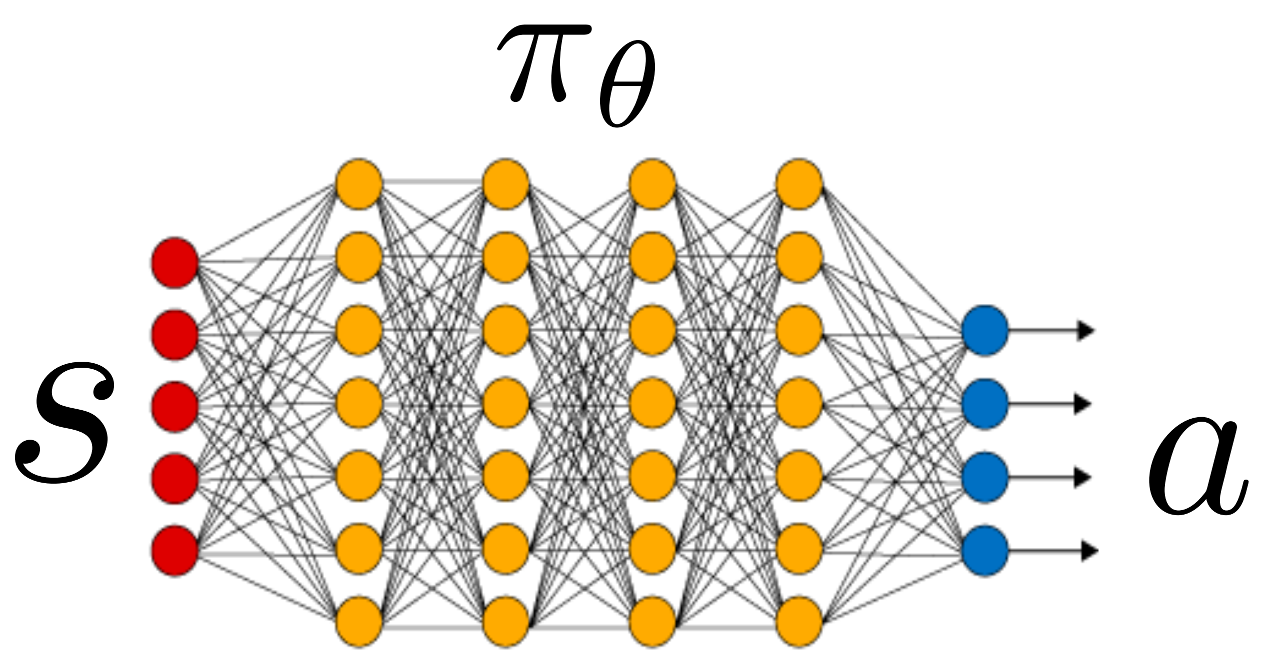

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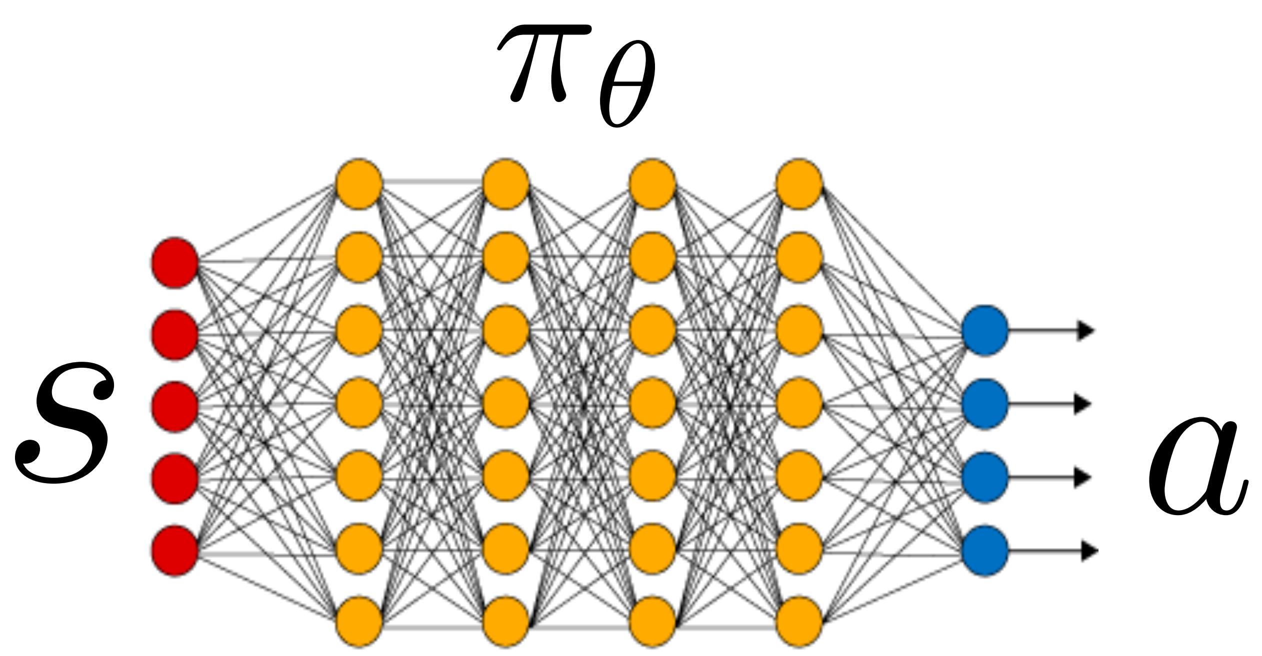

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Preconditioning w/ Fisher Information matrix

(TRPO and PPO are variants of it)

Advantages of Policy Gradient Methods

Strong Agnostic guarantee:

Compete to the best policy in the given class: $\widetilde{\pi} \in \Pi$

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Initial/reset dist

Q-learning, Fitted Q iteration:

Realizability (& Bellman Complete)

$$Q^\star \in \mathcal{Q}$$

Successful Story of PG:

Robot hand manipulation (OpenAI, 19)

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A notable technique

Domain Randomization:

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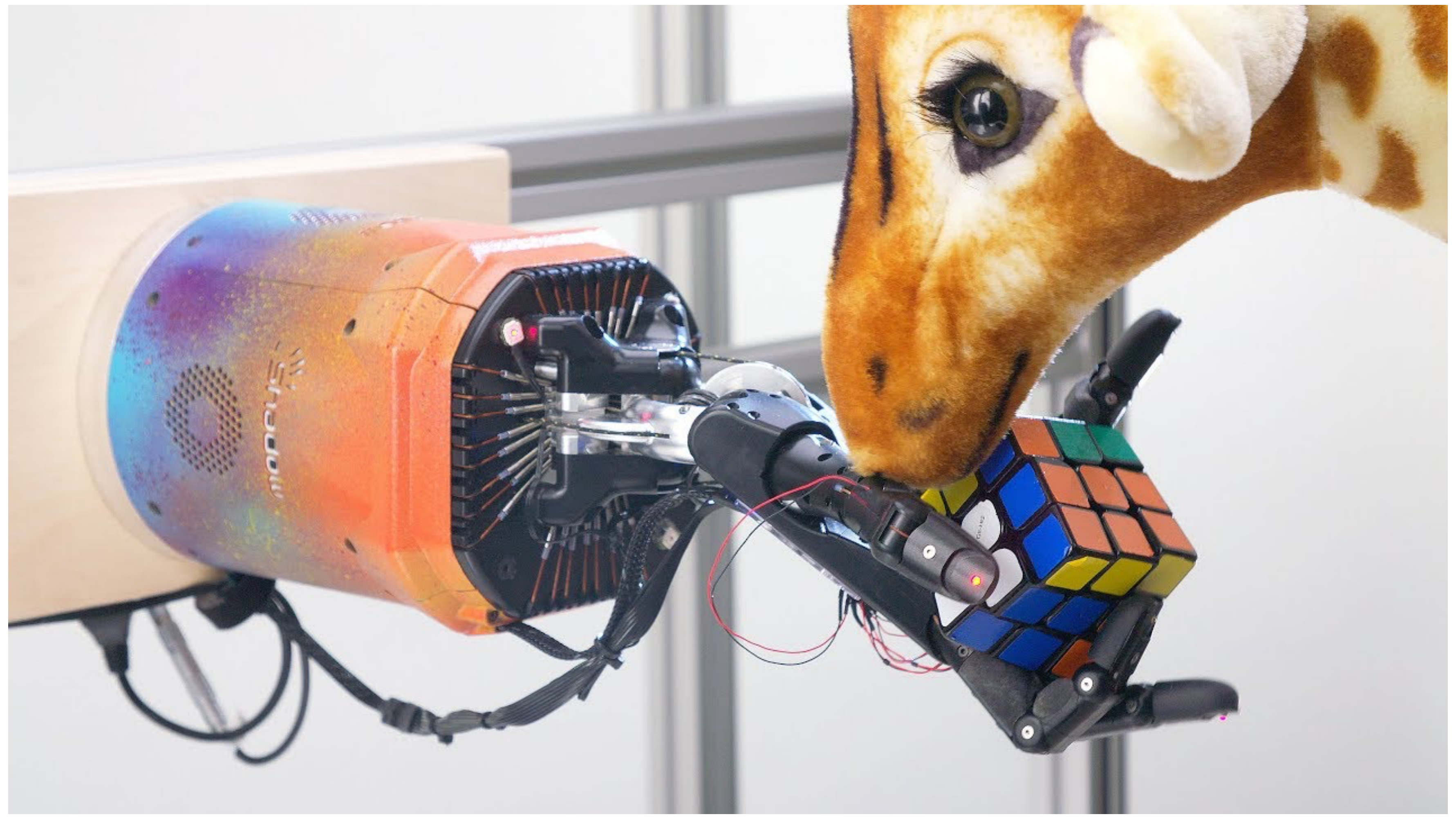

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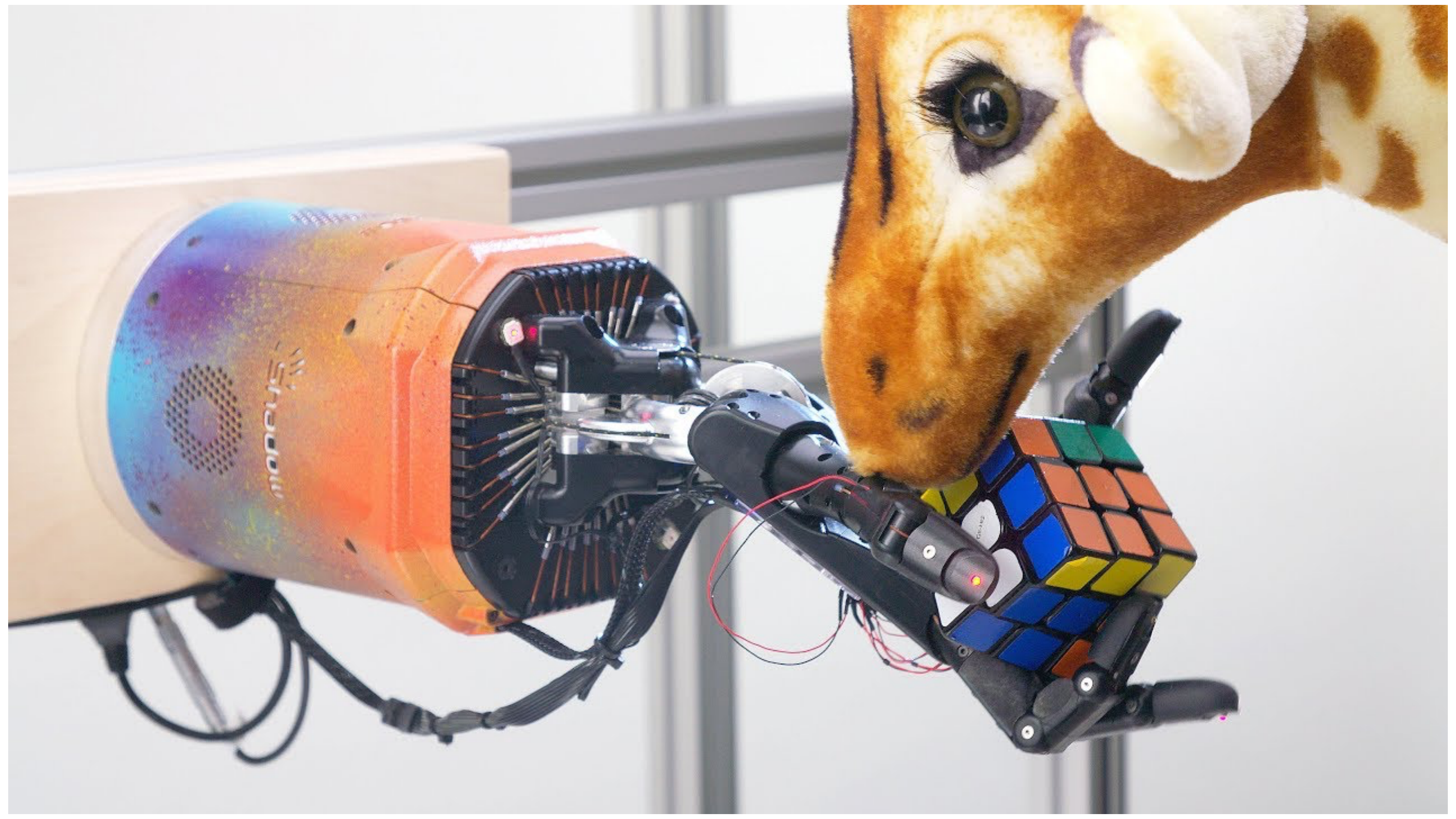

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Domain Randomization:

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Initial/reset dist

Make μ_0 as "wide" as possible !!

But PG Fails if initial condition does not hold, provably

Initialization: s_0 + random walk

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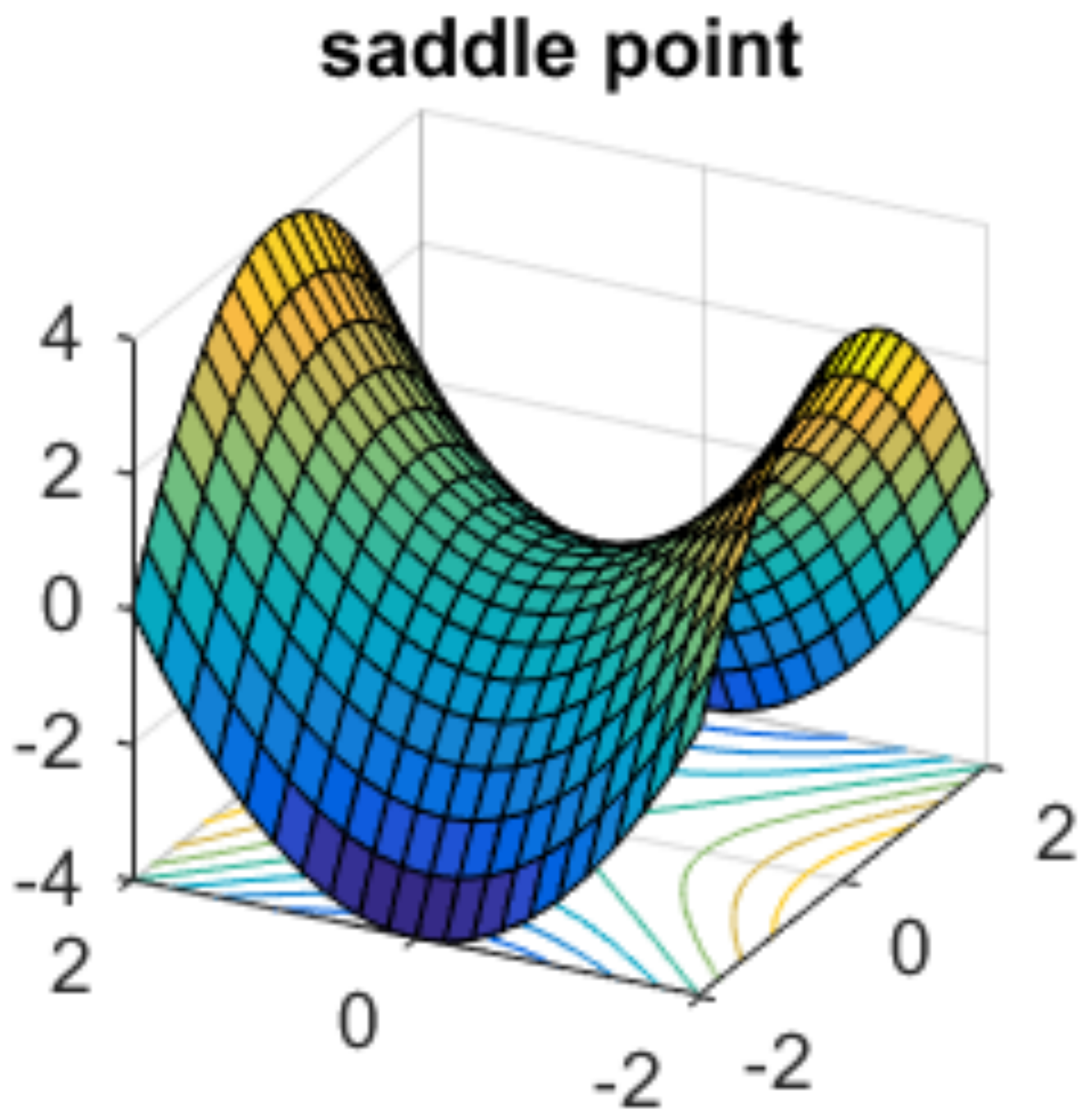
Extremely flatten gradient:

magnitude of gradient is exponentially small (even in higher order): 2^{-H}

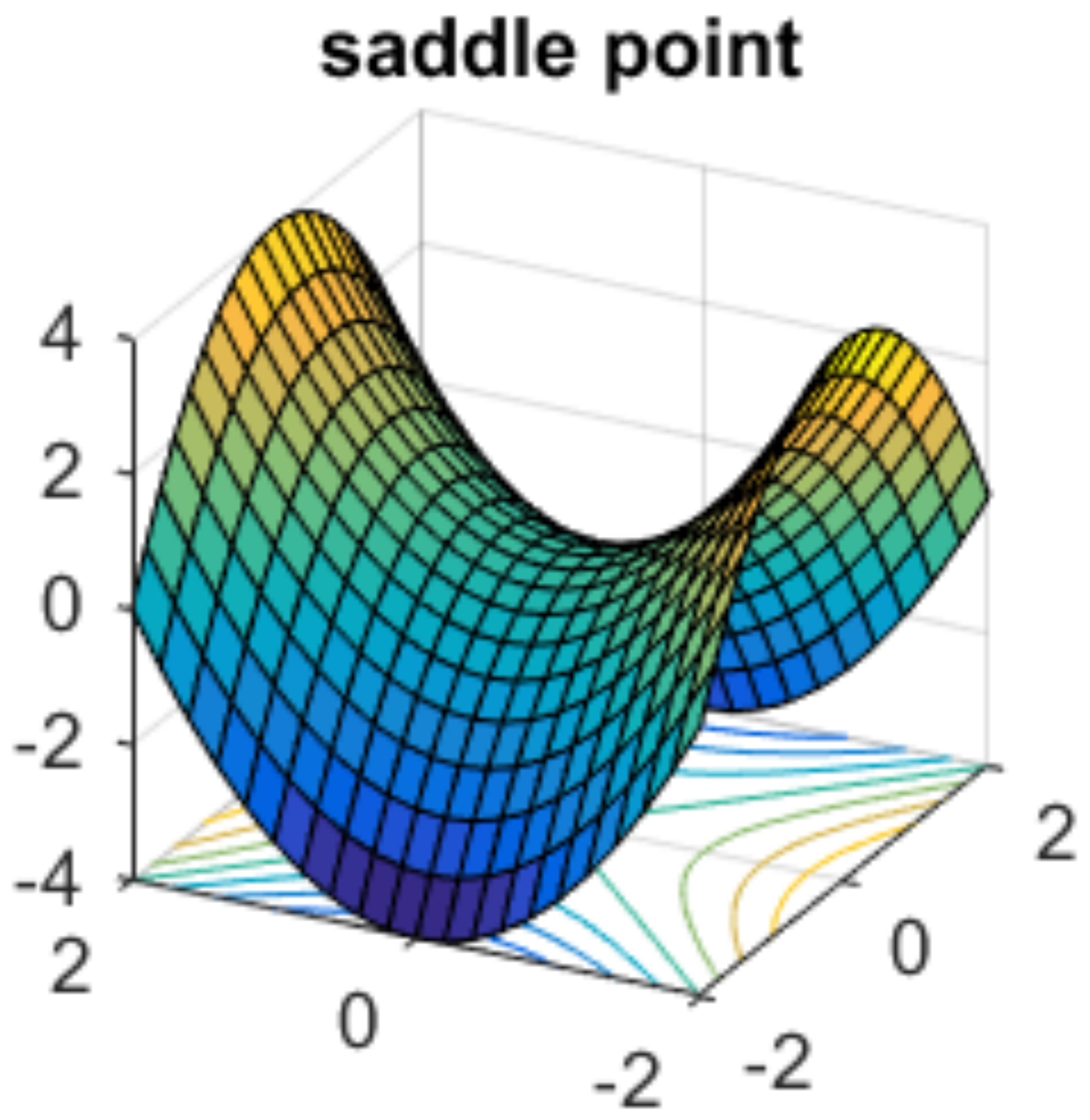
(e.g., see CPI from Kadade & Langford 02, and Agarwal et al 19)

The Optimization Landscape



Supervised Learning

The Optimization Landscape



Supervised Learning

- Gradient descent tends to just work
- Not sensitive to initialization
- Saddle point is not a problem

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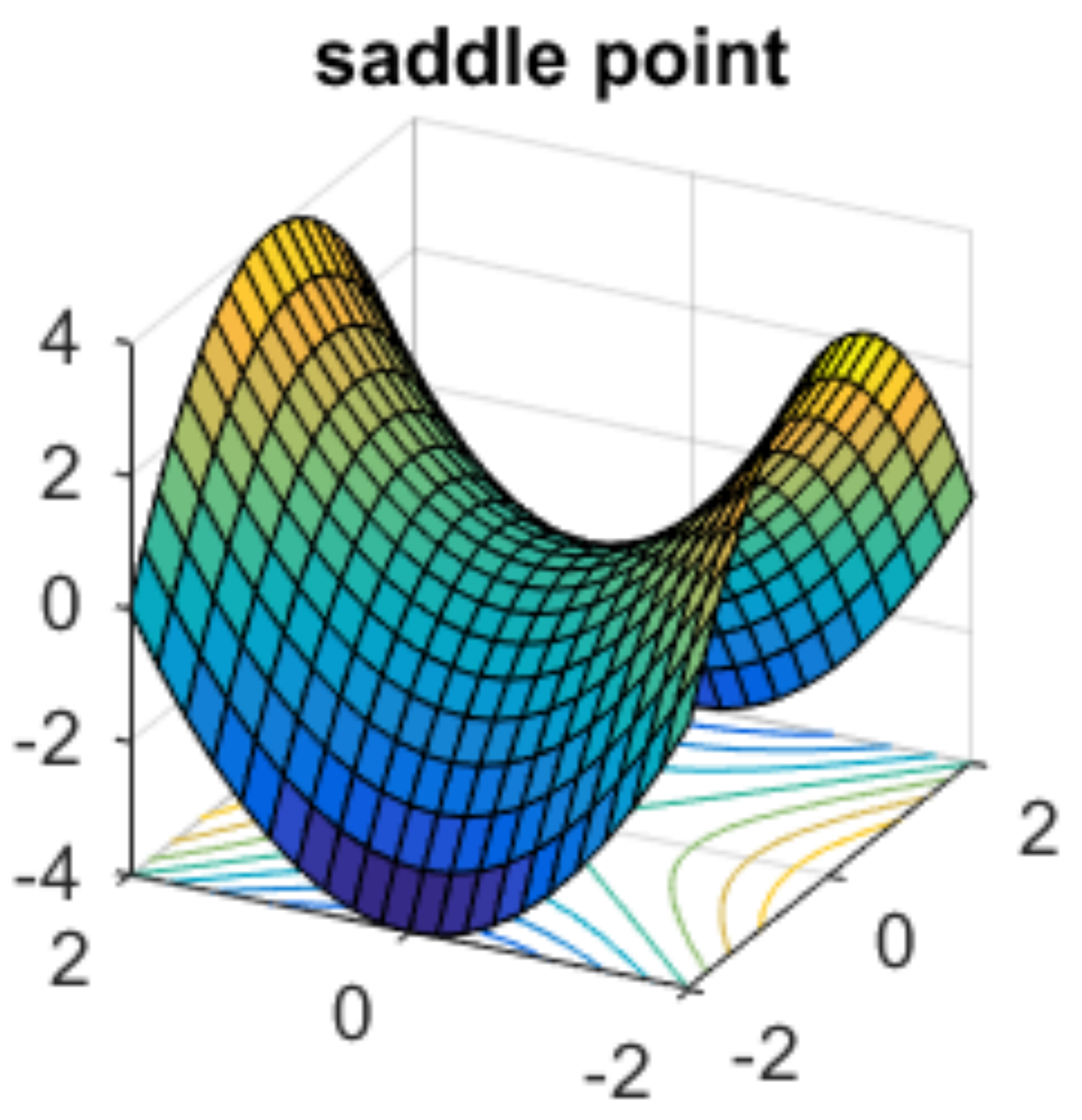

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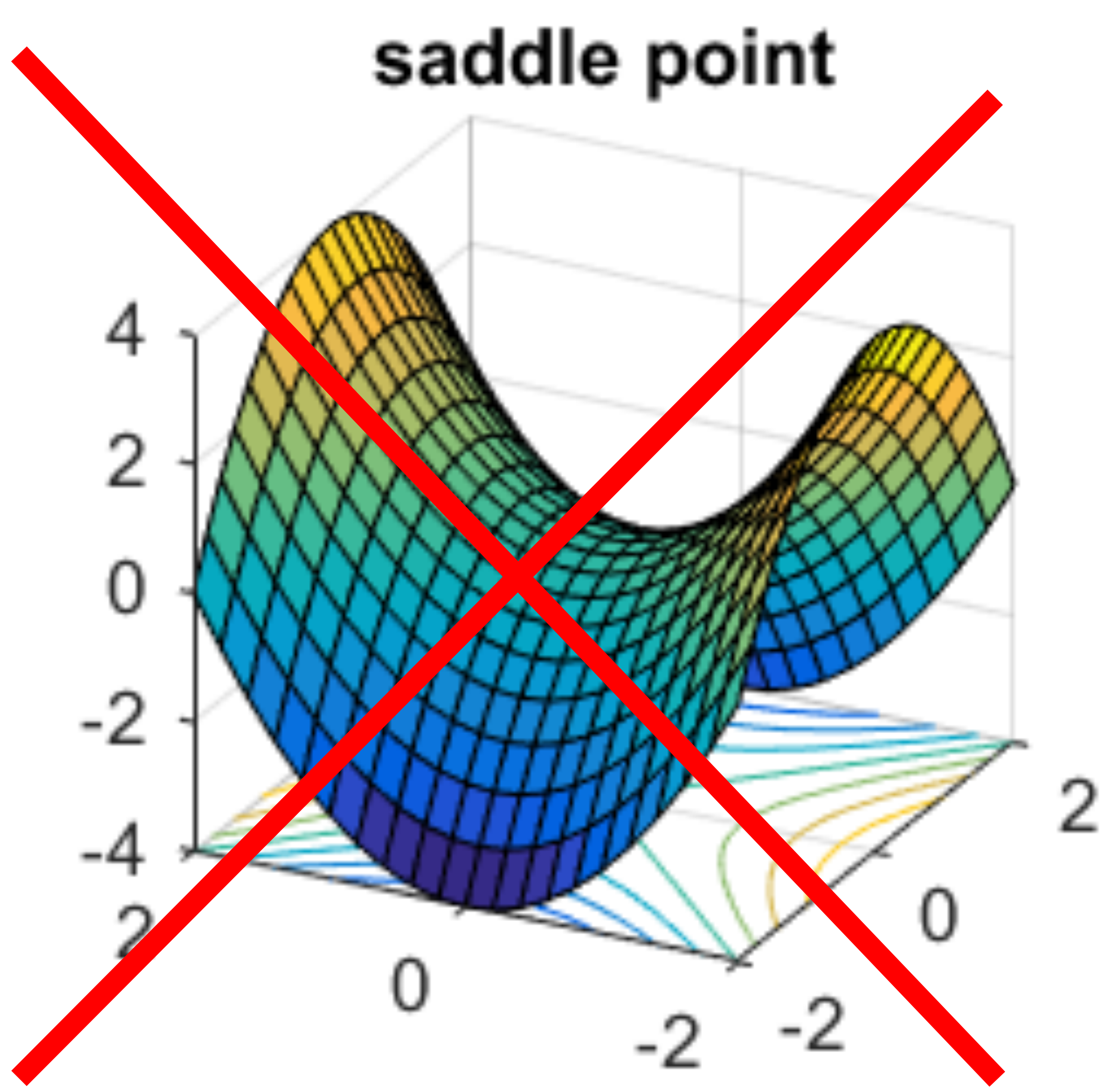

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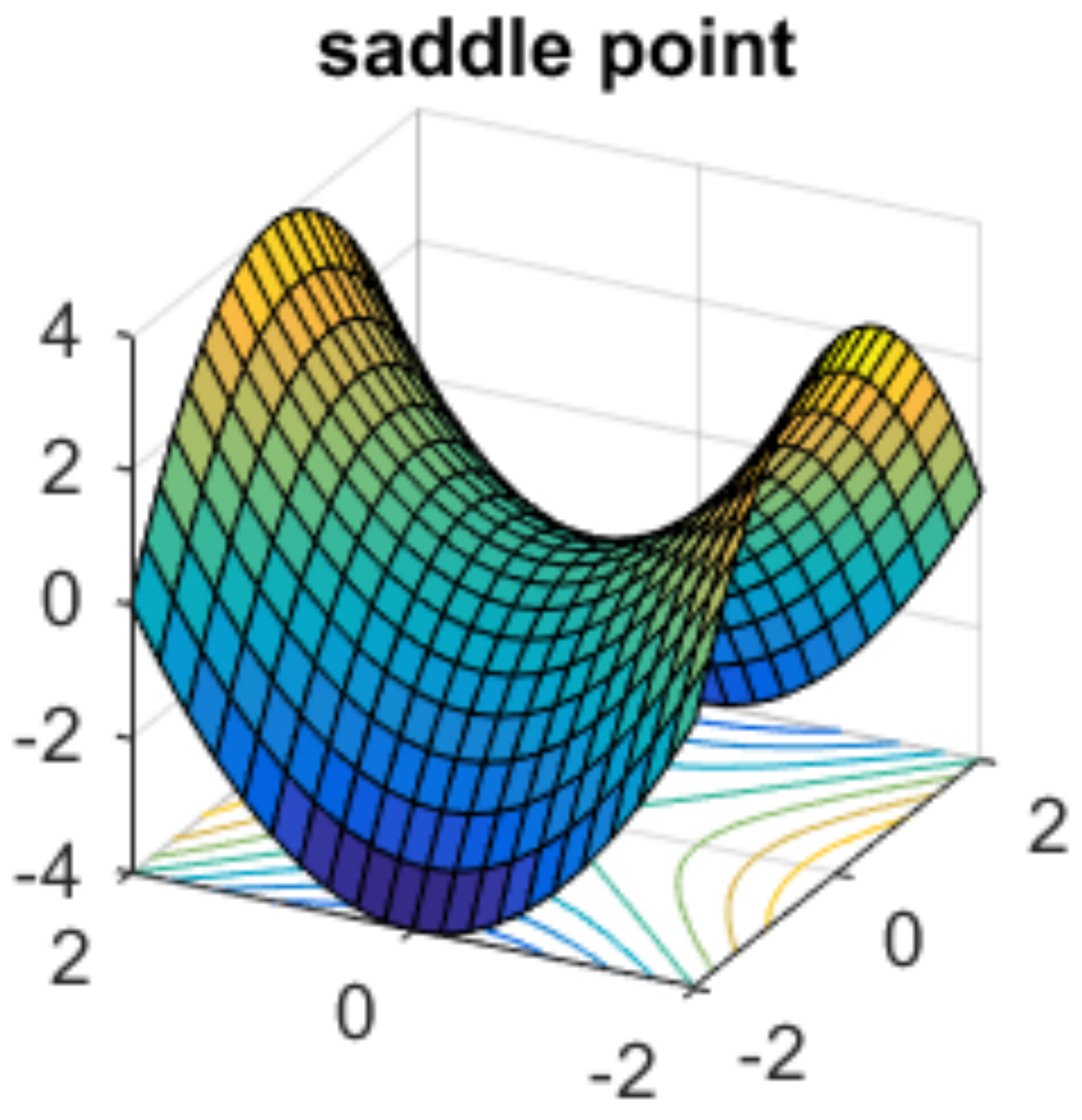

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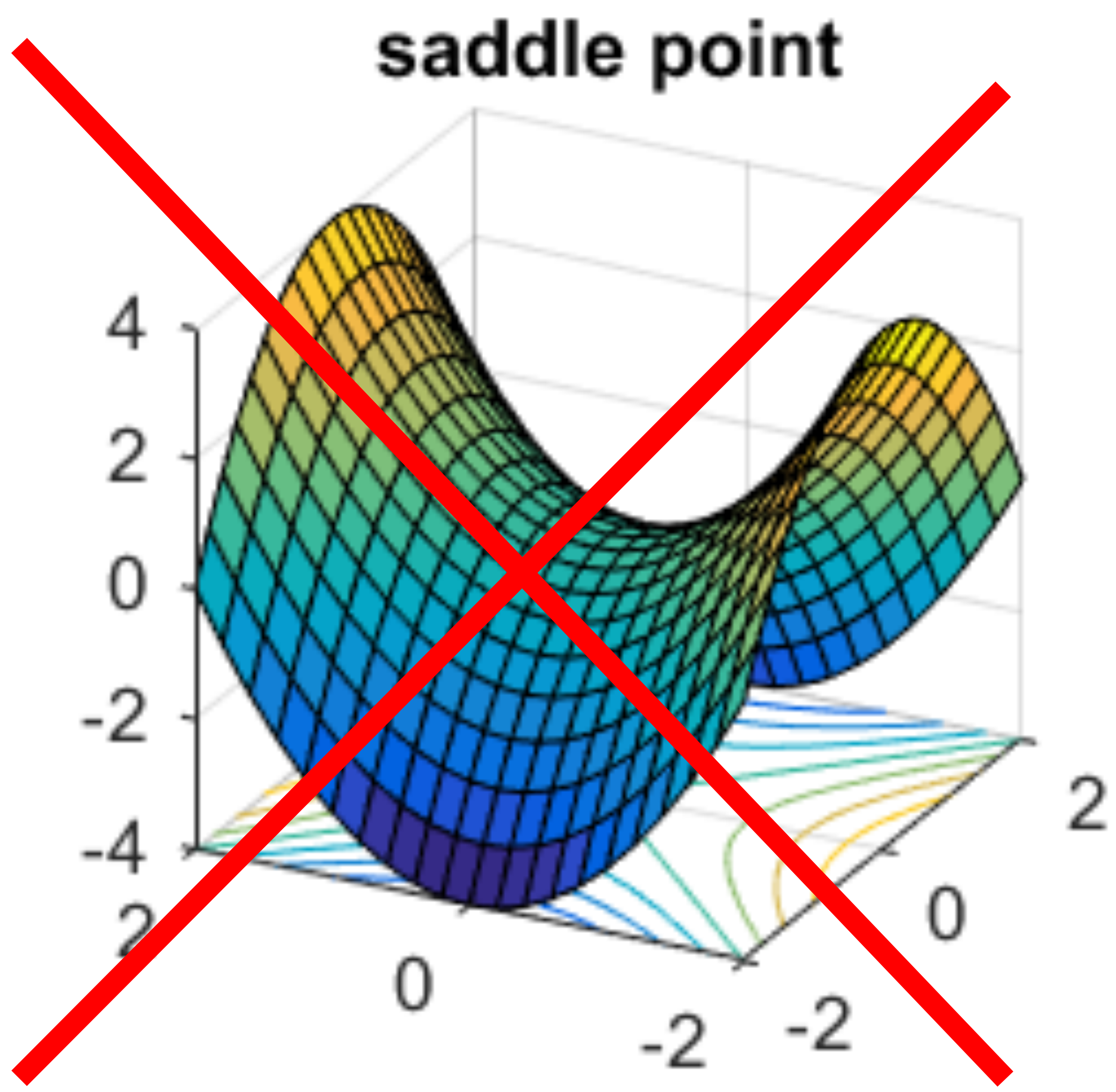
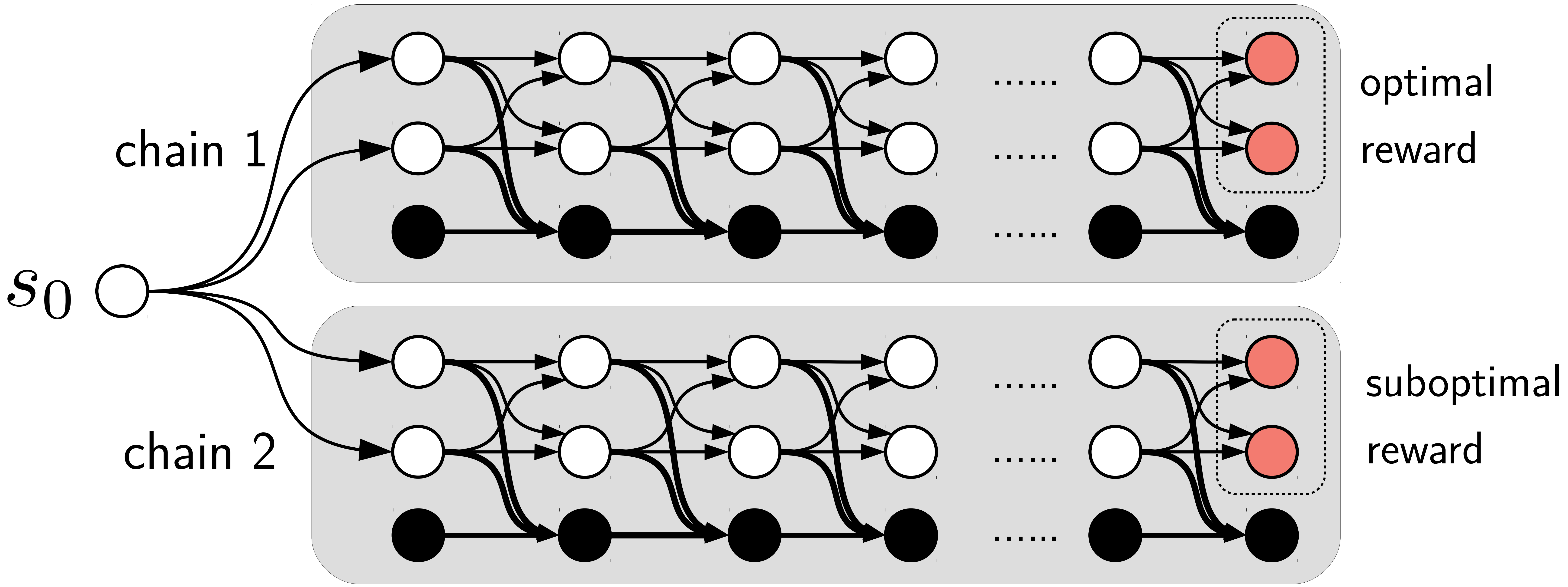


Reinforcement Learning:

- Extremely flatten region even at initialization
- Due to lack of exploration

Bidirectional Combination Lock



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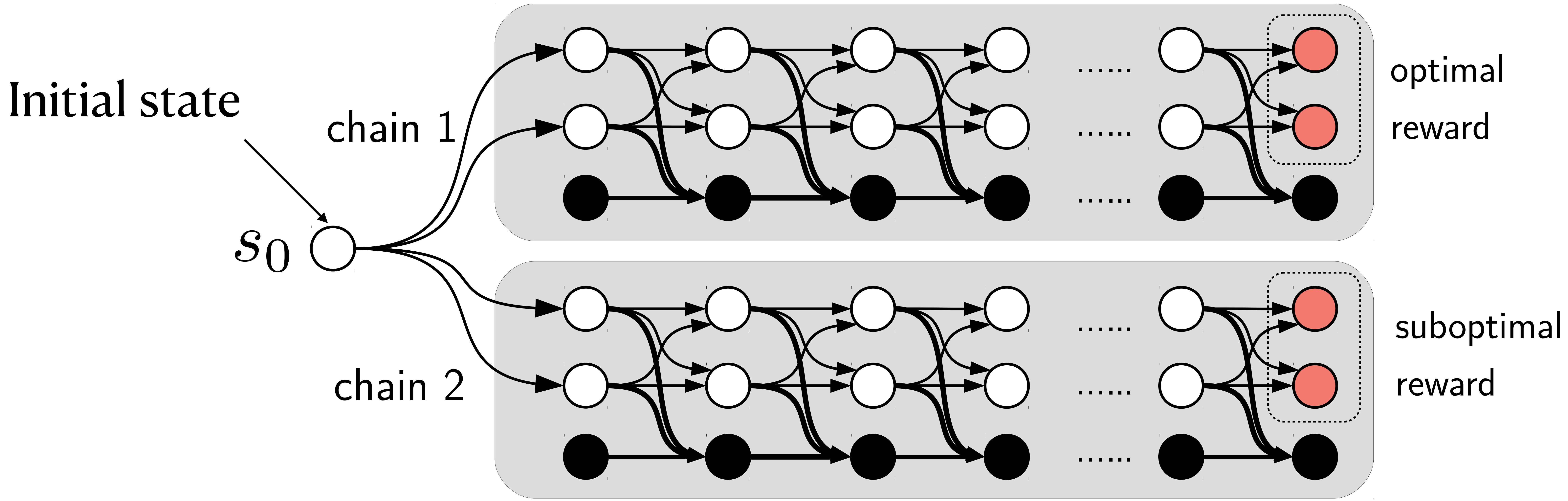

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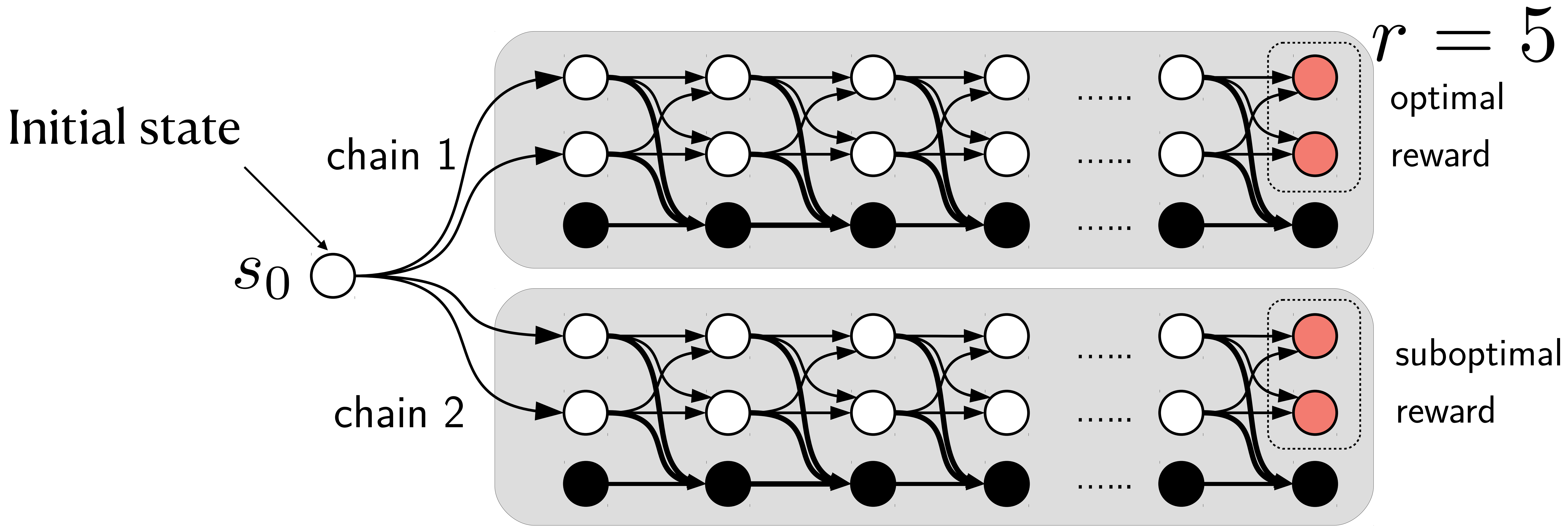

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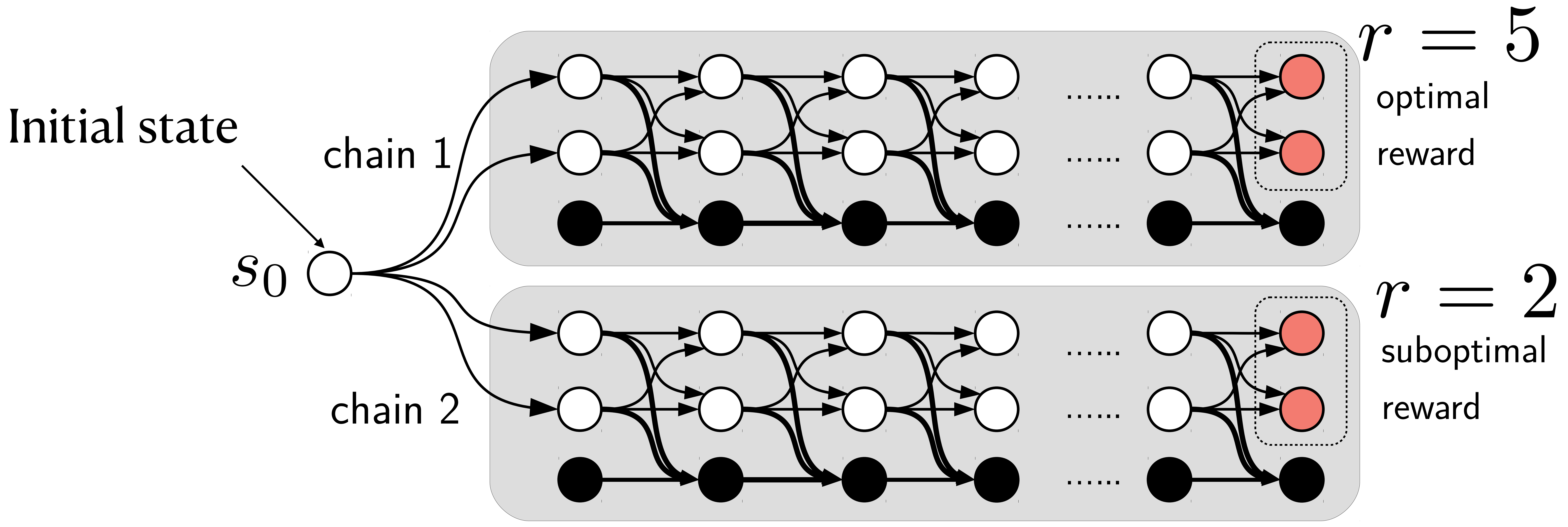
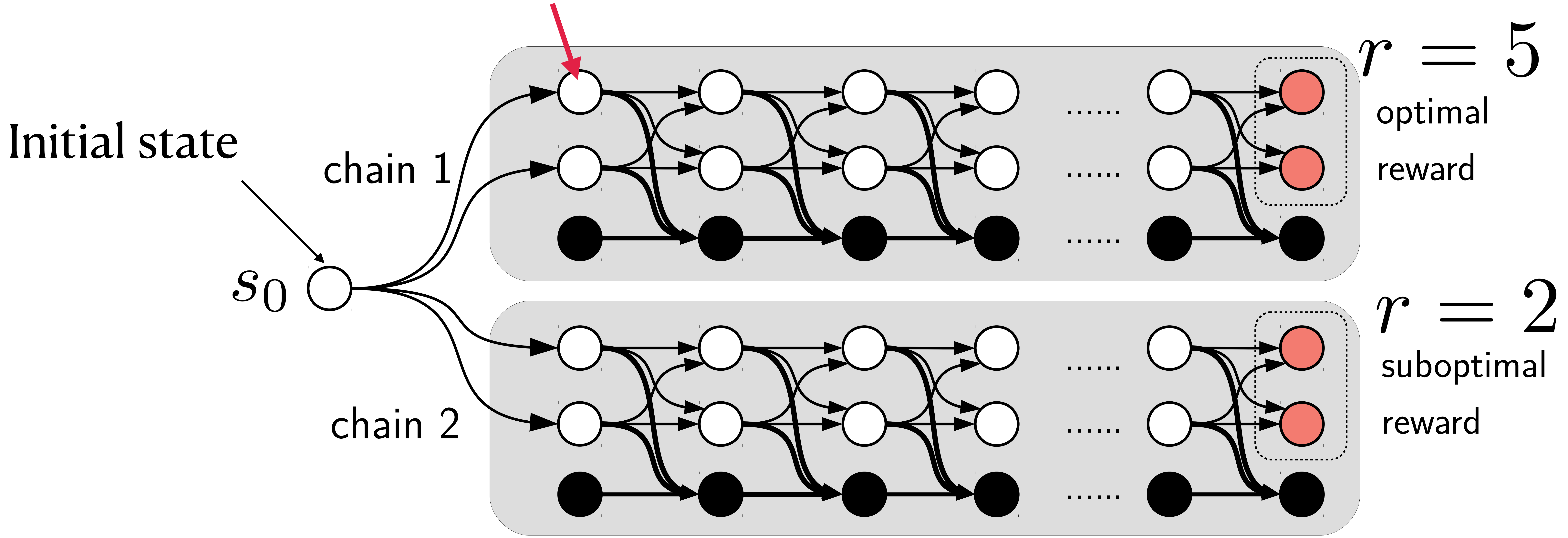


Bidirectional Combination Lock

"survived" state (white): 9 out of 10 actions go to bad state (black)



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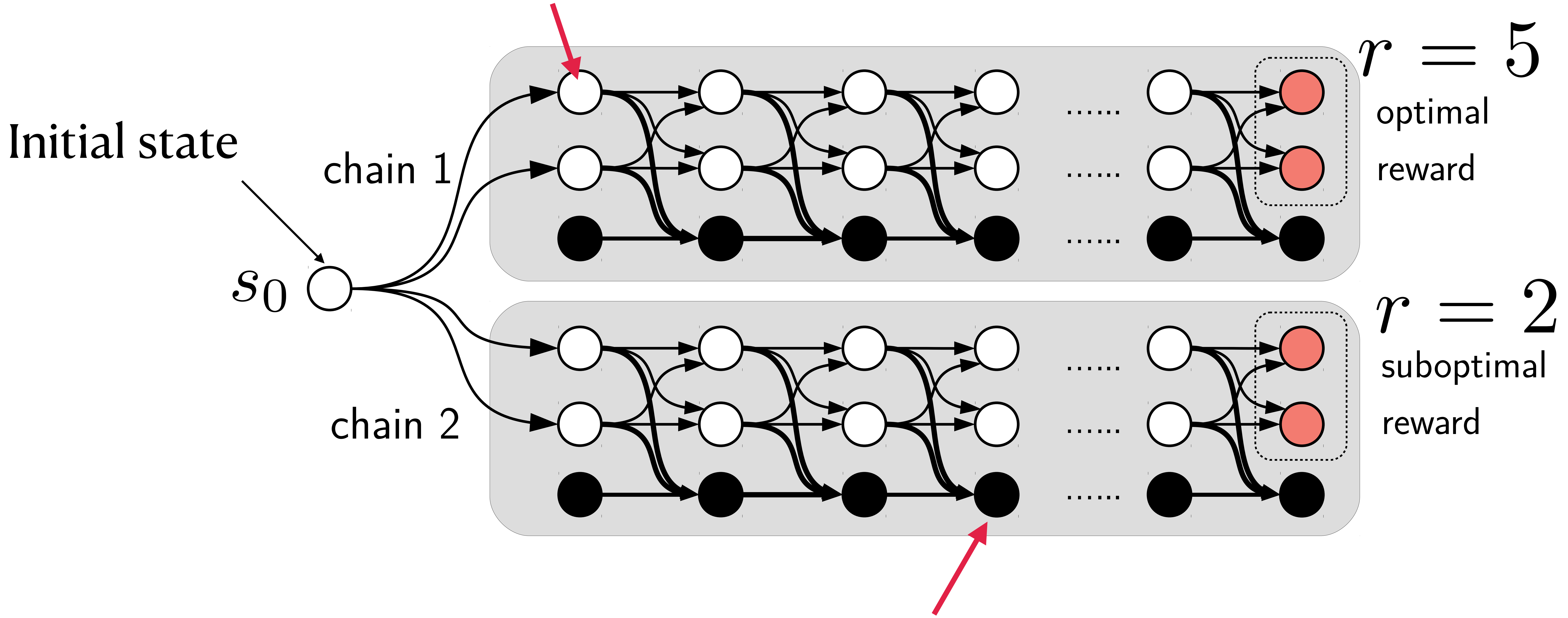

Bad state (cannot recover) has Anti-shaped reward:

$$r = 1/H$$

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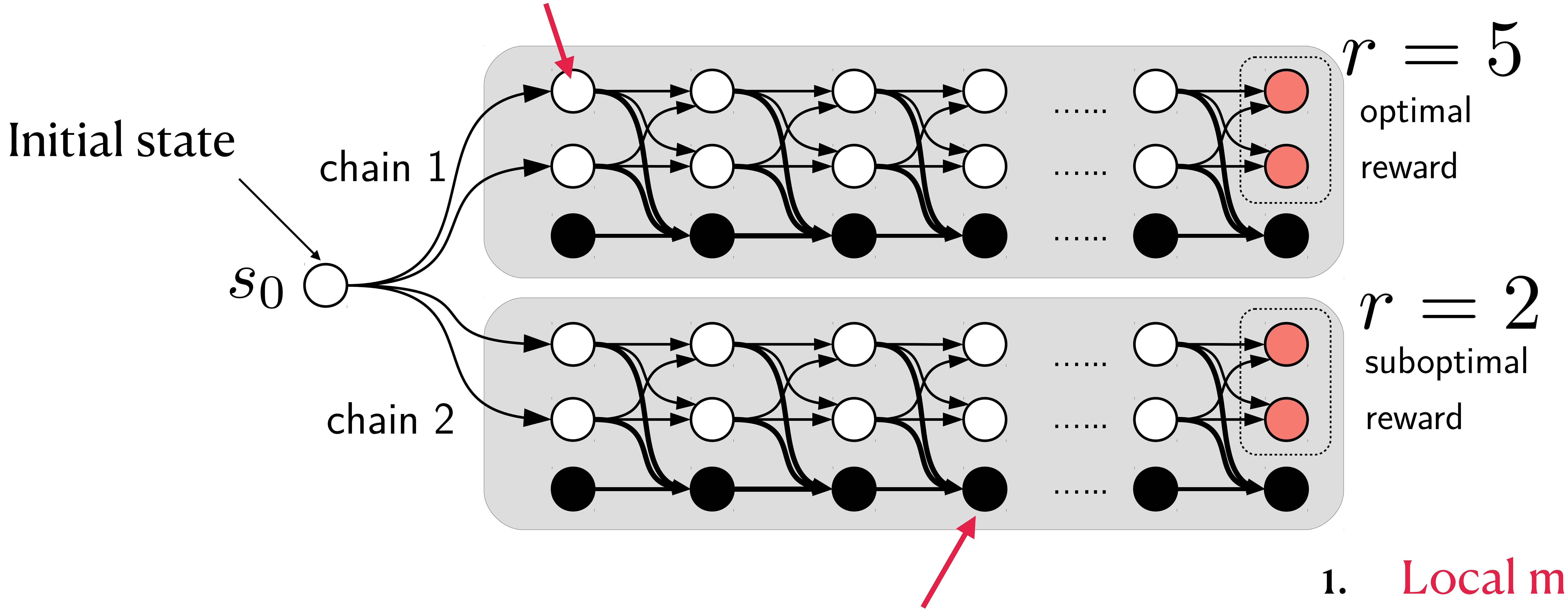

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2. Forgetting (policy becomes deterministic

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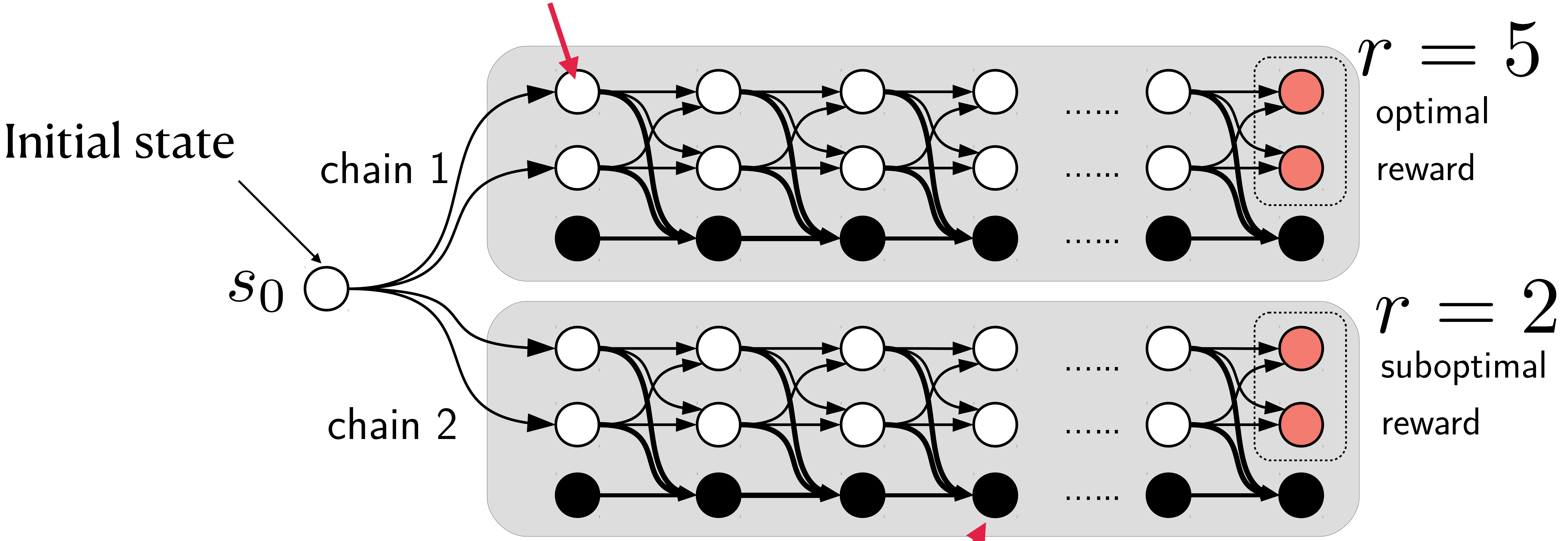

$$\left(\frac{1}{10}\right)^H$$

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Experiments on Bi-directional Comb Lock

Success Rate (visit the better chain):

Algorithm	Horizon			
	2	5	10	15
PPO	1.0	0.0	0.0	0.0
PPO+RND	0.75	0.40	0.50	0.55
PC-PG	?	?	?	?

PPO+RND: Random Network Distillation [Burda et.al, 19]

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Forgets to visit the other chain !

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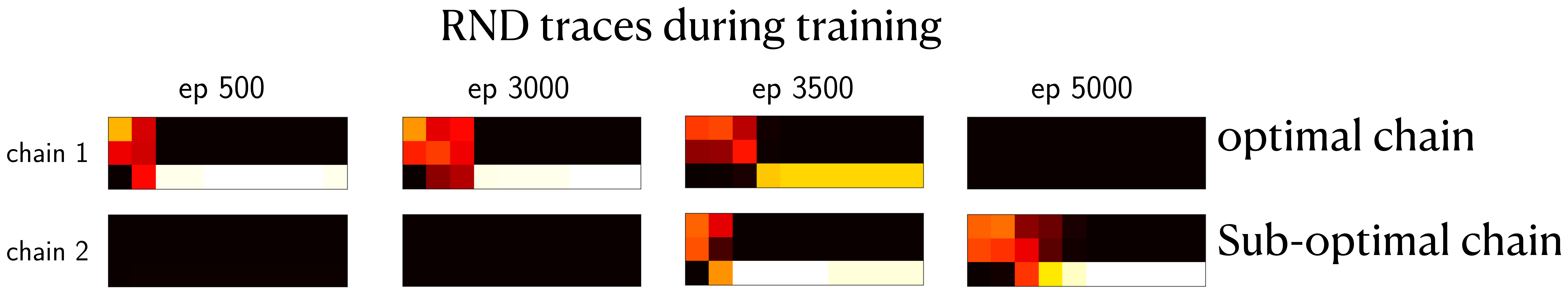

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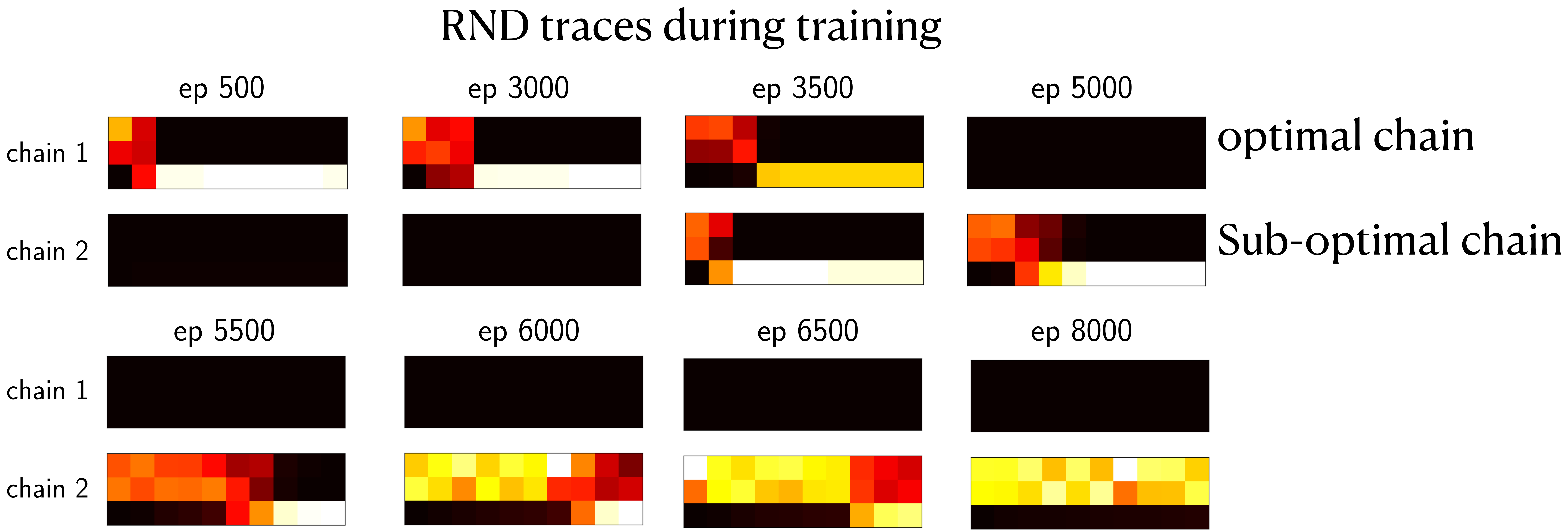
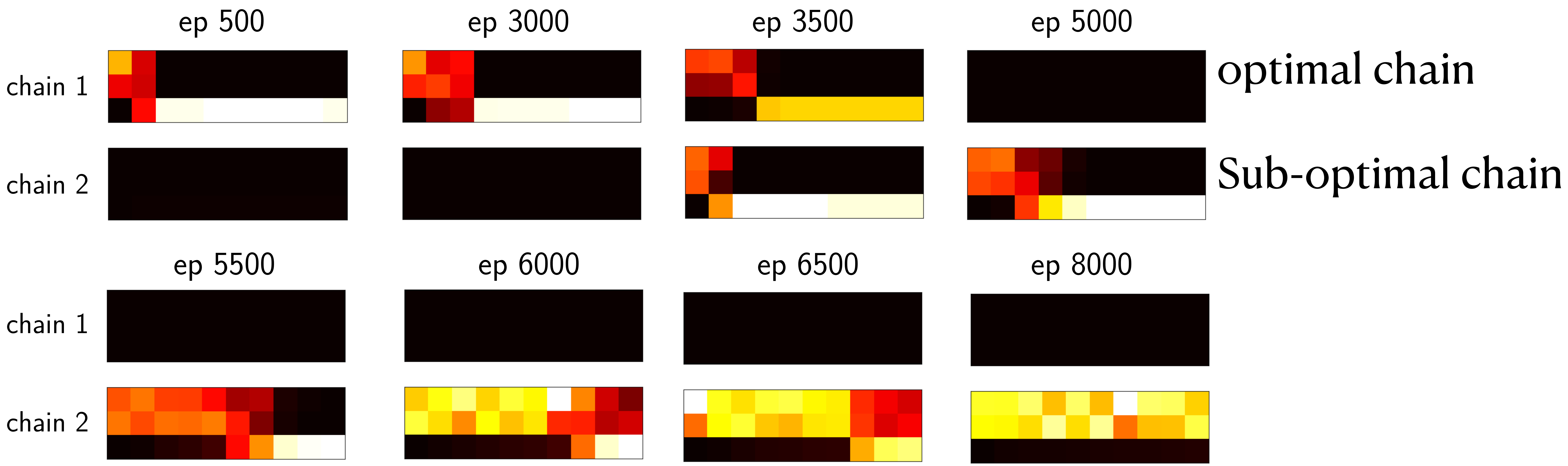


Experiments on Bi-directional Comb Lock

RND traces during training



Policy quickly becomes too deterministic and
forgets to explore the other (better!) chain

Summary of PG methods' common issues

1. Lack ability to explore

1. Catastrophic forgetting (even w/ reward bonus)

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Next:

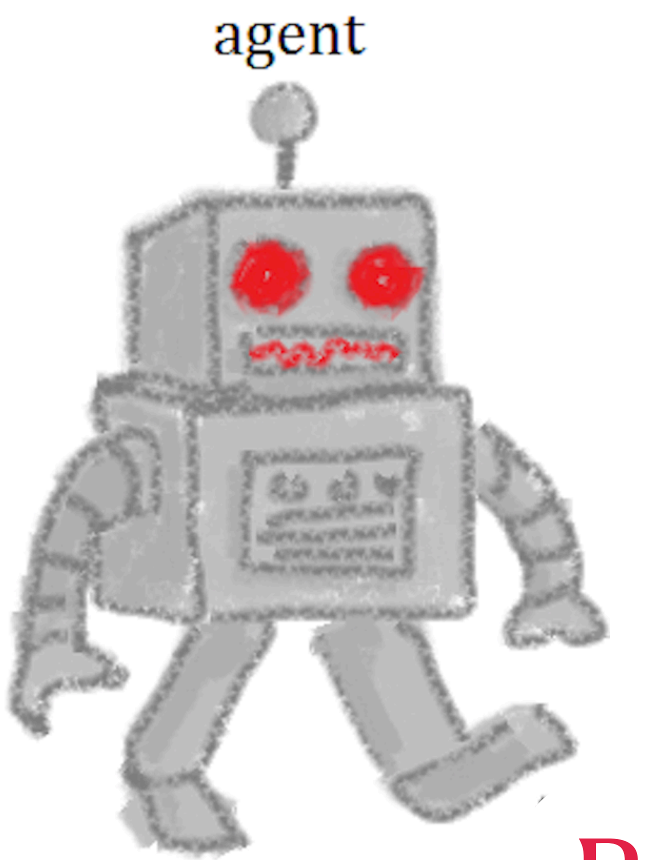
Our Solution: Policy Cover Policy Gradient (PC-PG)

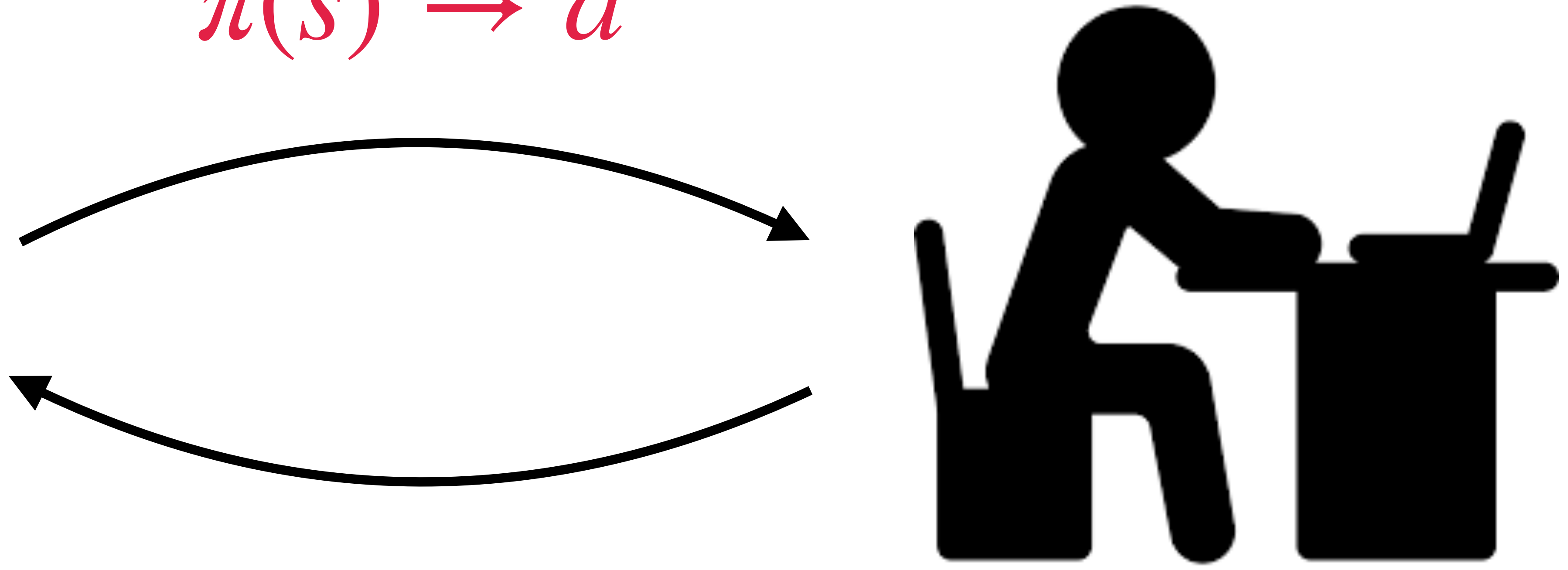
Policy Ensemble + Reward Bonus

Notations

Policy: state to action

$$\pi(s) \rightarrow a$$





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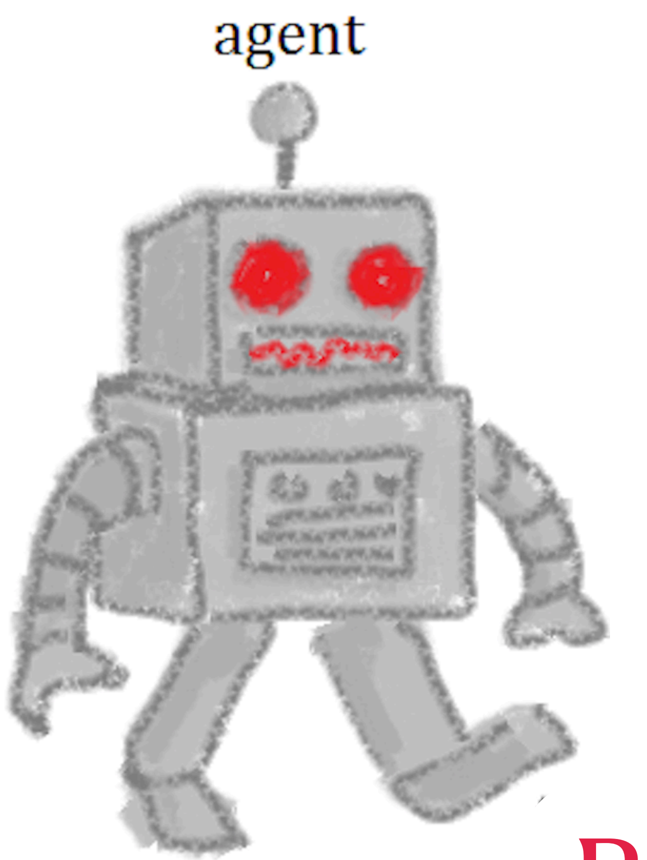

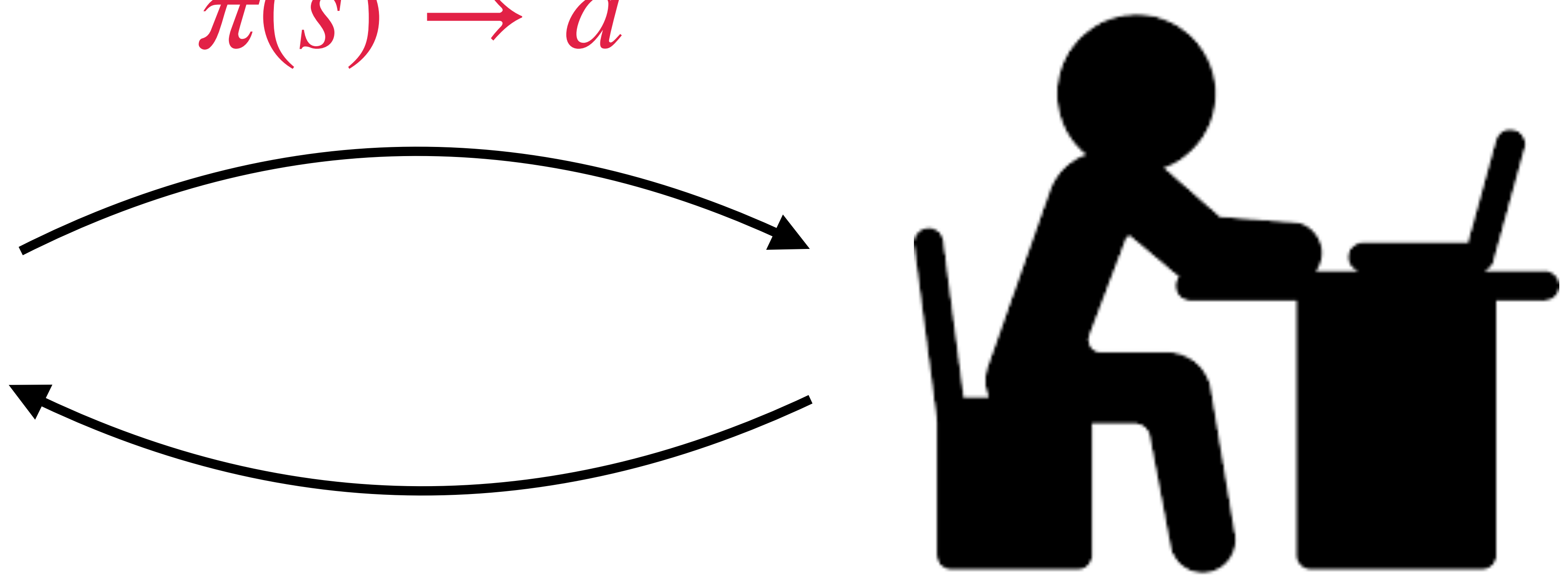

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Value and Q function

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$$Q^\pi(s,a) = r(s,a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)} V^\pi(s')$$

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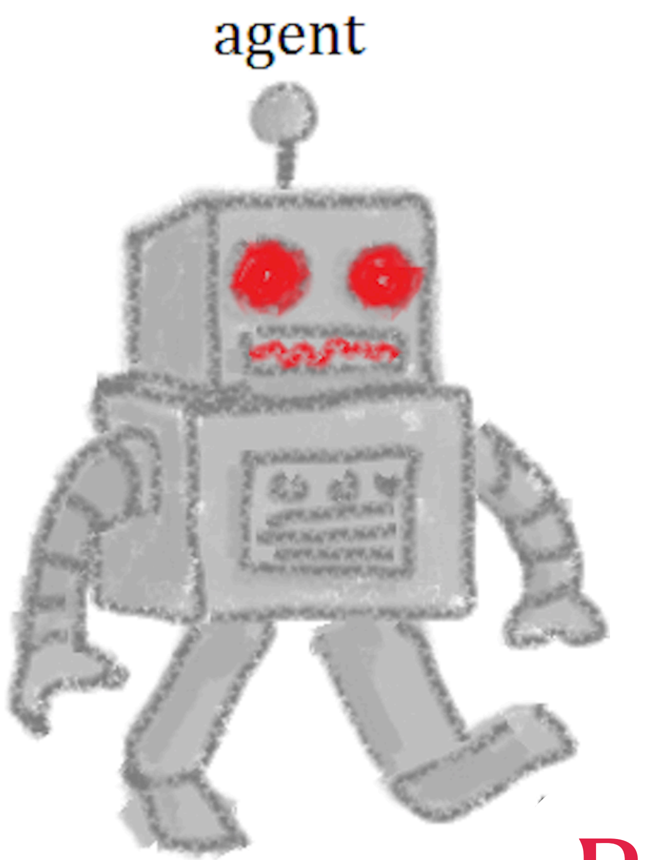

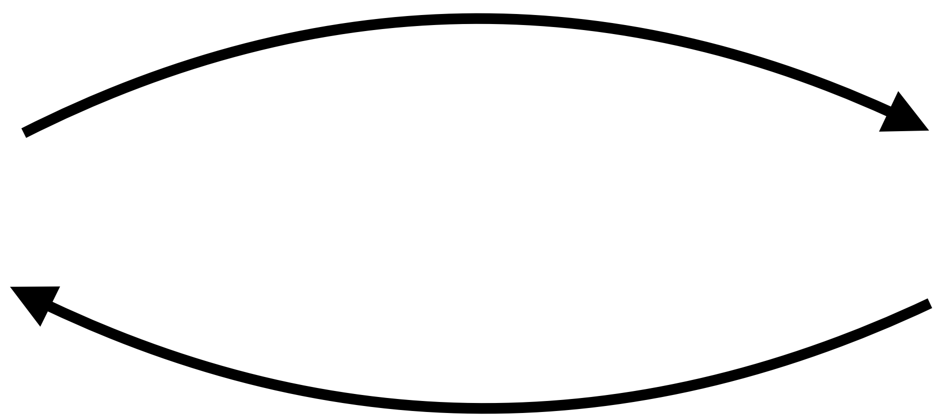

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Policy's state-action distribution

$$d^\pi(s, a) = (1 - \gamma) \sum_{h=0}^{\infty} \gamma^h \mathbb{P}^\pi\left((s_h, a_h) = (s, a)\right)$$

PC-PG: Policy Cover - Policy Gradient

1. **Form Cover:**

$$\{\pi_1, \pi_2, \ldots, \pi_n\}$$

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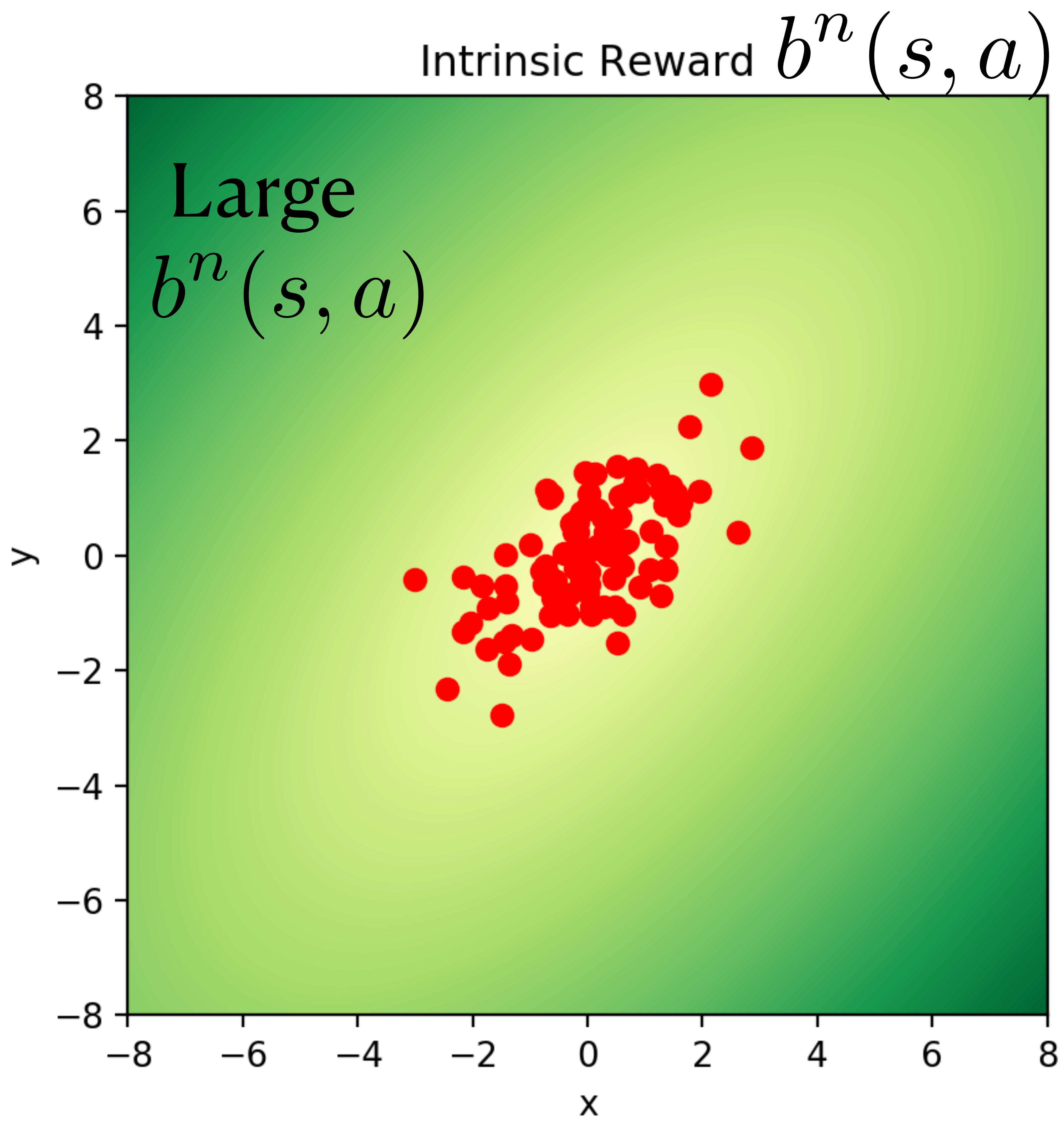

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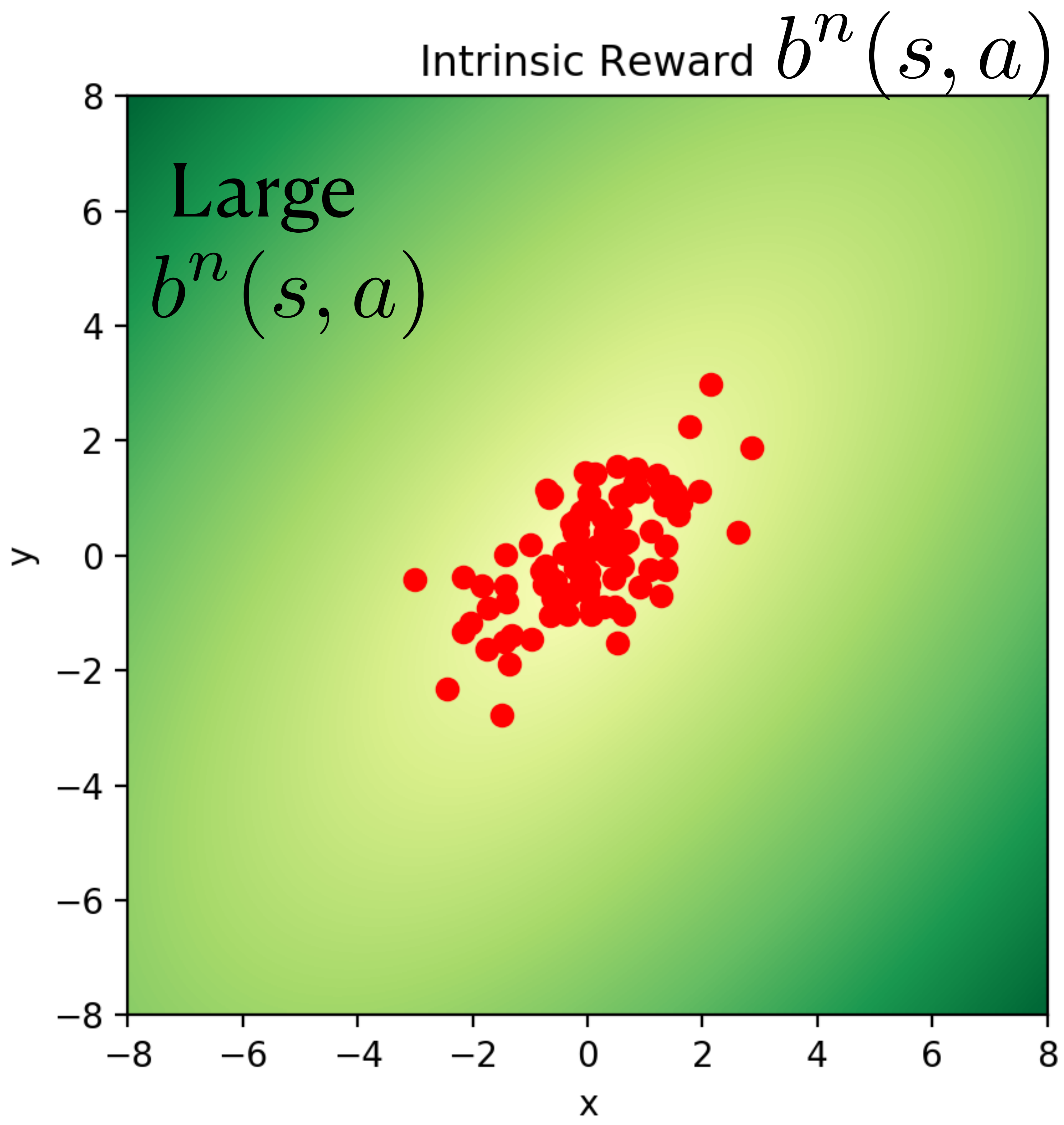

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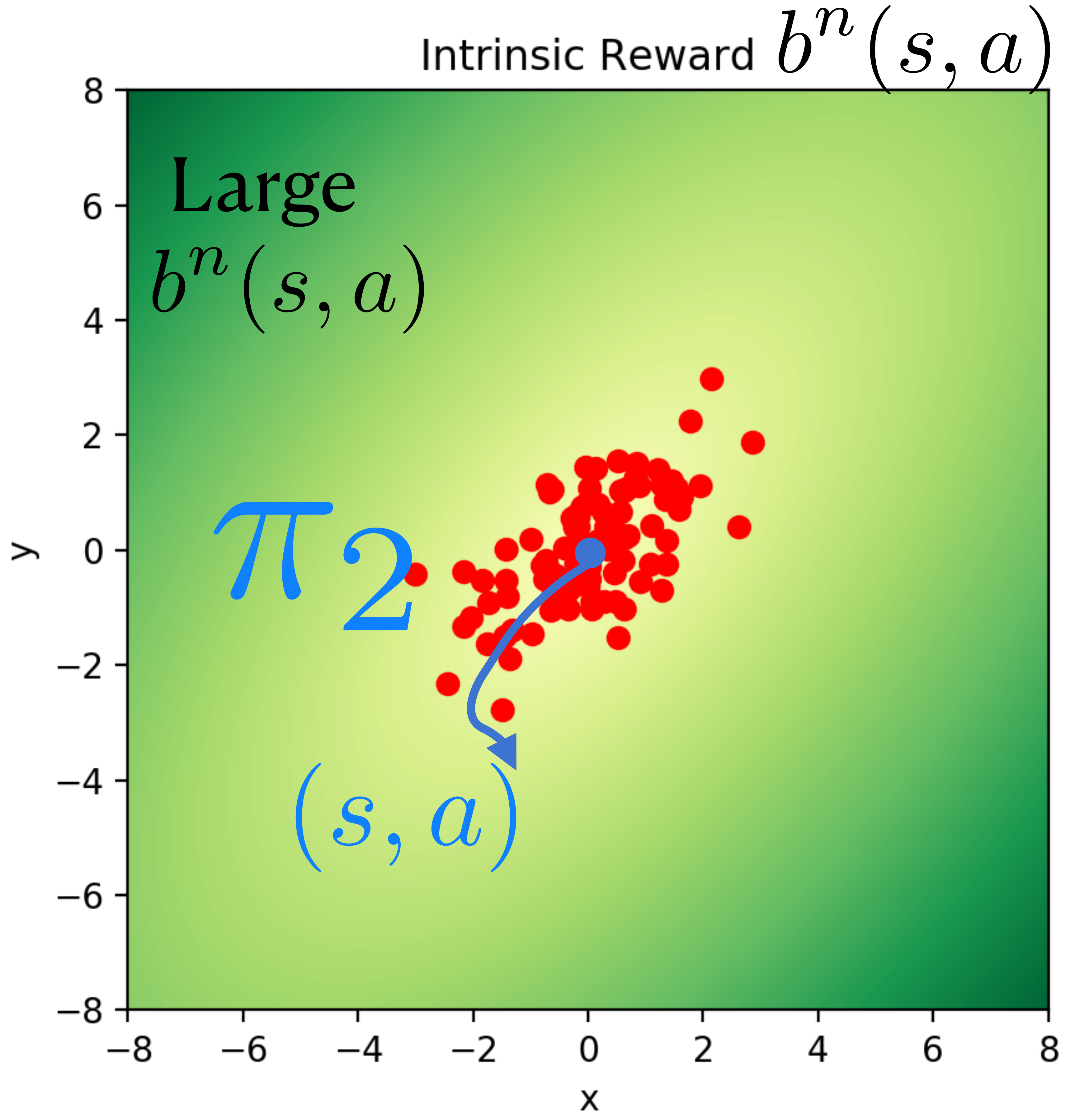

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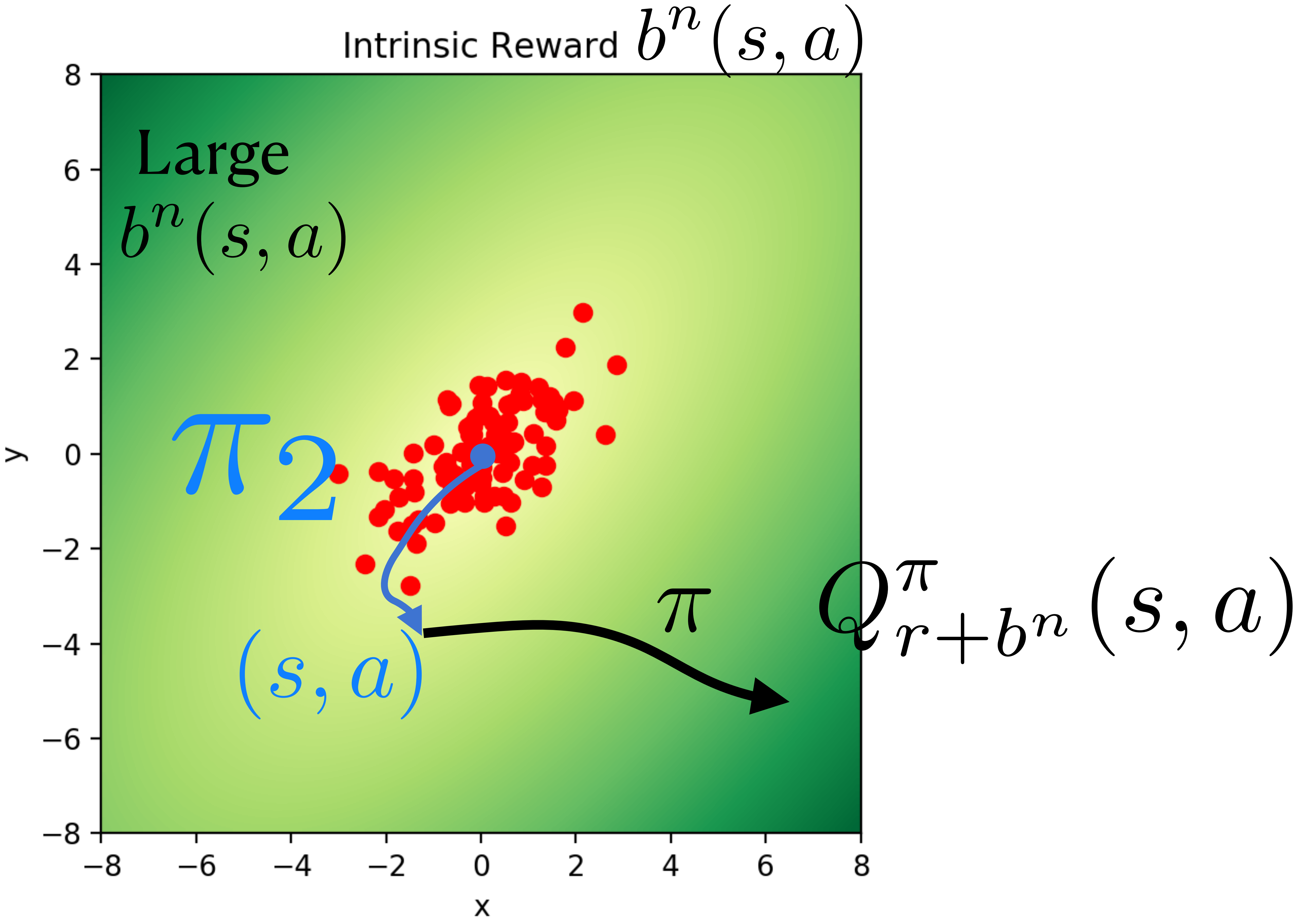

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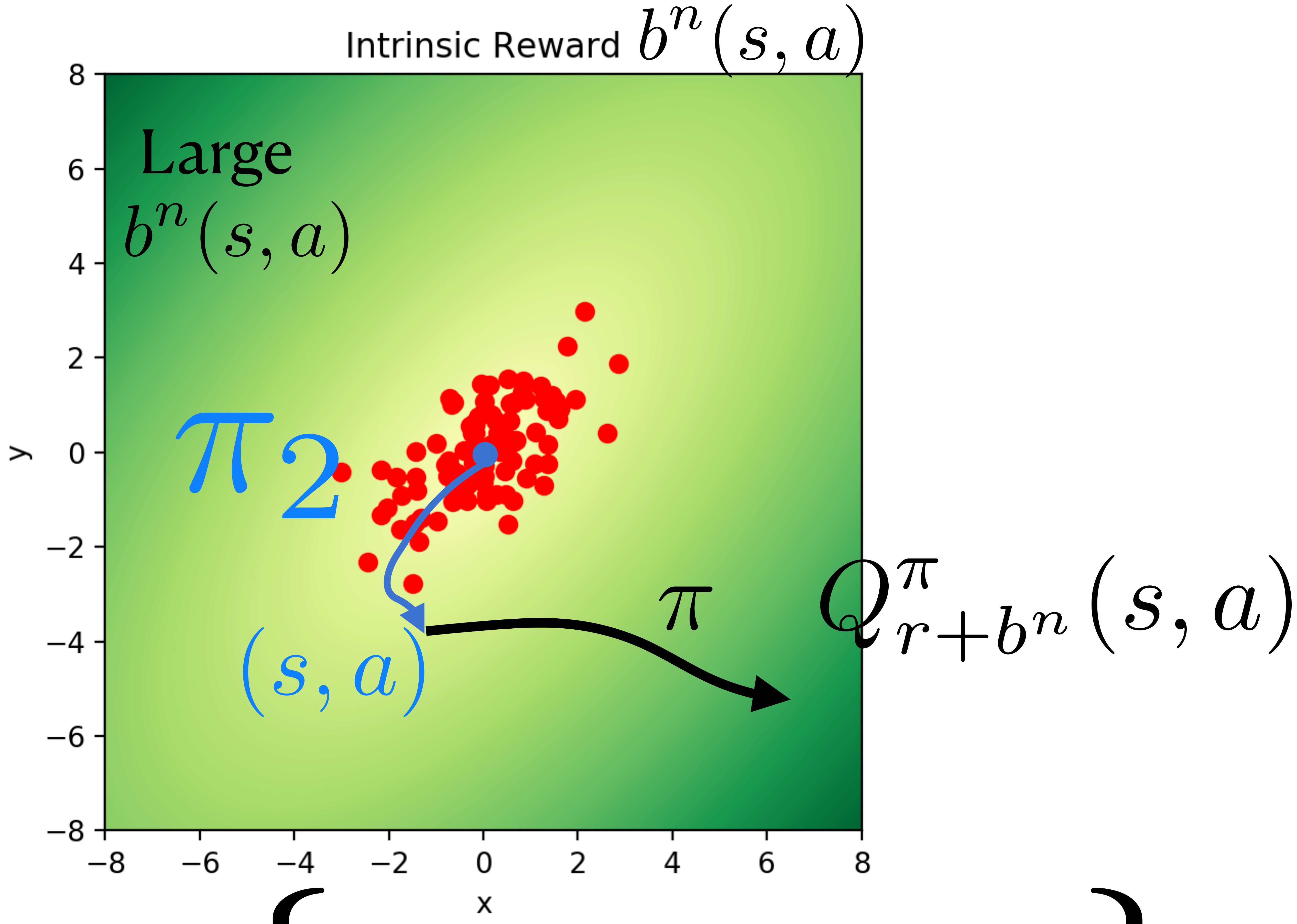

$$\left\{ (s_i, a_i), Q^{\pi}_{r+b^n}(s_i, a_i) \right\}$$

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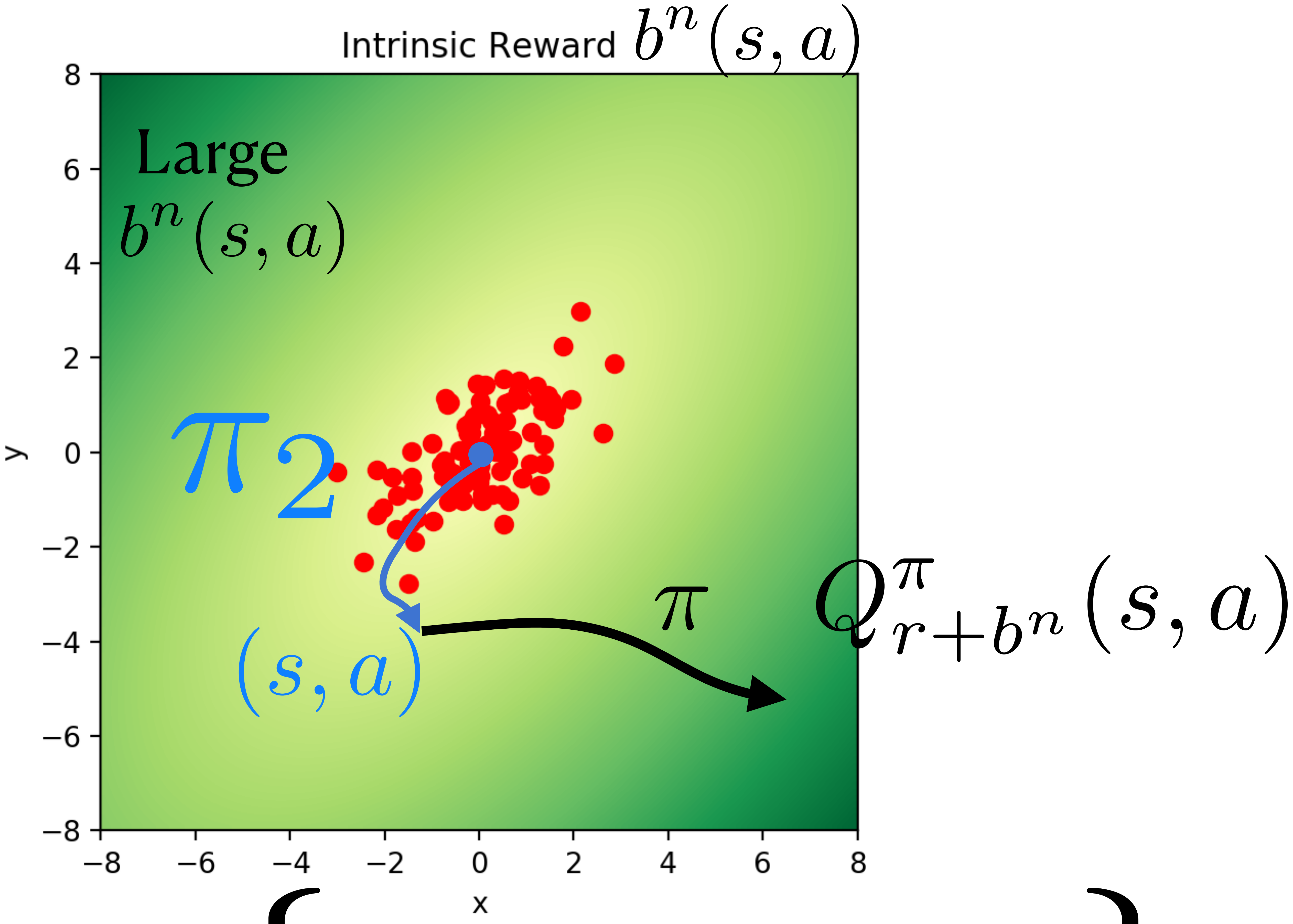

$$\left\{(s_i, a_i), Q^\pi_{r+b^n}(s_i, a_i)\right\}$$

$$\hat{f} = \max_{f\in\mathcal{F}} \sum_i \left(f(s_i, a_i) - Q^\pi_{r+b^n}(s_i, a_i)\right)^2$$

3. On-Policy Critic Fit (least square)

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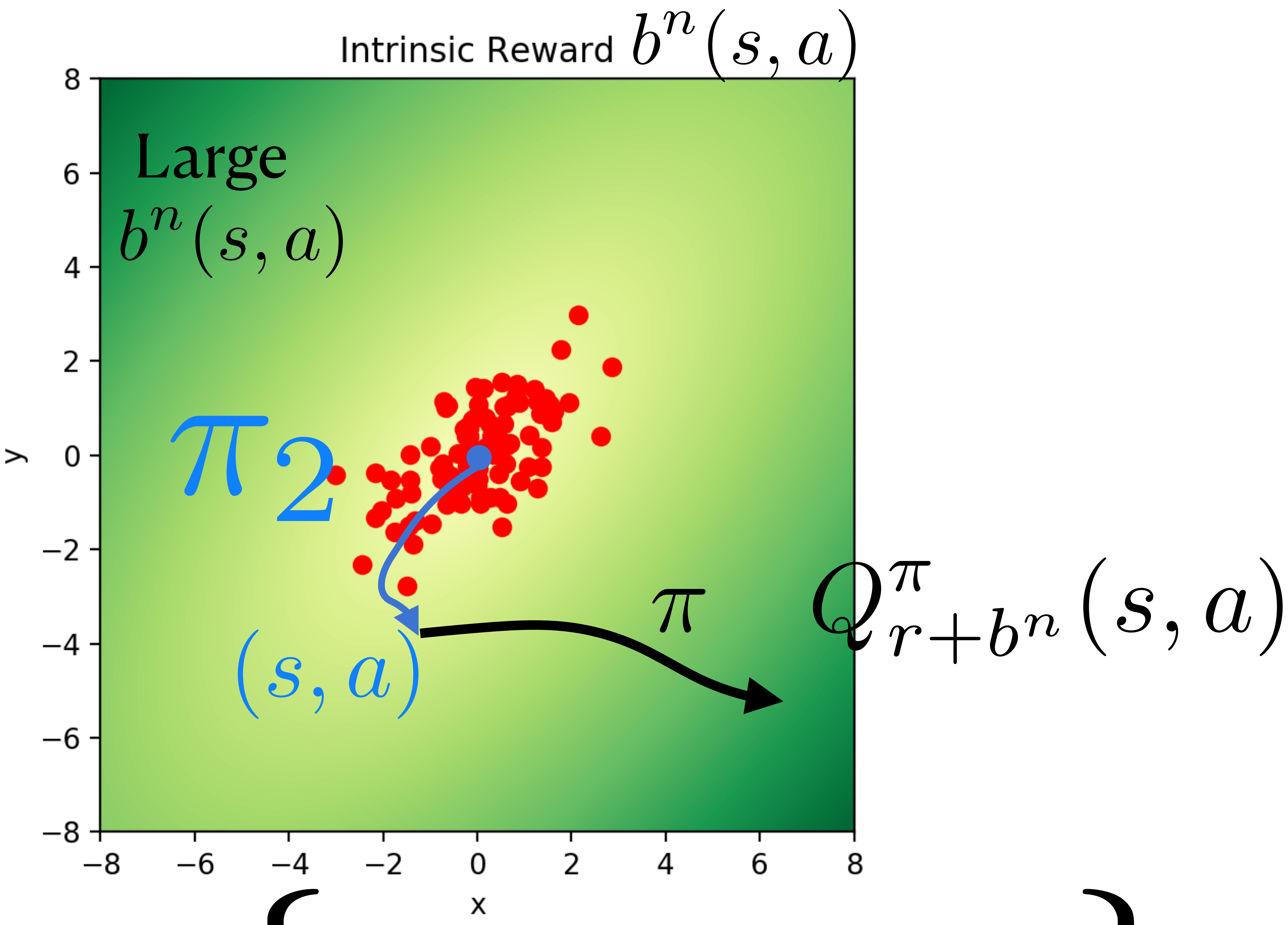

$$\pi(s,a) \Leftarrow \pi(s,a)\exp(\eta \hat{f}(s,a))$$

4. Actor NPG update (mirror descent)

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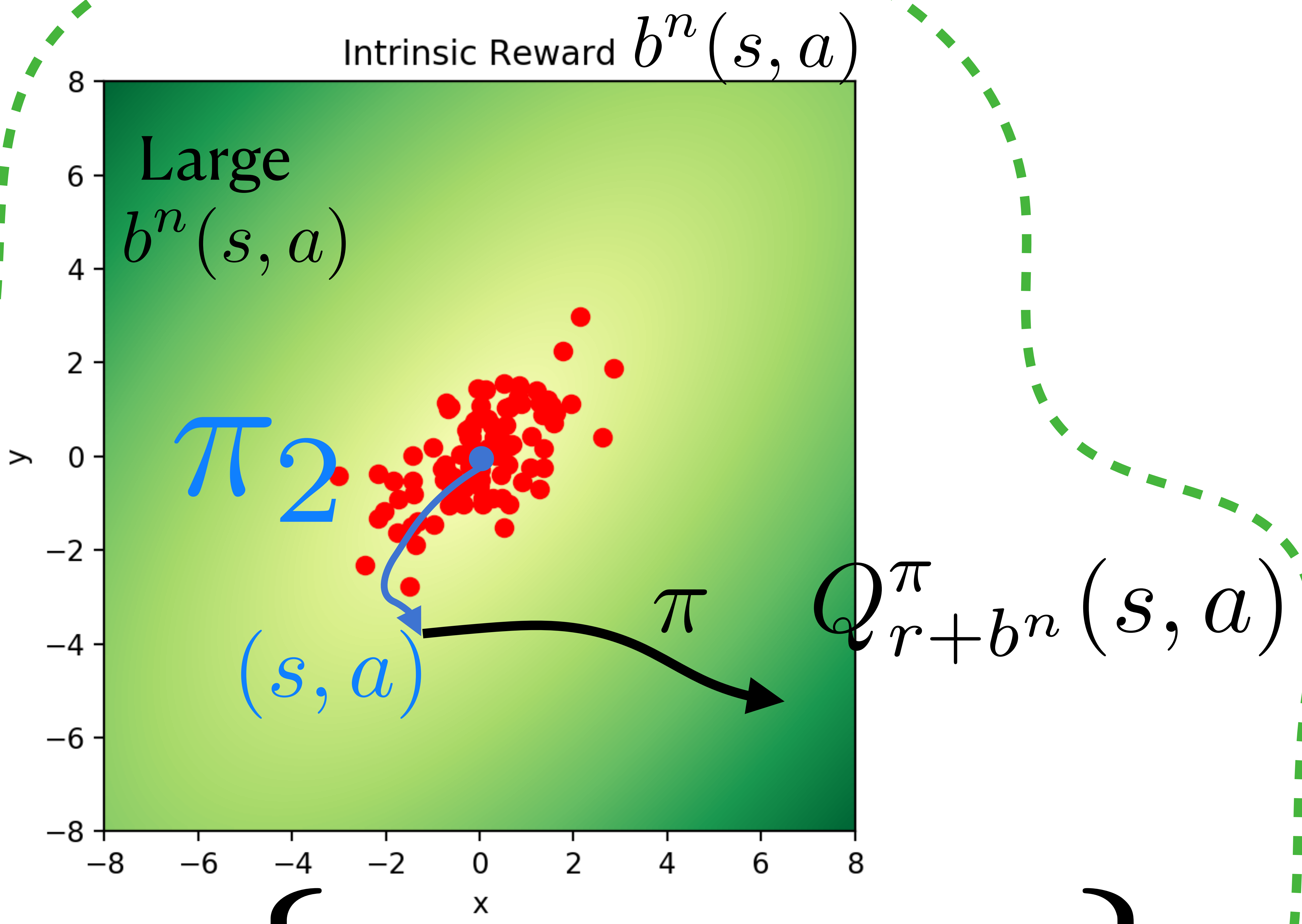

$$\left\{ (s_i, a_i),\, Q^{\pi}_{r+b^n}(s_i, a_i) \right\}$$

$$\hat{f} = \max_{f \in \mathcal{F}} \sum_i \left(f(s_i, a_i) - Q^{\pi}_{r+b^n}(s_i, a_i) \right)^2$$

3. On-Policy Critic Fit (least square)

$$\pi(s,a) \Leftarrow \pi(s,a) \exp(\eta \hat{f}(s,a))$$

4. Actor NPG update (mirror descent)

PC-PG: Policy Cover - Policy Gradient

1. Form Cover:

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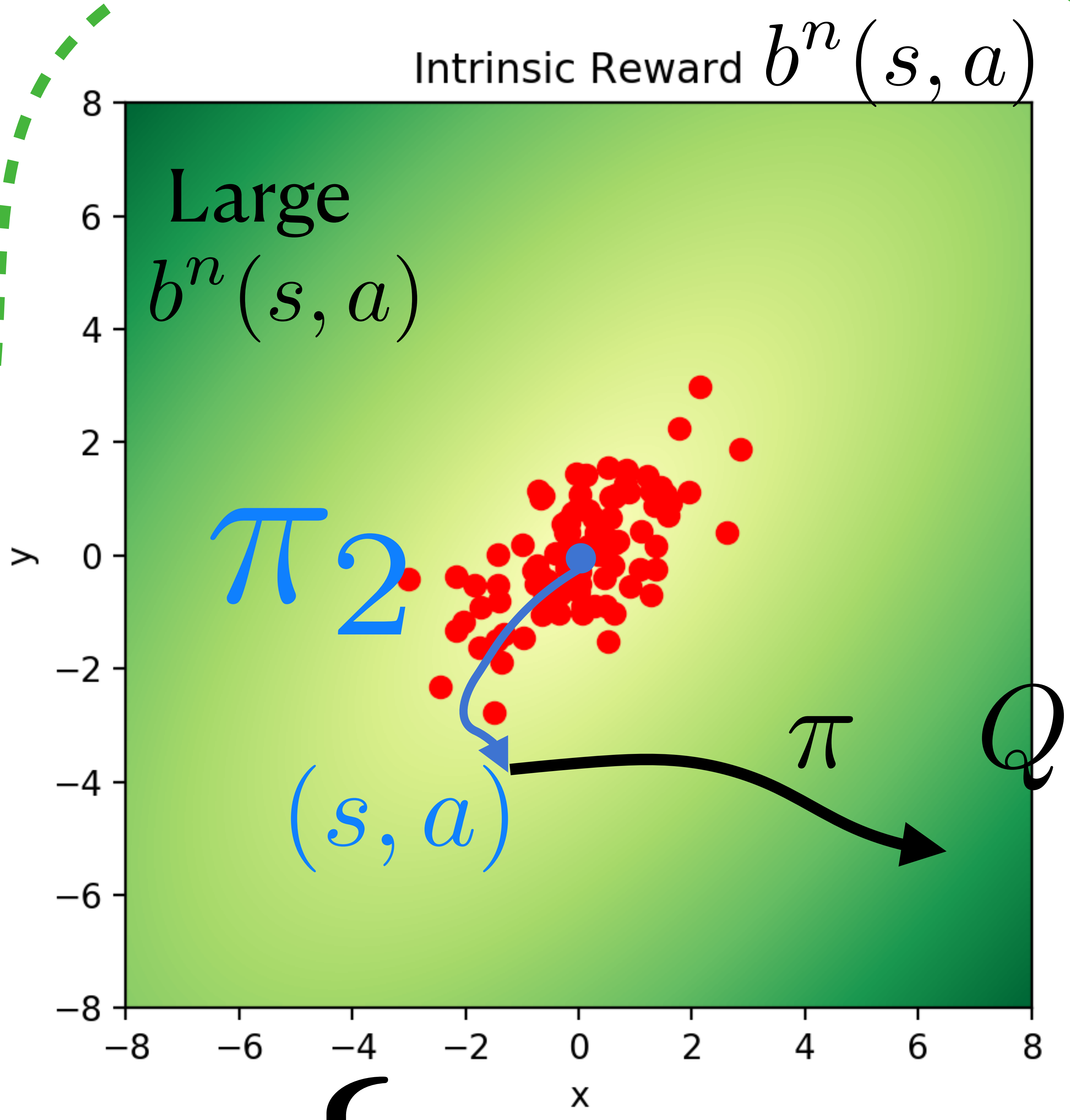

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5. Append

What objective function PC-PG is trying to optimize?

At episode n: Natural PG is optimizing

$$\mathbb{E}_{s_0, a_0 \sim \rho_n} \left[\sum_{t=0}^{\infty} \gamma^t \left(r(s_t, a_t) + b^n(s_t, a_t) \right) \right]$$

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Use the cover and roll-in via policies in the cover

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No more sparse reward!

PC-PG Specialized to Linear Function Approximation

Use linear function $\theta \cdot \phi(s, a)$ to approximate Q^π

1. Form Cover:

$$\{\pi_1, \pi_2, \ldots, \pi_n\}$$

$$\Sigma_{\pi_i} = \mathbb{E}_{s,a \sim d_{\pi_i}} \phi(s, a) \phi(s, a)^\top$$

$$\Sigma_n = \sum_{i=1}^{n} \Sigma_{\pi_i} + \lambda I$$

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→

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Rewarding (s, a) whose feature $\phi(s, a)$ aligns with small eigenvectors

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One-hot vector: $\phi(s, a) \in \mathbb{R}^{SA}$

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Rewarding state that has low probability of being covered

$$\phi(s, a)^\top \Sigma_n^{-1} \phi(s, a) \Leftarrow \frac{1}{\rho_n(s, a)}$$

Well-Specified Setting: Linear MDPs (and Tabular MDPs)

Reward and transition in RKHS:

[RKHS version of the Linear mdp model from Jin et al, 19]

$$r(s,a) = \theta \cdot \phi(s,a), P(\cdot \,|\, s,a) = \mu\phi(s,a)$$

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with # of samples

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Dim of feature

(extendable to RKHS w/ Information Gain)

Comparison to vanilla Natural PG in Linear MDPs

[Agarwal et al 19]

A wide initial distribution:

$$\kappa = 1/\sigma_{\min}\left(\mathbb{E}_{s,a\sim\mu_0}\phi(s,a)\phi(s,a)^\top\right) < \infty$$

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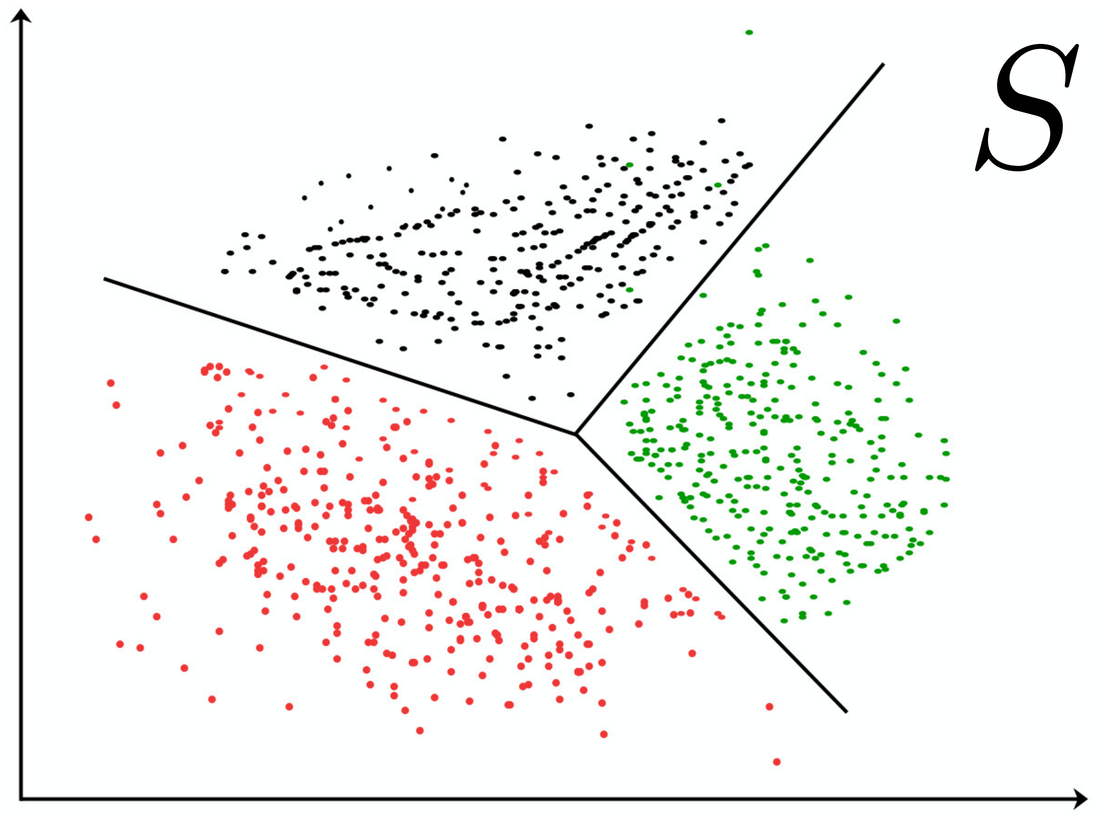
PC-PG eliminates the condition number by actively **exploring** and building **policy cover**

Robustness to Model-Misspecification

Average VS ℓ_∞

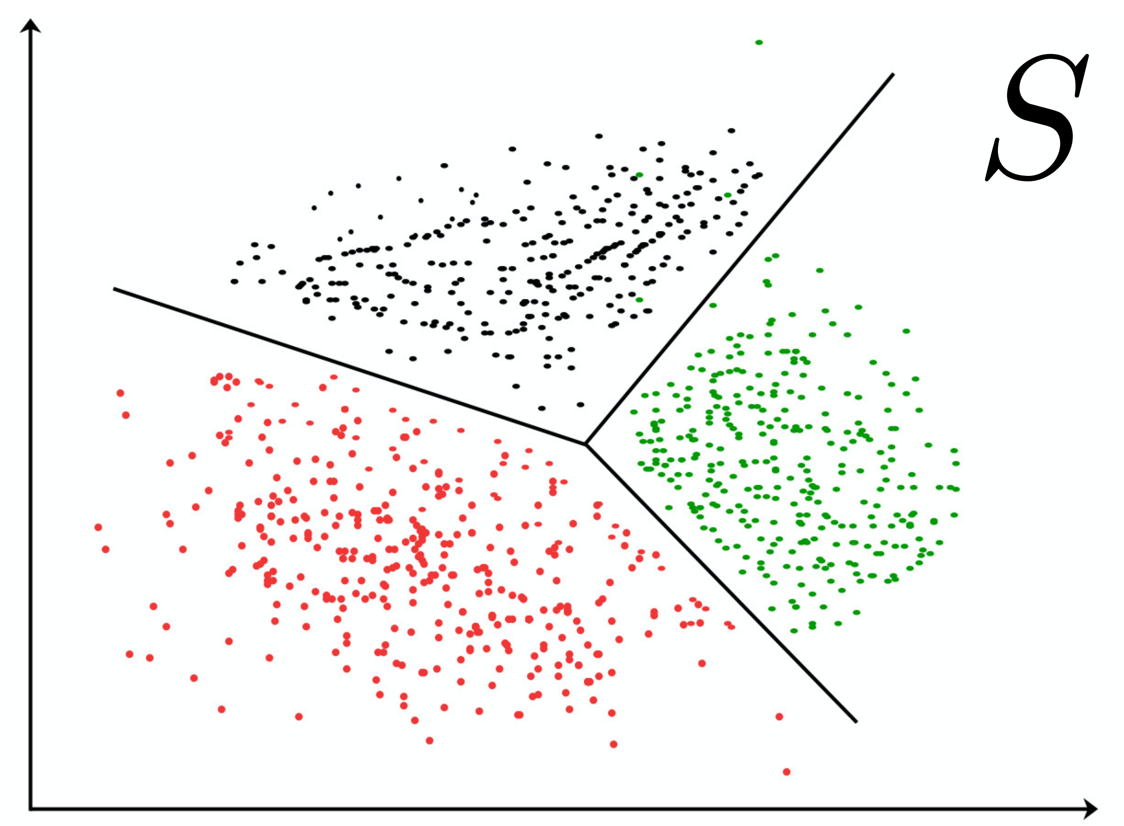
Model-Misspecification from State-Abstraction



$$\phi : S \to Z$$

Group "similar" states (s) into an abstracted state (z)

Model-Misspecification from State-Abstraction



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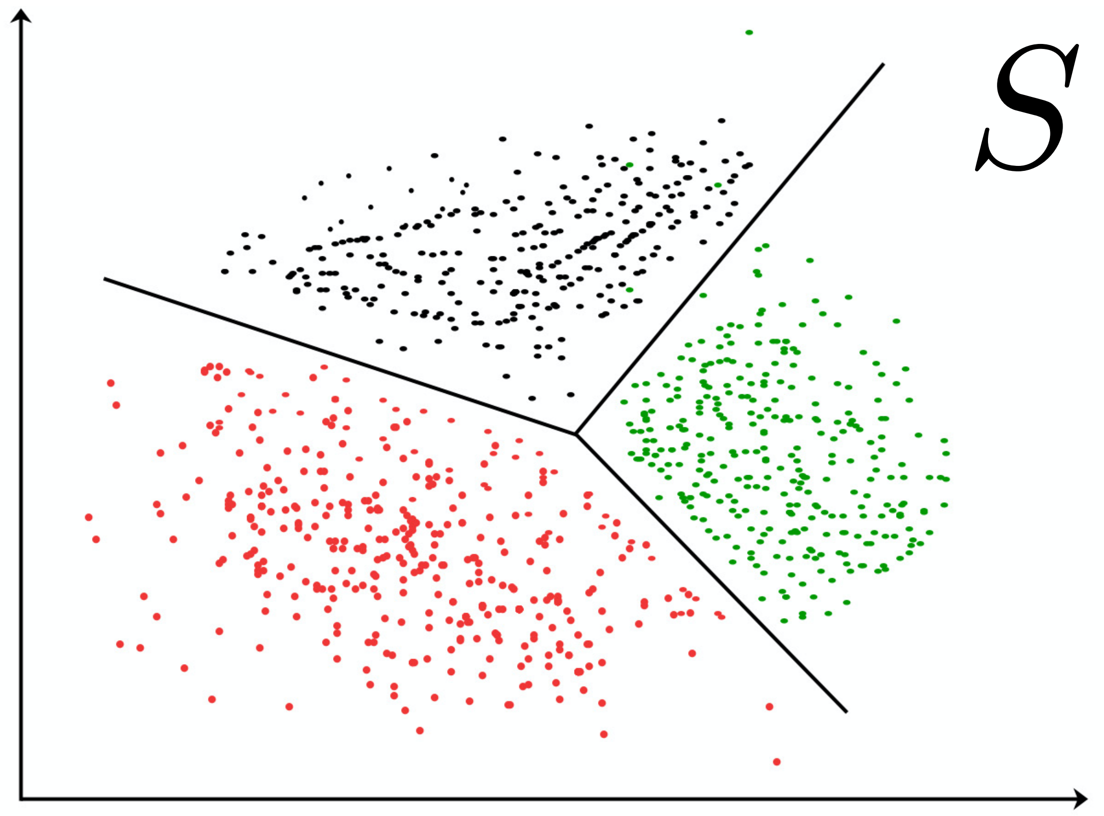

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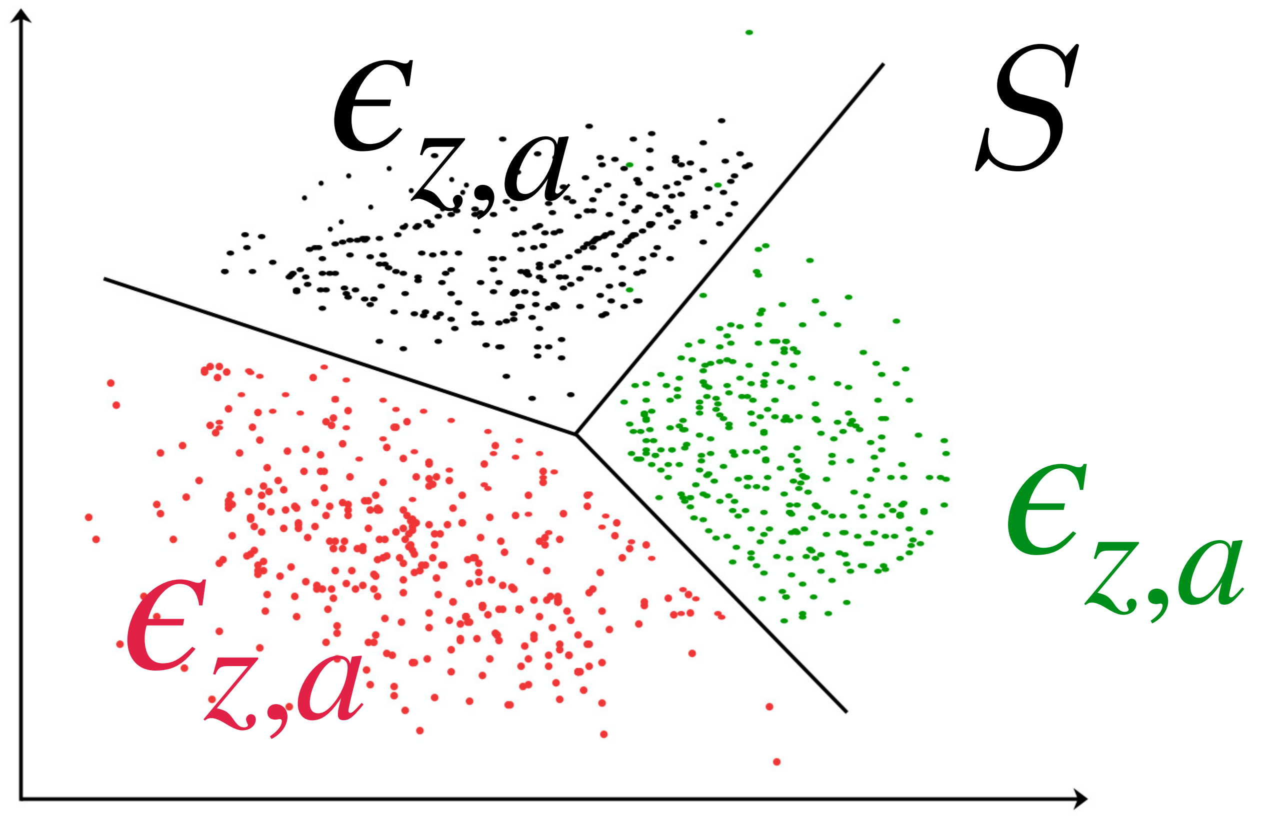

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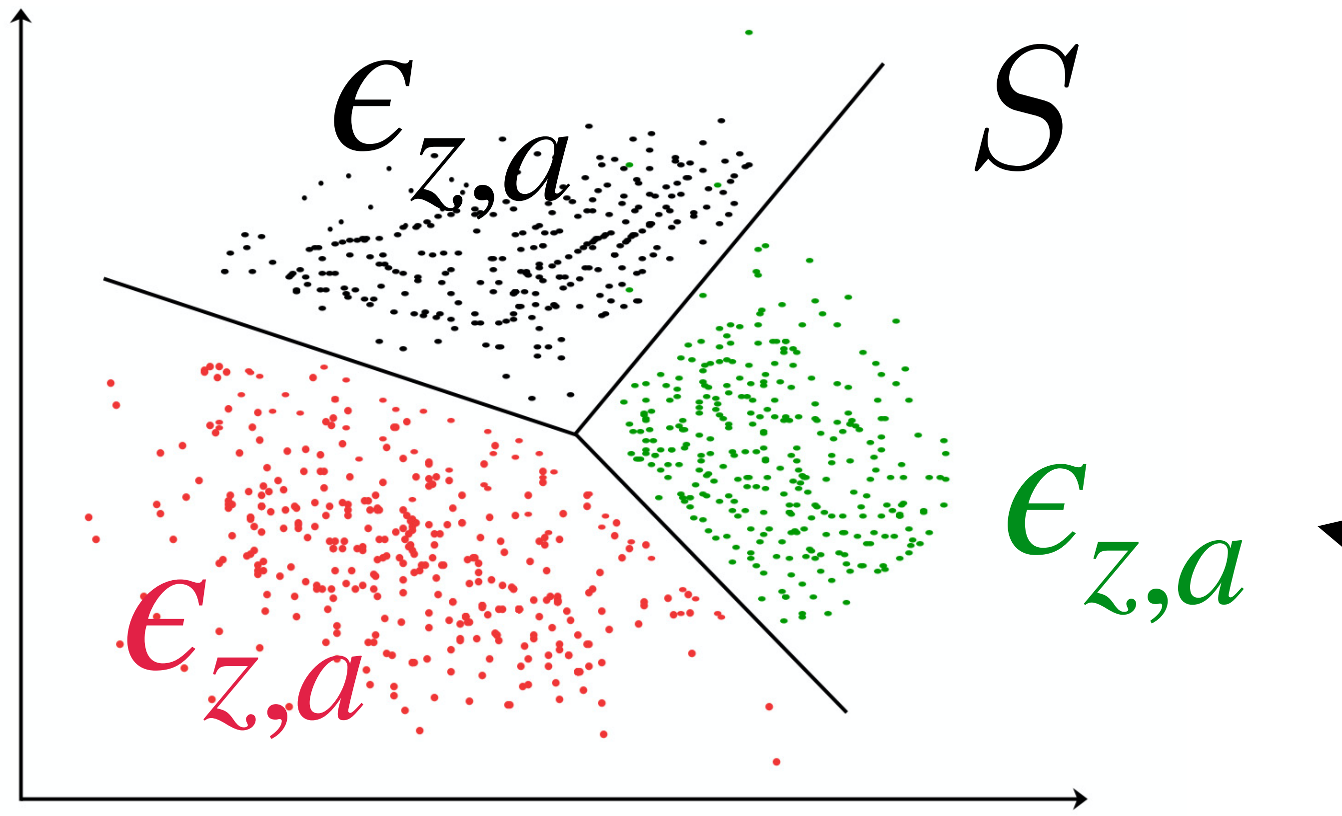

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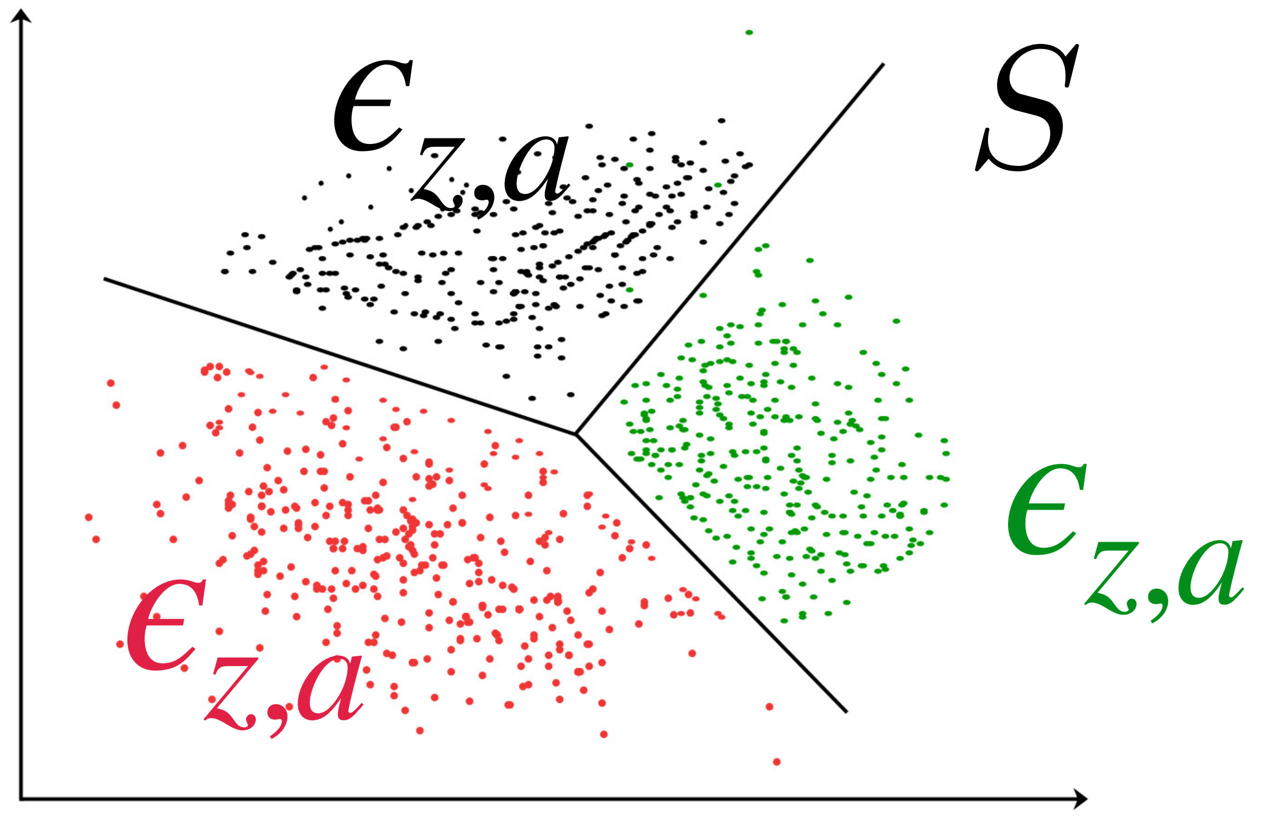

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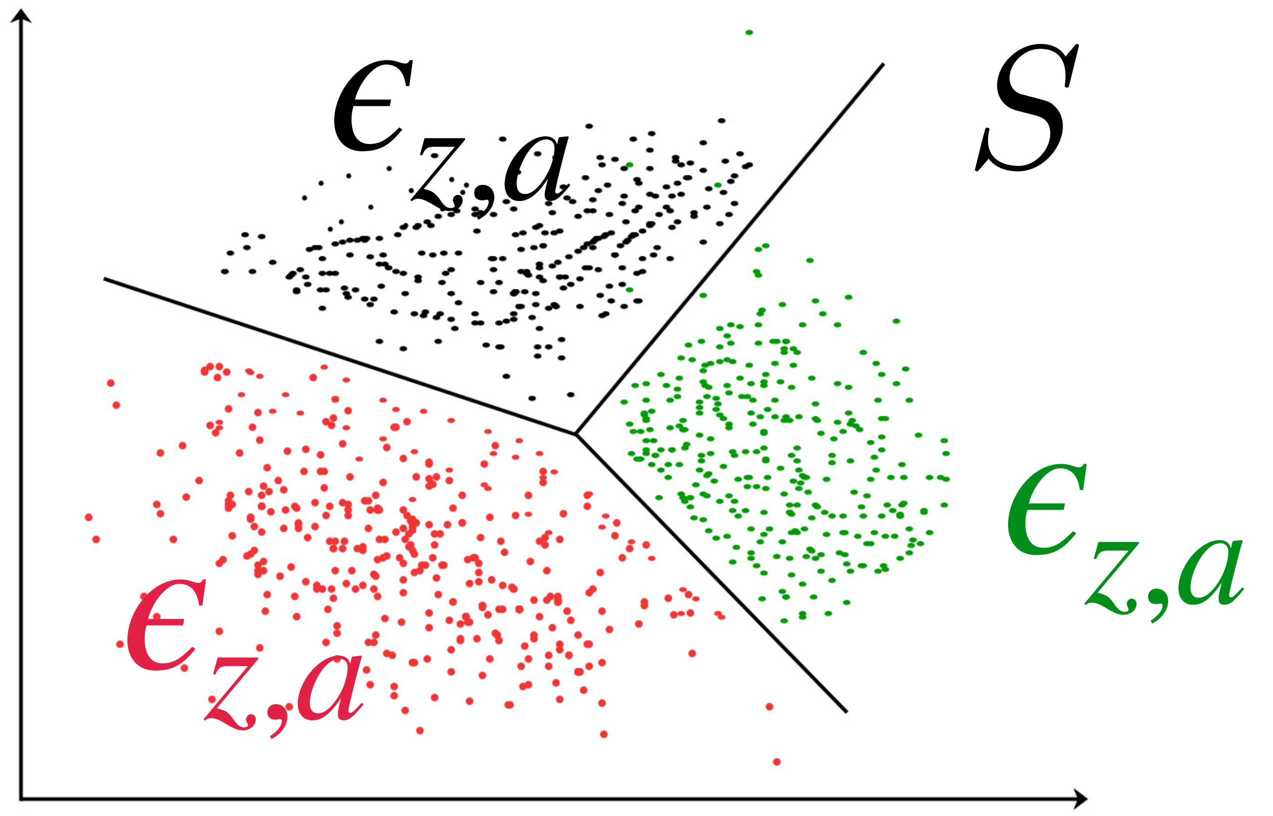

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PC-PG

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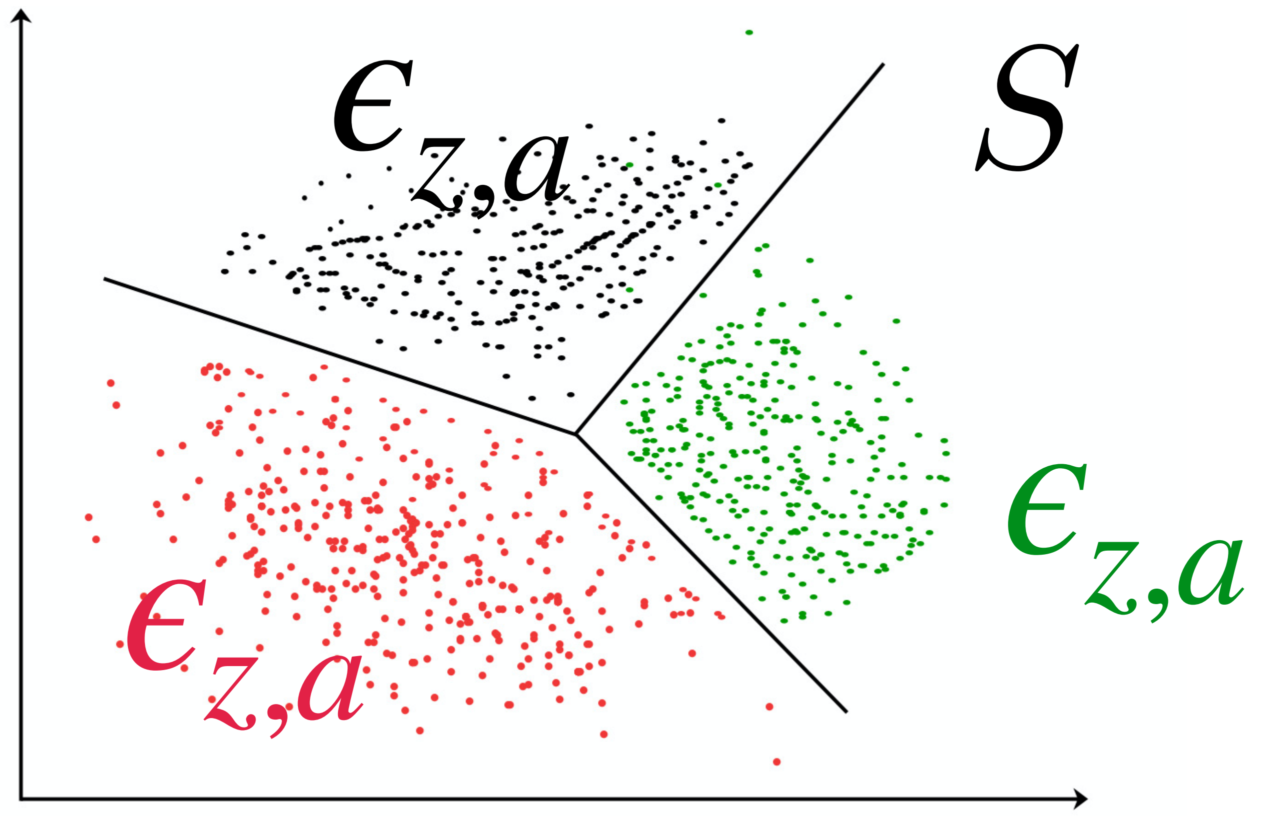

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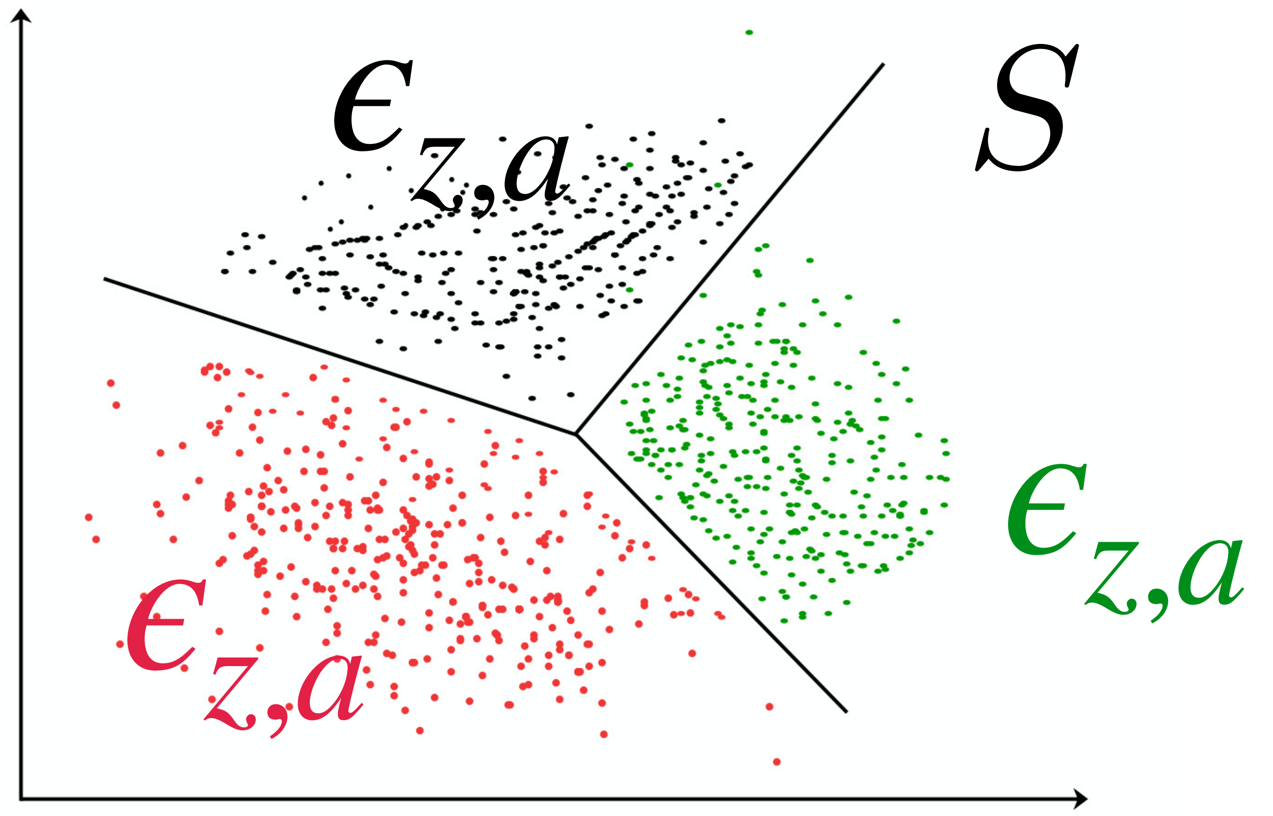

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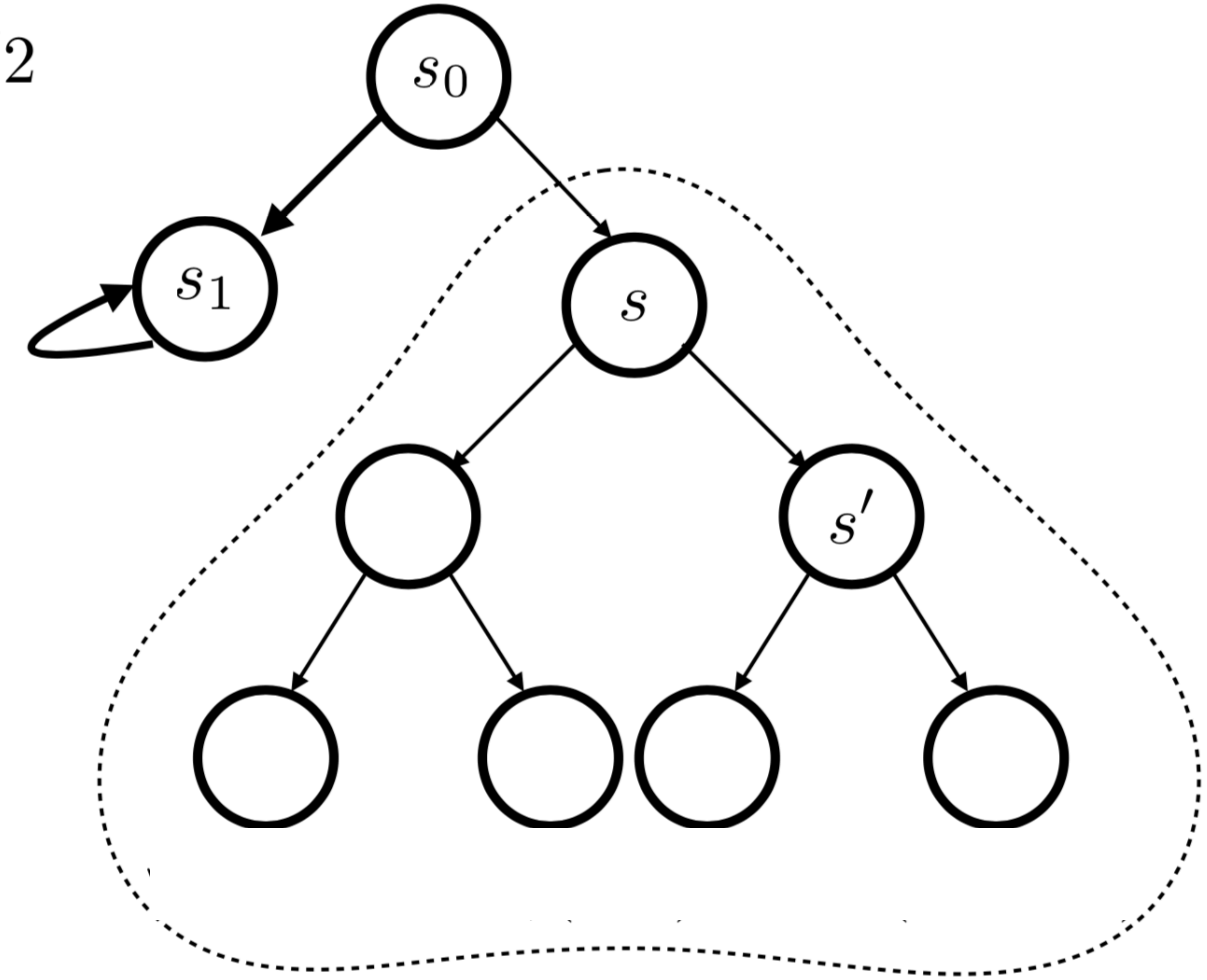
Average over comparator's distribution

Additional Example w/ linear approximation

$r(s_0, L) = 1/2$



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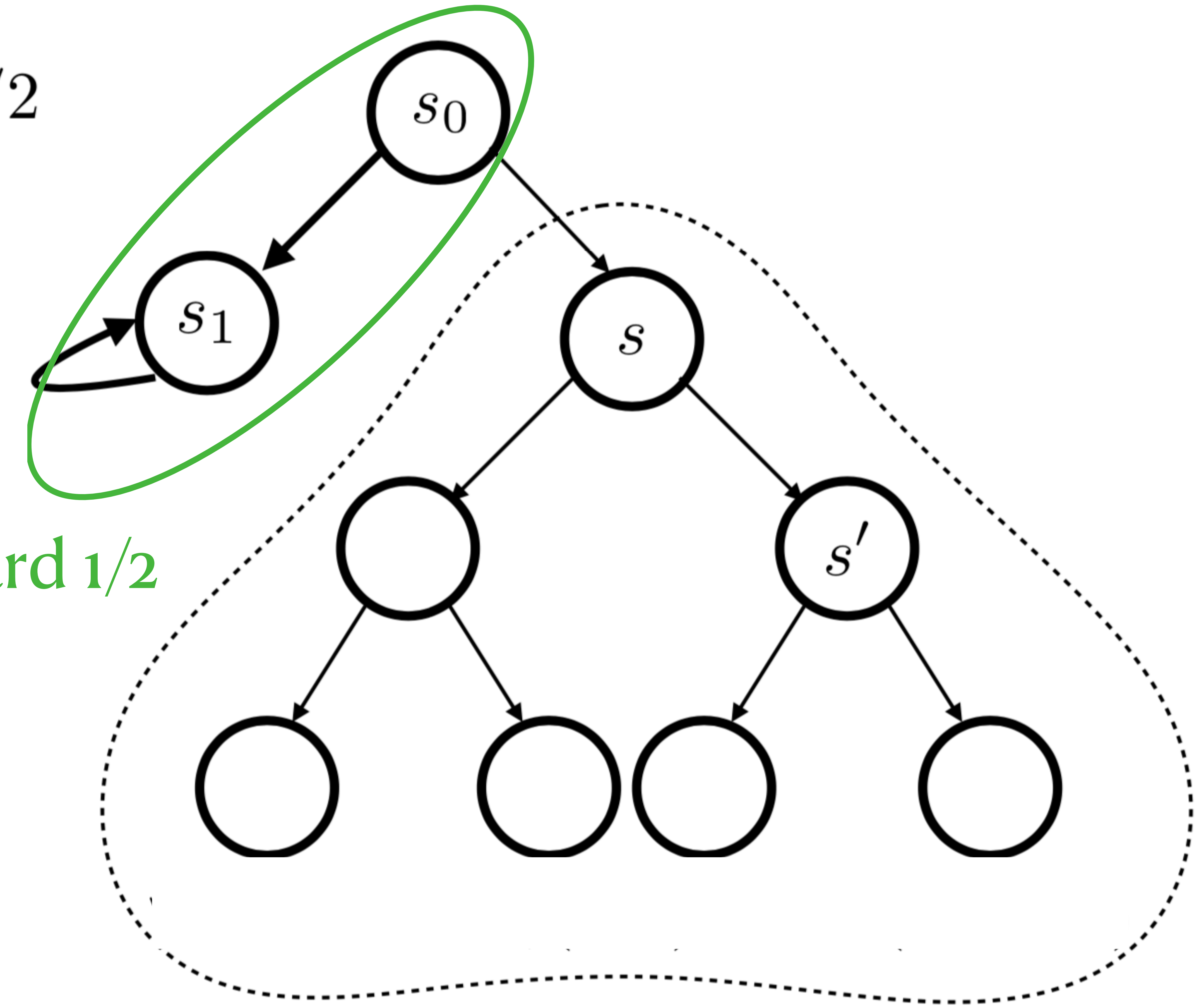

Path with reward 1/2

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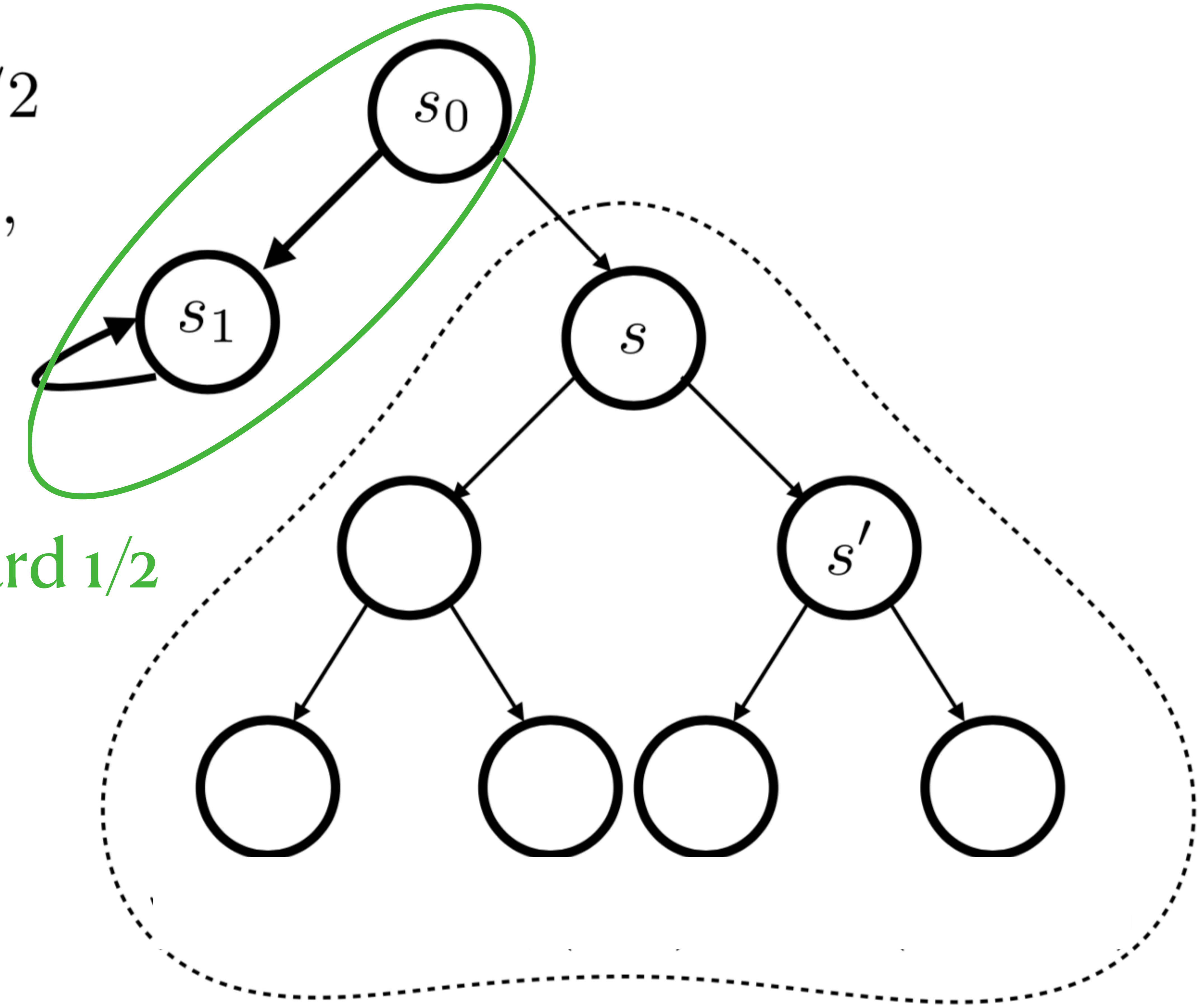
$$r(s_0, L) = 1/2$$
$$\phi(s_0, L) = e_1,$$



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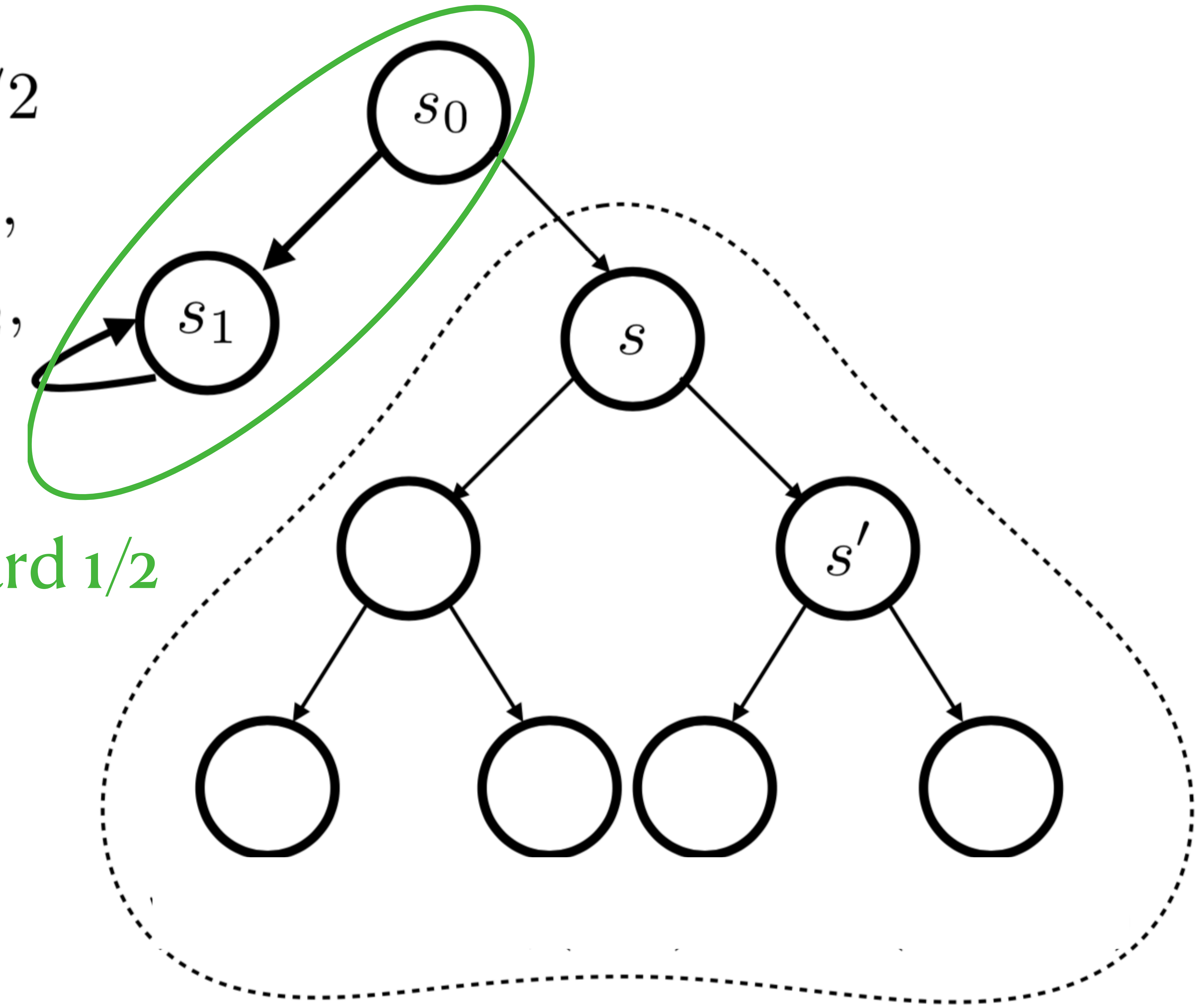
$\phi(s_0, R) = e_2,$



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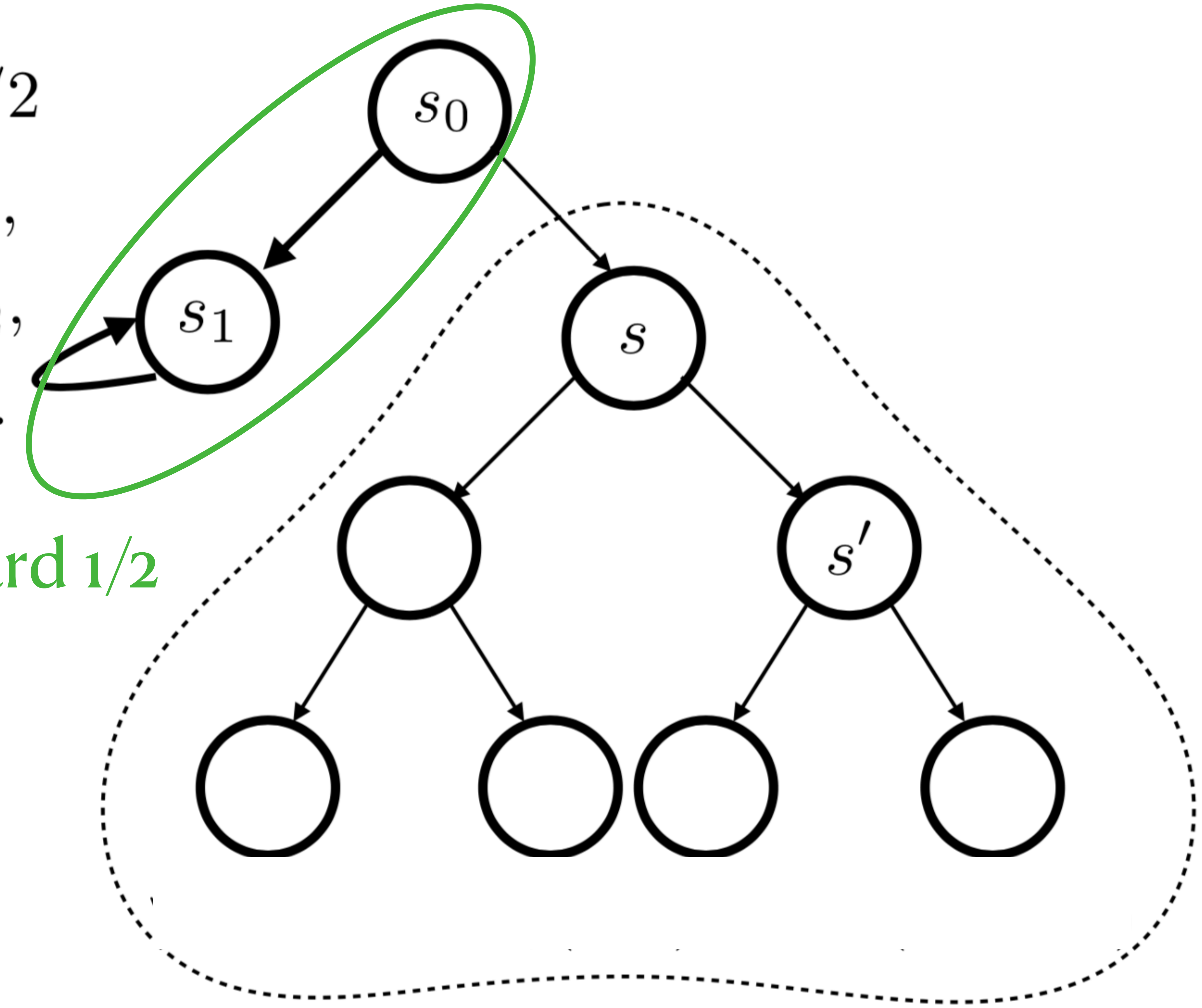

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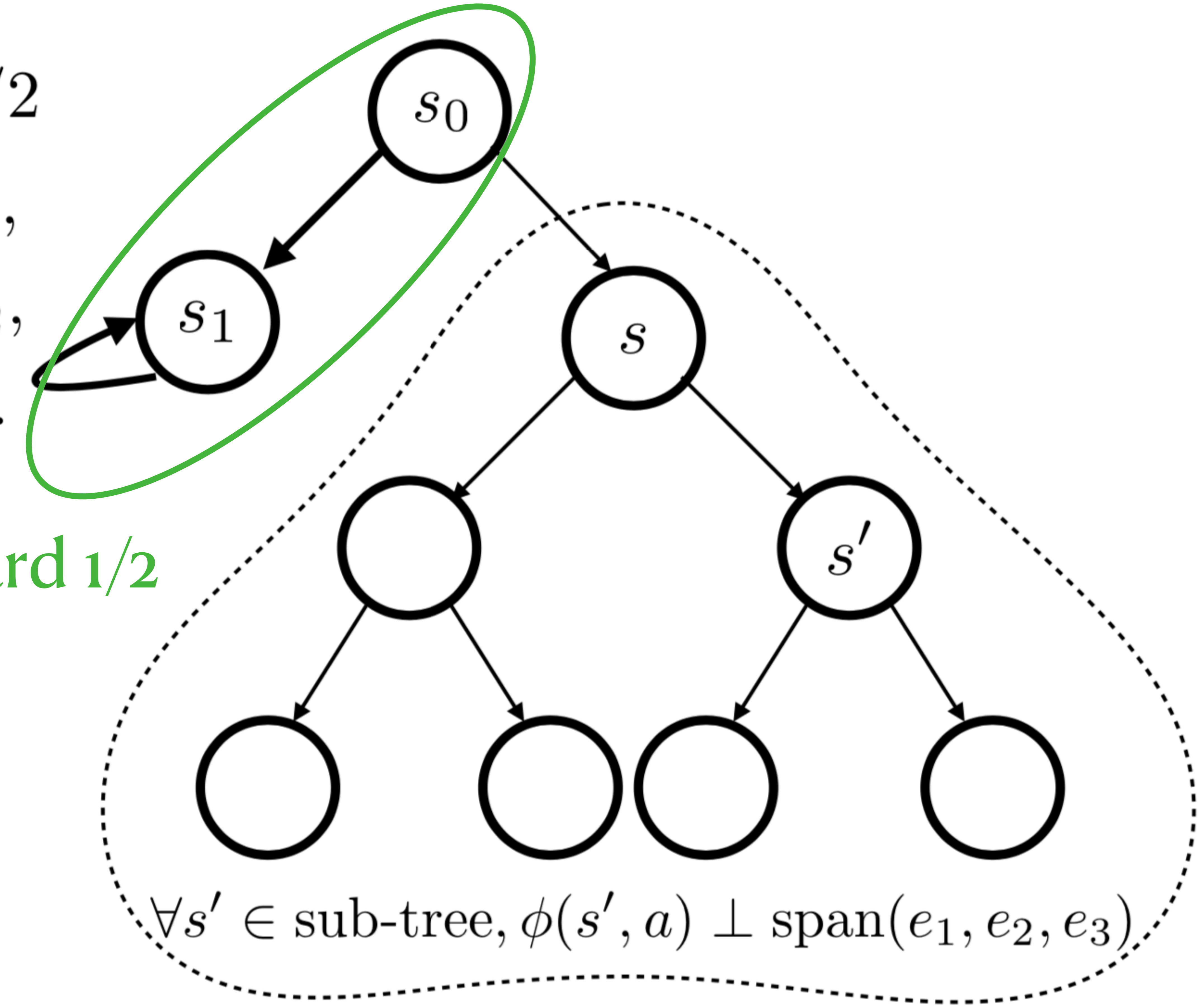

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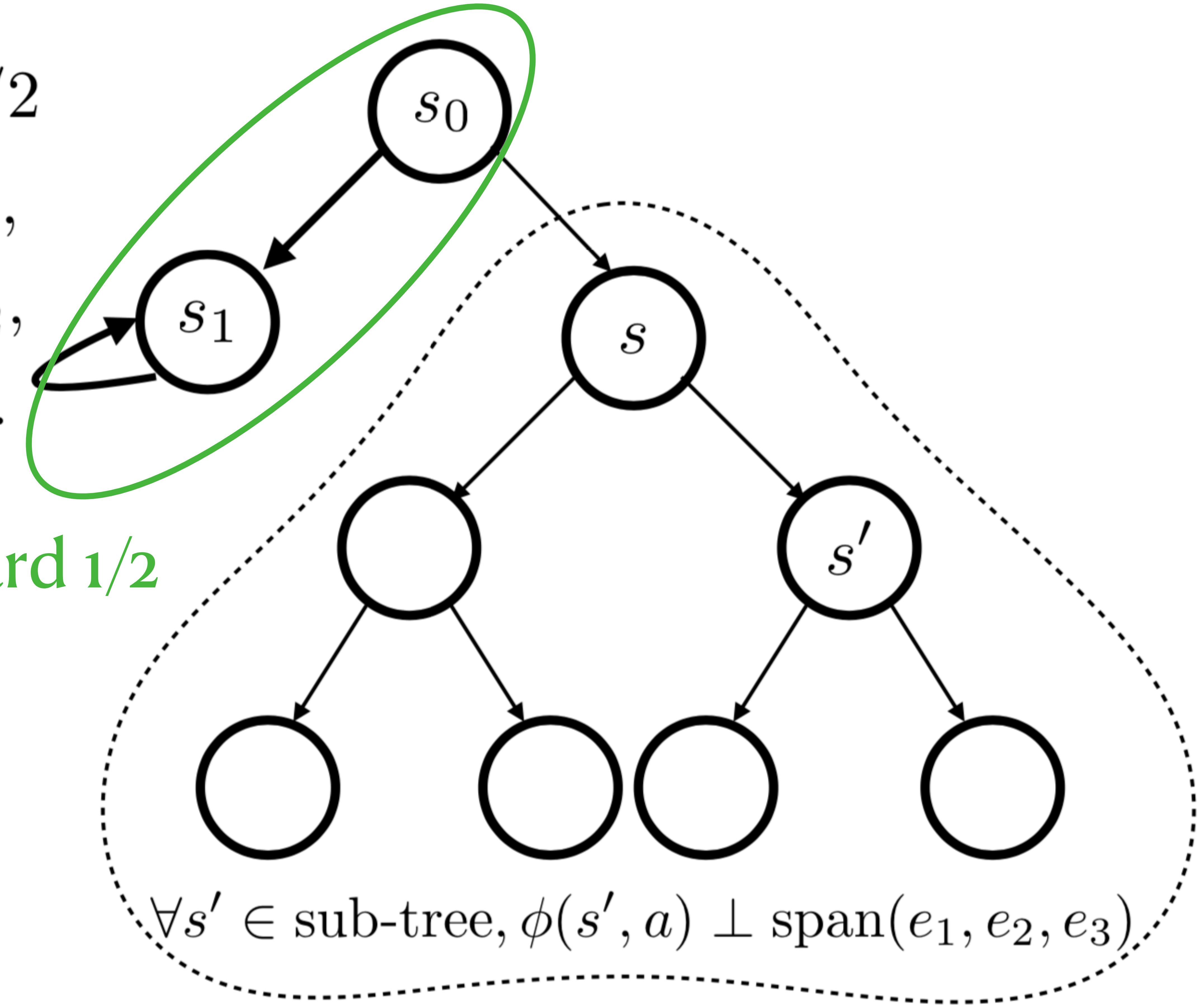

Path with reward 1/2

Features is **arbitrary** in the tree, i.e., model-misspecification can be very **serious**.

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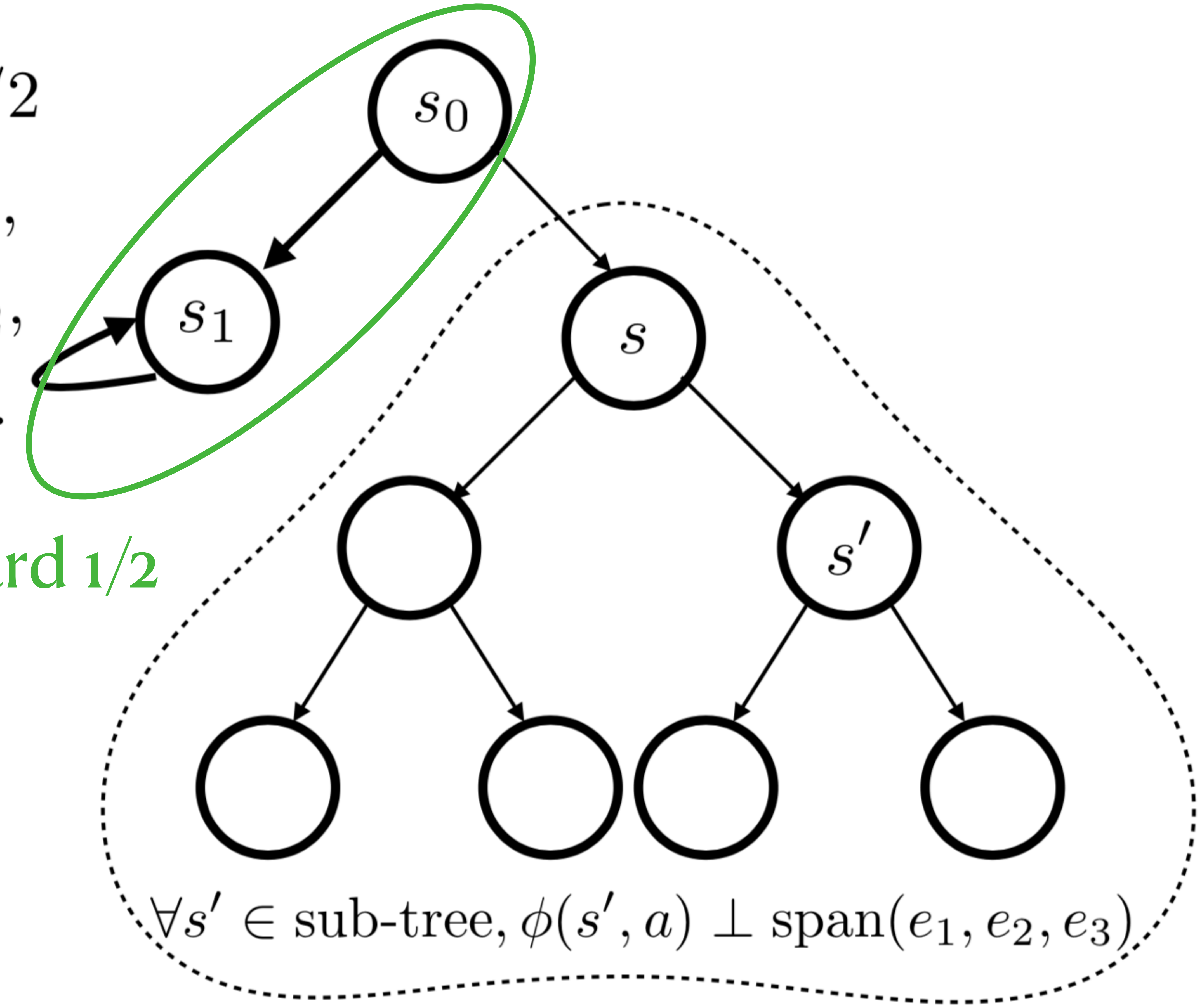

Path with reward 1/2

PC-PG is at least as good as the green trajectory

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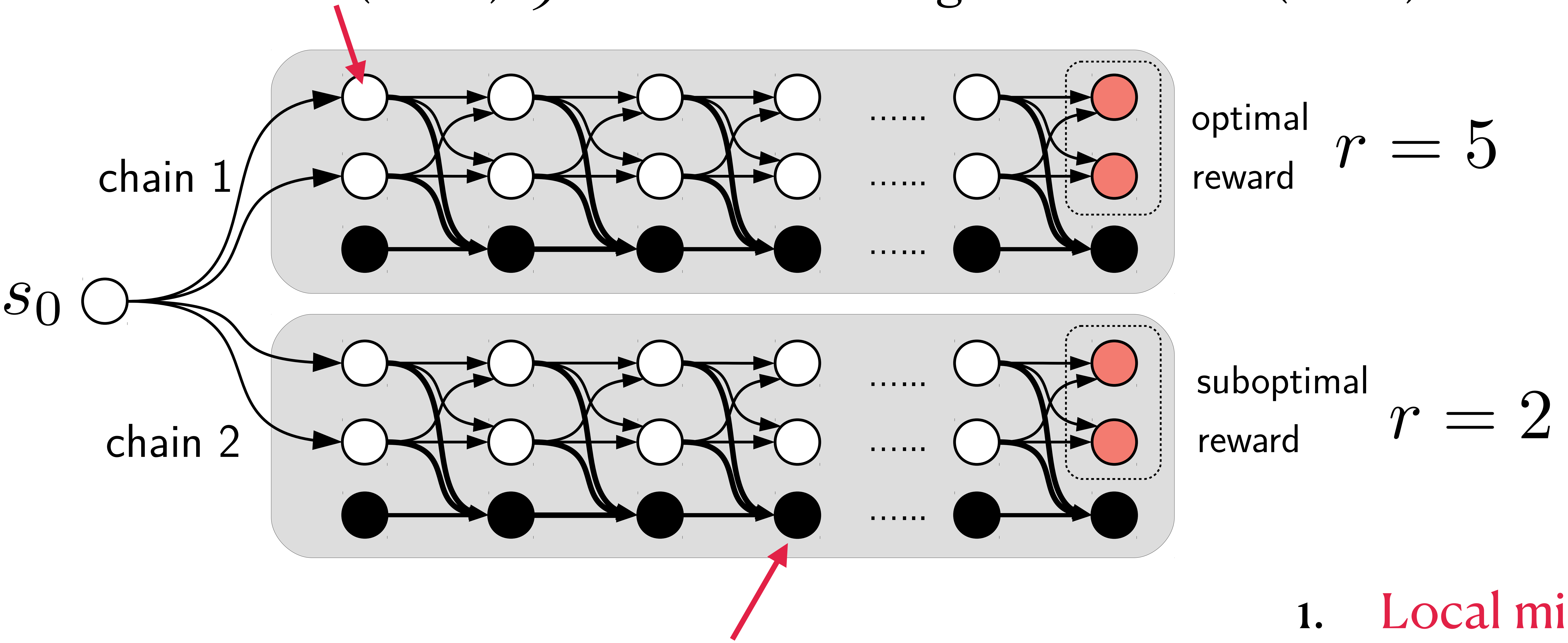
Experiments

Bidirectional Combination Lock

Good state (white): 9 out of 10 actions go to bad state (black)



Bad state has Anti-shaped reward: $r = 1/H$

1. Local minima
2. Forgetting

Experiments on Bi-directional Comb Lock

Feature vector: binary vector indicating state-action and time step

Policy Opt procedure: PPO w/ NN policy

Bonus: $\phi^\top \Sigma_n^{-1} \phi$

Experiments on Bi-directional Comb Lock

Feature vector: binary vector indicating state-action and time step

Policy Opt procedure: PPO w/ NN policy

Bonus: $\phi^\top \Sigma_n^{-1} \phi$

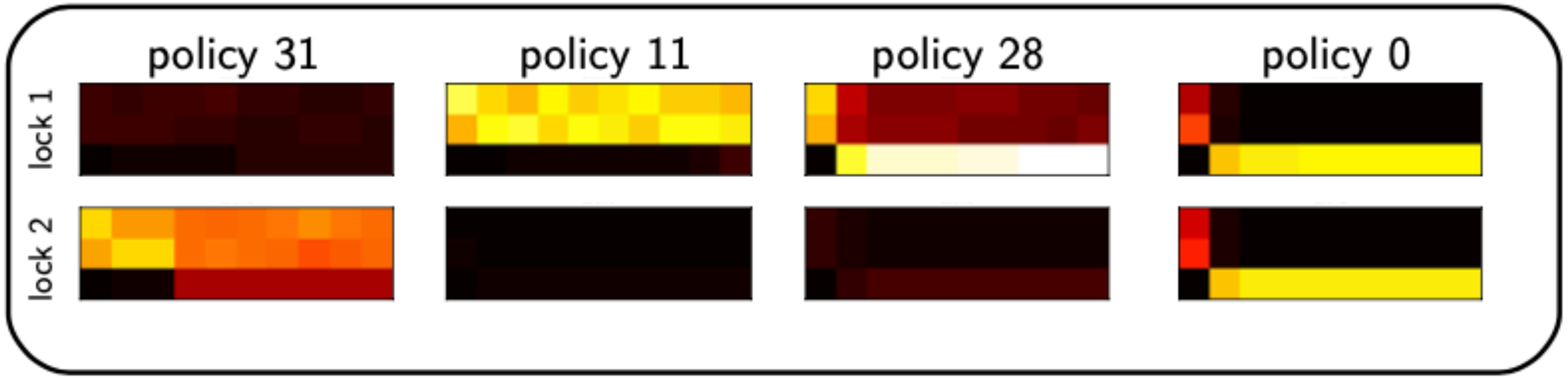
Success Rate (visit two chains):

Algorithm	Horizon			
	2	5	10	15
PPO	1.0	0.0	0.0	0.0
PPO+RND	0.75	0.40	0.50	0.55
PC-PG	1.0	1.0	1.0	1.0

Experiments on Bi-directional Comb Lock

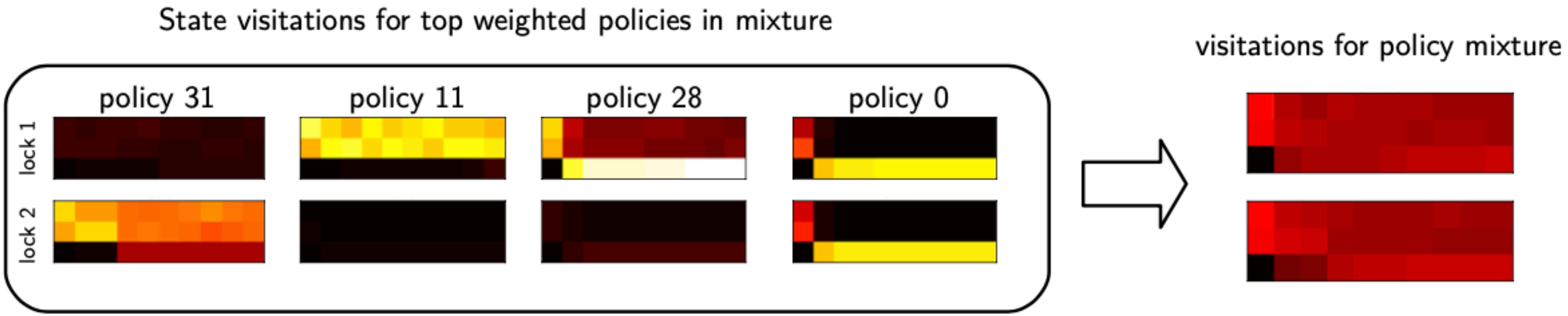
Due to the policy cover PC-PG maintains..

State visitations for top weighted policies in mixture



Experiments on Bi-directional Comb Lock

Due to the policy cover PC-PG maintains..



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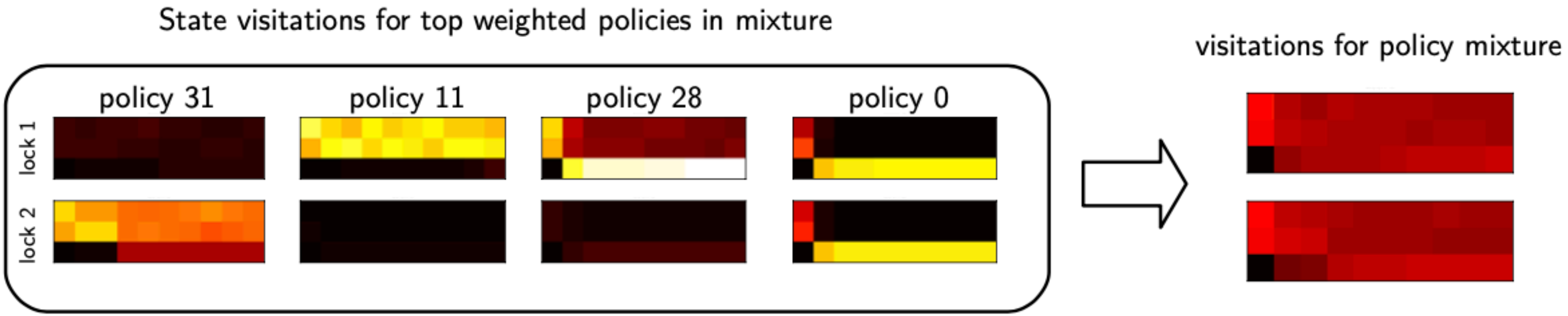

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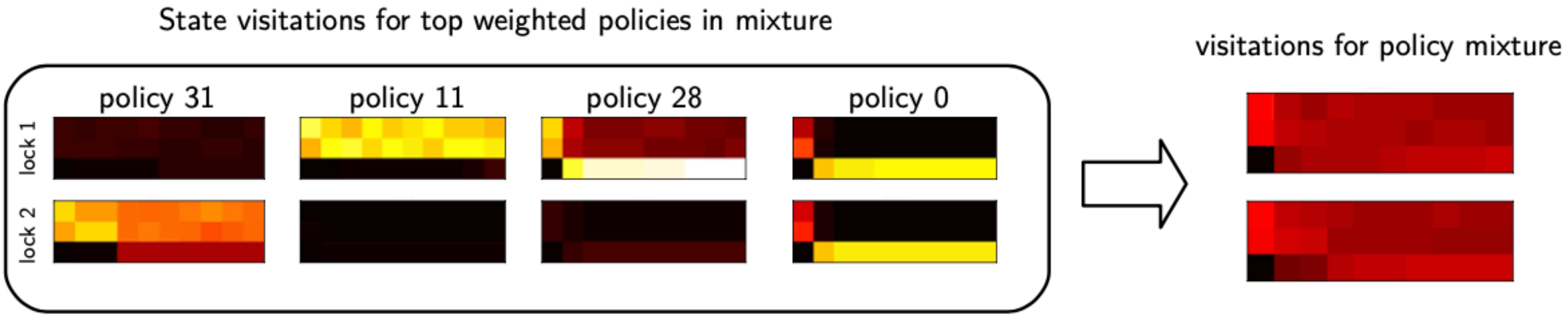


Cover ρ_n near uniformly cover both chains

PG with ρ_n will succeed!

Reward-Free Explore in Maze

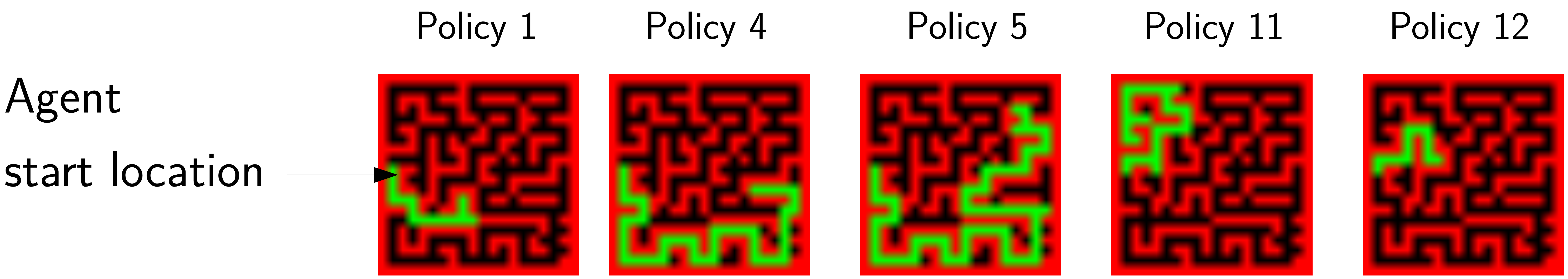
$$r(s, a) = 0$$

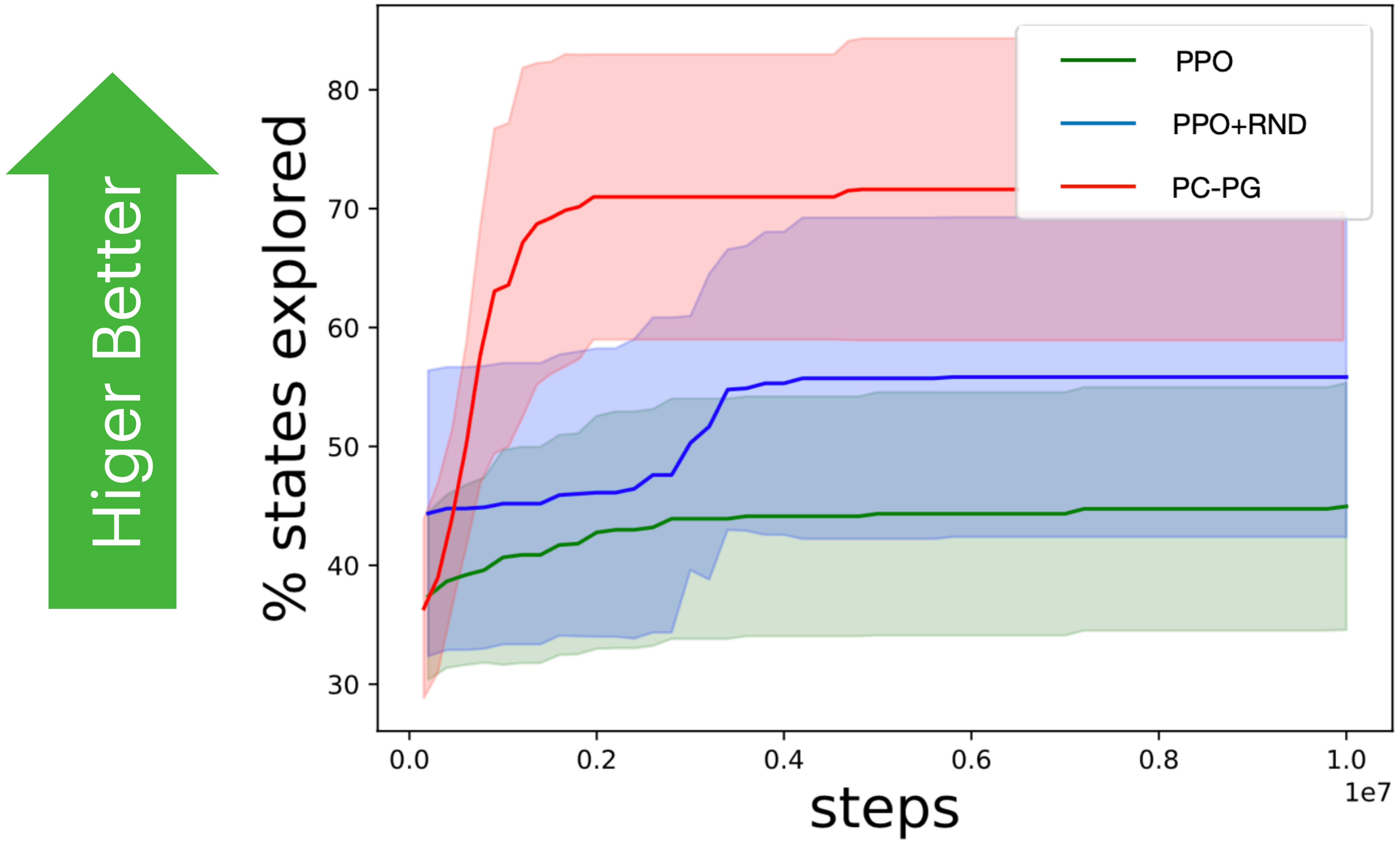
$\phi(s, a)$: Random initialized CovNet

Policy Opt procedure: PPO w/ CovNet-based policy

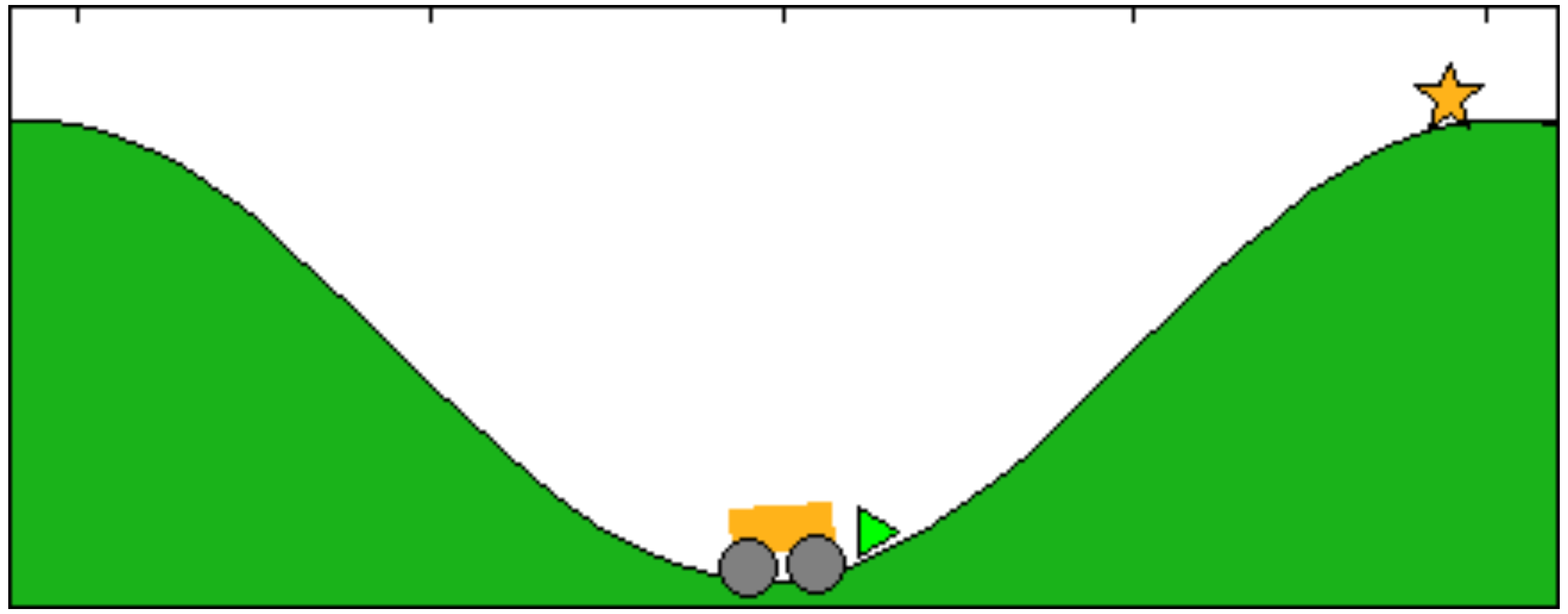
Reward-Free Explore in Maze

Traces of policies in the policy cover:





Continuous Control w/ sparse reward



Sparse reward: large reward at the goal;

Anti-shaped reward: penalize control inputs

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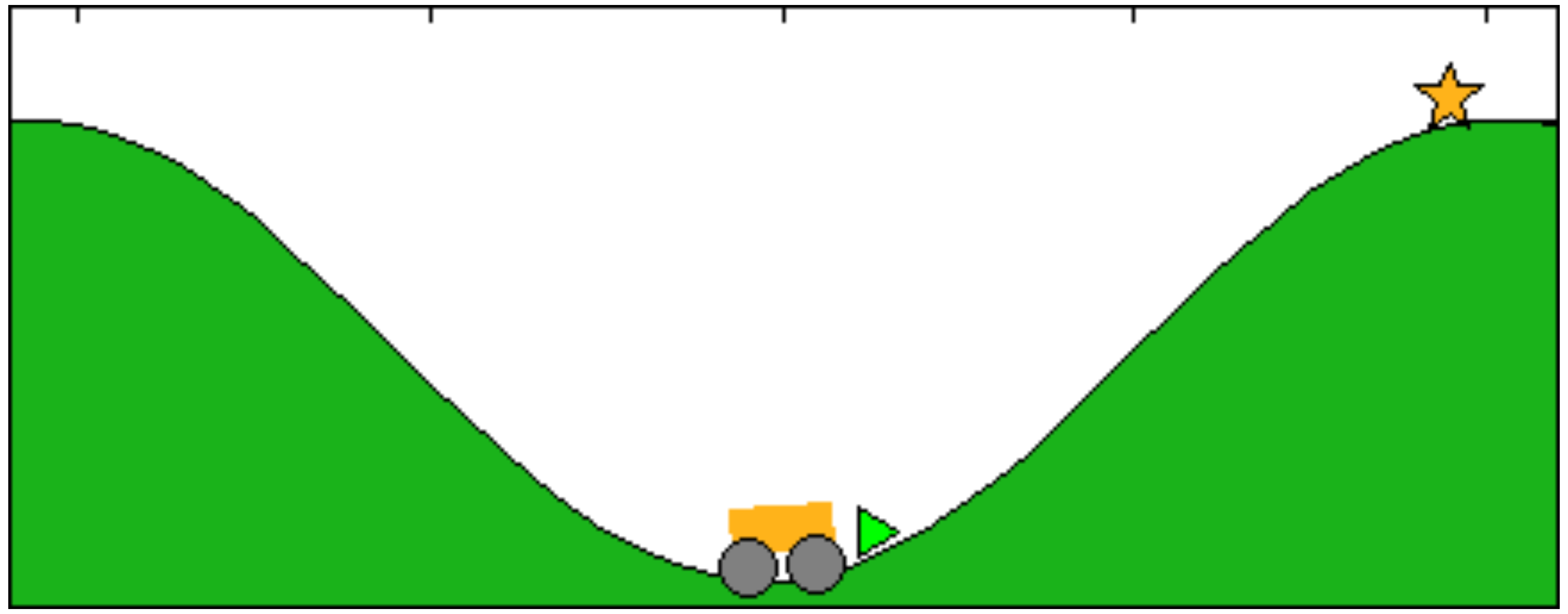

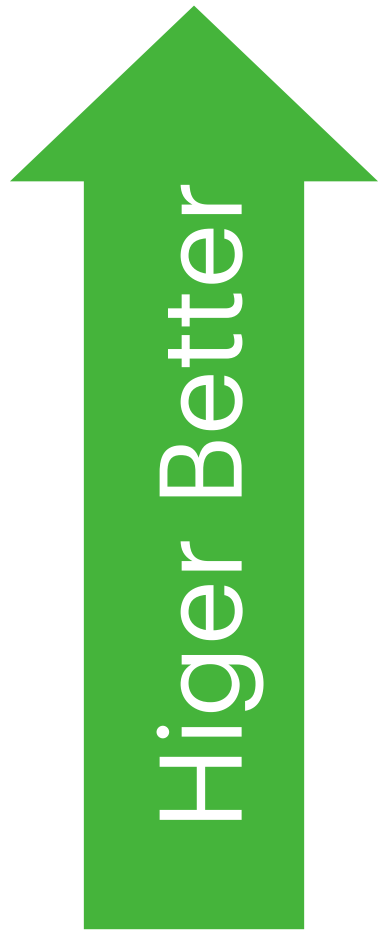

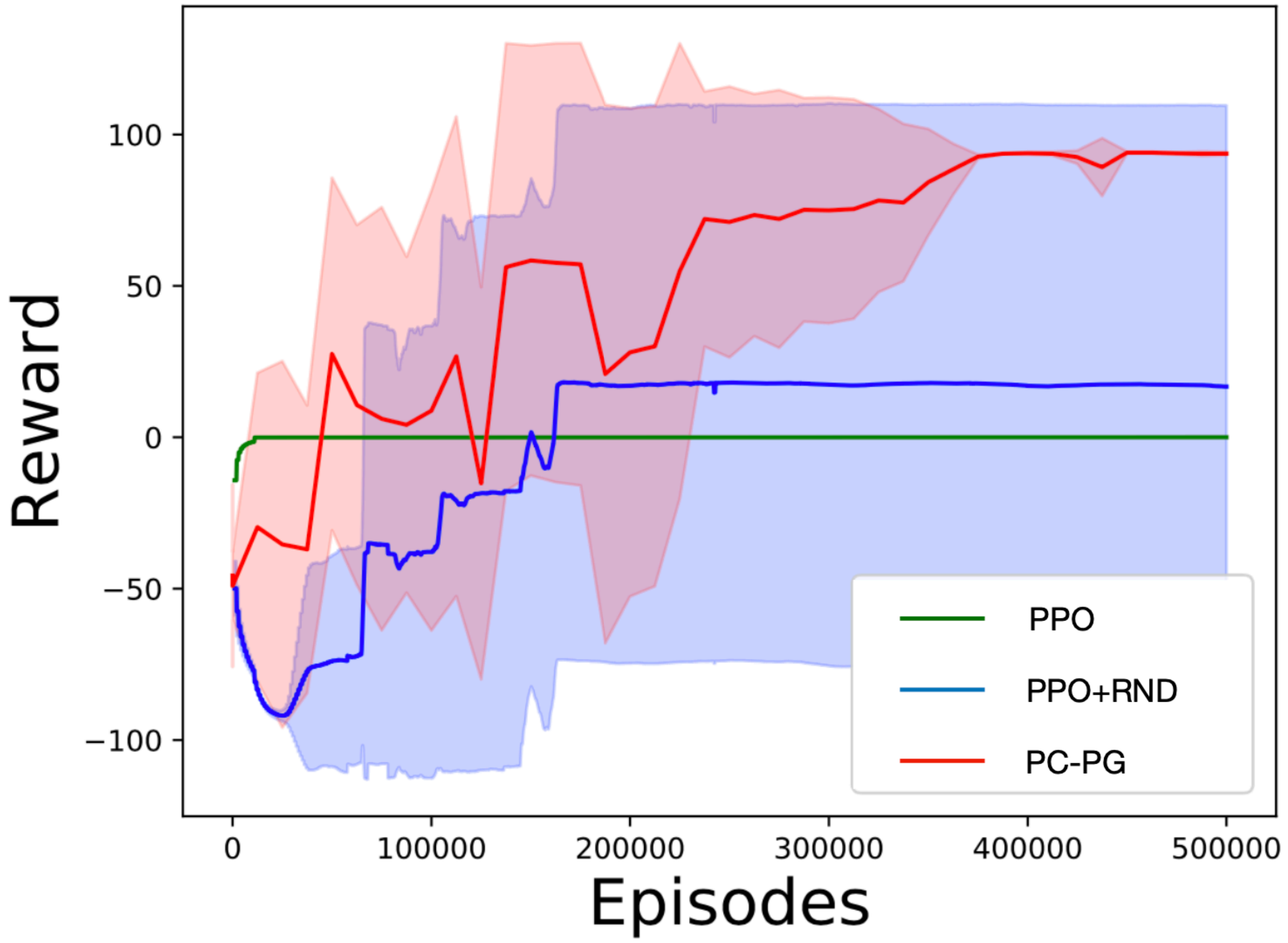

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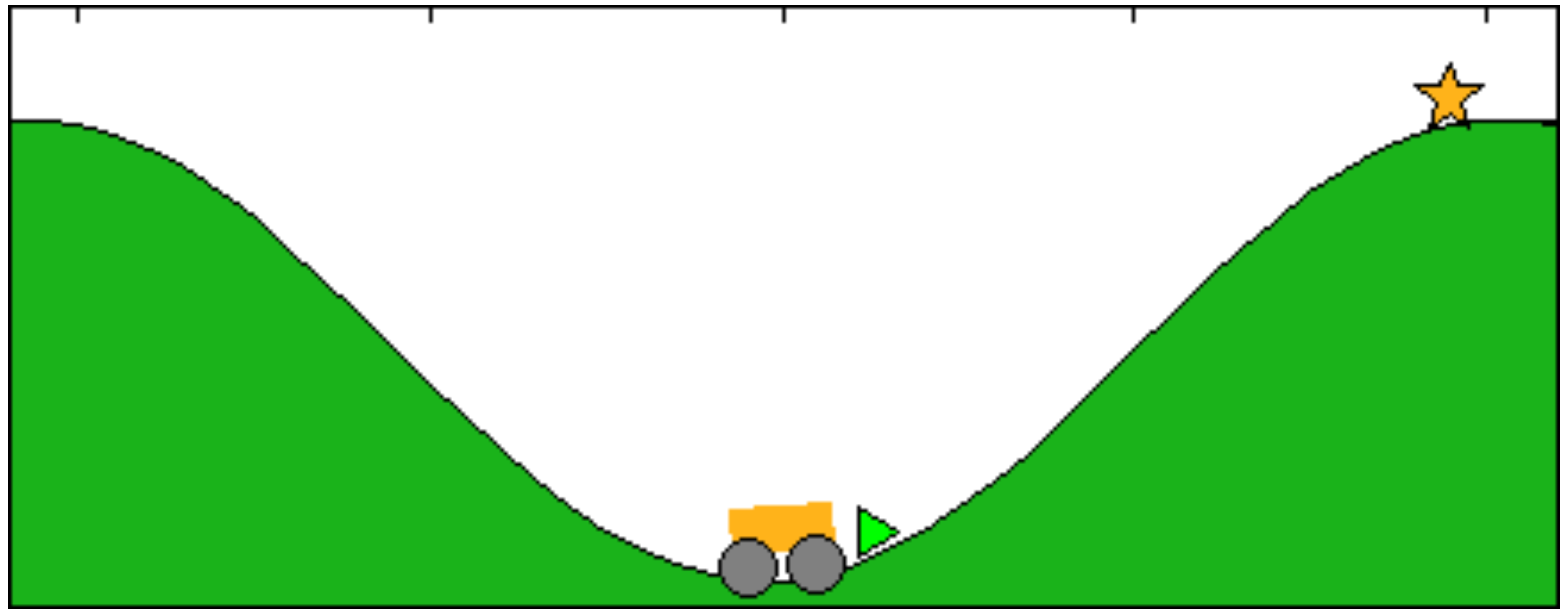

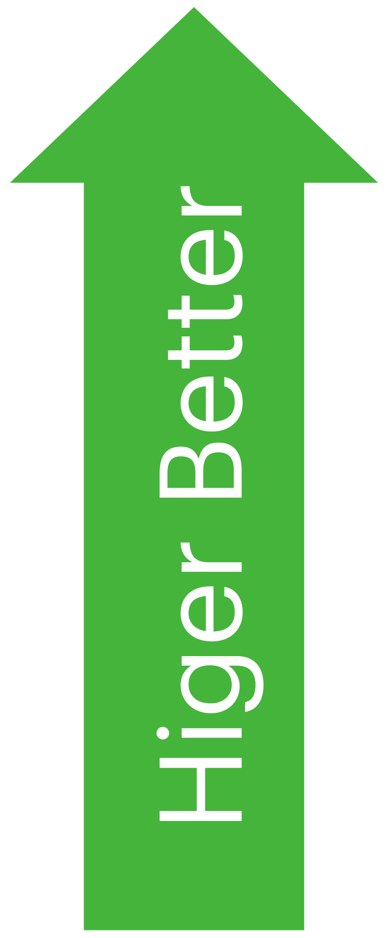

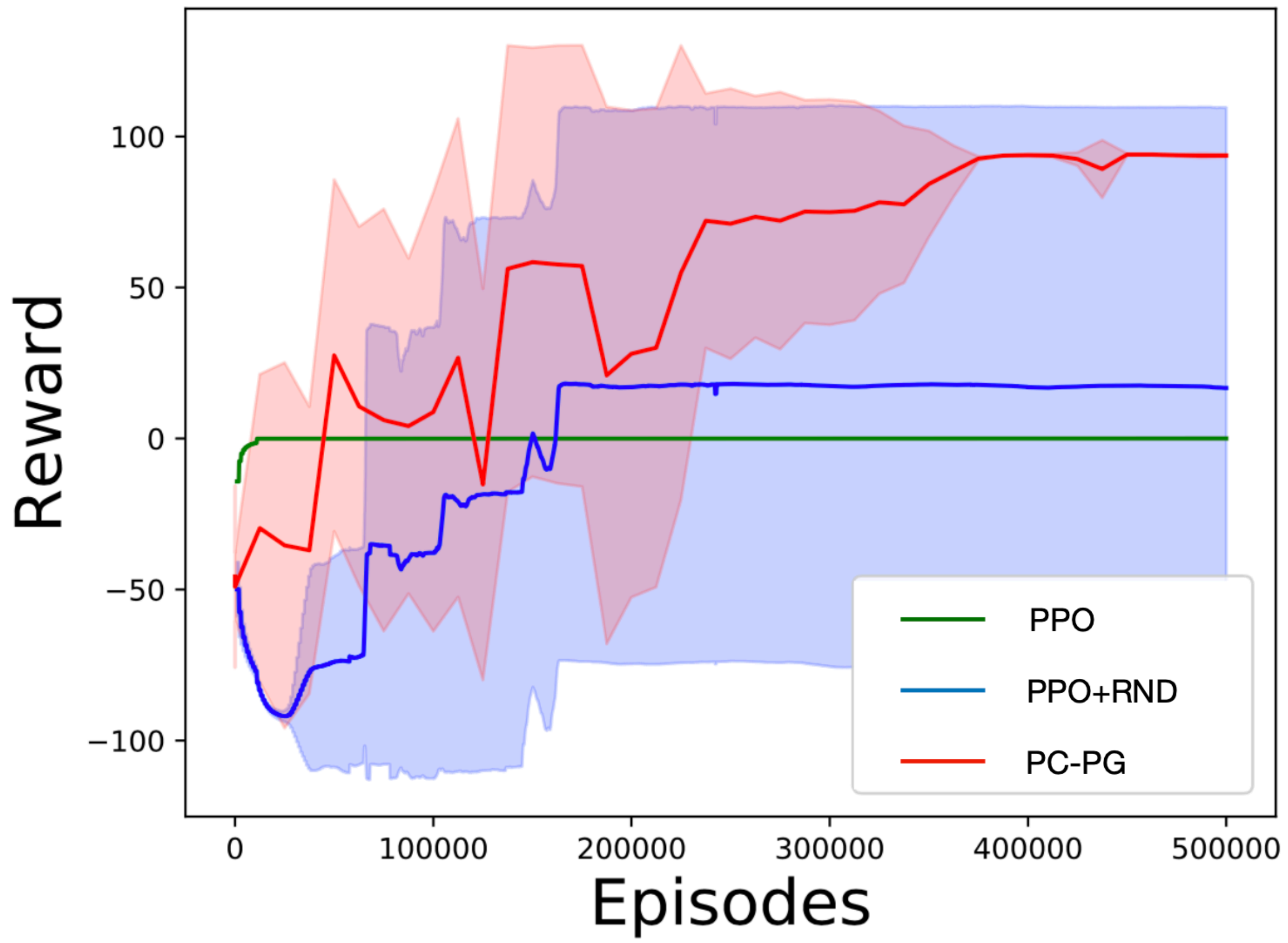

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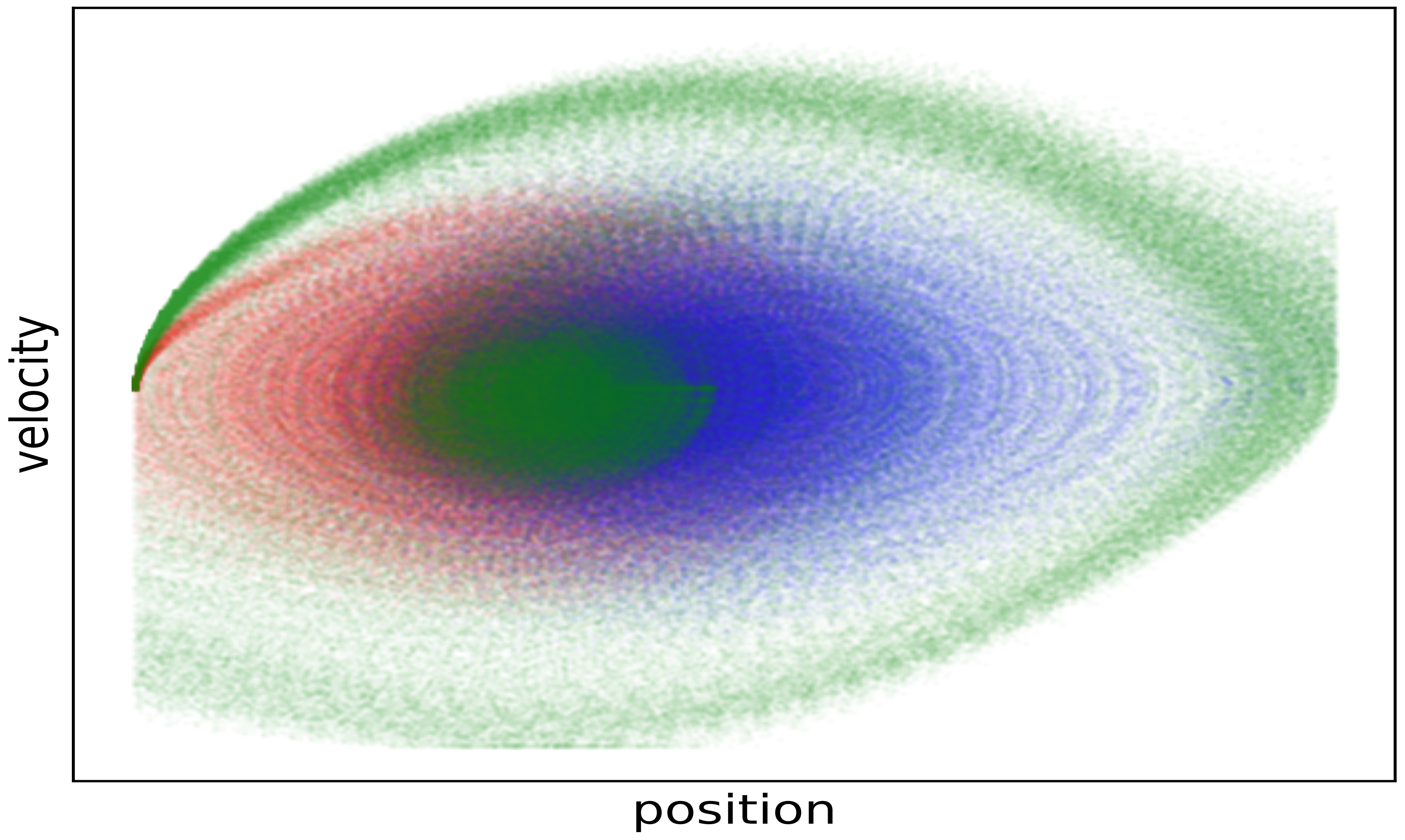

Conclusion

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Strong agnostic results

Average model-misspecification VS ℓ_∞

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Polynomial Sample Complexity in well-specified case:

Linear MDPs (RKHS) & Tabular MDPs

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Flexibility to leverage existing deep learning/RL tools

Vanilla implementation explores 4 to 5 rooms in M-Revenge